Recall from lecture that Sam is either fit or unfit

\[ S = \{ \text{fit, unfit} \} \]

and has to decide whether to exercise or relax

\[ A = \{ \text{exercise, relax} \} \]

on the basis of the following (probability, reward)-matrices \((p(s, a, s'), r(s, a, s'))\)

for row \(s\), column \(s'\) in table with corner \(a\)

<table>
<thead>
<tr>
<th></th>
<th>exercise</th>
<th>fit</th>
<th>unfit</th>
<th>relax</th>
<th>fit</th>
<th>unfit</th>
</tr>
</thead>
<tbody>
<tr>
<td>fit</td>
<td>.99, 8</td>
<td>.01, 8</td>
<td></td>
<td>.7, 10</td>
<td>.3, 10</td>
<td></td>
</tr>
<tr>
<td>unfit</td>
<td>.2, 0</td>
<td>.8, 0</td>
<td></td>
<td>0, 5</td>
<td>1, 5</td>
<td></td>
</tr>
</tbody>
</table>

The \(\gamma\)-discounted value of \((s, a)\) is

\[
\lim_{n \to \infty} q_n(s, a)
\]

where

\[
q_0(s, a) := p(s, a, \text{fit})r(s, a, \text{fit}) + p(s, a, \text{unfit})r(s, a, \text{unfit})
\]

\[
V_n(s) := \max(q_n(s, \text{exercise}), q_n(s, \text{relax}))
\]

\[
q_{n+1}(s, a) := q_0(s, a) + \gamma (p(s, a, \text{fit})V_n(\text{fit}) + p(s, a, \text{unfit})V_n(\text{unfit}))
\]

In particular, \(\gamma = 0.9\) leads to the following \(q_n(s, a)\) for \(n = 0, 1, 2\)

<table>
<thead>
<tr>
<th></th>
<th>exercise</th>
<th>relax</th>
<th>\pi</th>
</tr>
</thead>
<tbody>
<tr>
<td>fit</td>
<td>8, 16.955, 23.812</td>
<td>10, 17.65, 23.685</td>
<td>relax, relax, exercise</td>
</tr>
<tr>
<td>unfit</td>
<td>0, 5.4, 10.017</td>
<td>5, 9.5, 13.55</td>
<td>relax, relax, relax</td>
</tr>
</tbody>
</table>

Your task is to write a program that given

a positive integer \(n\), a \(\gamma\)-setting \(G (0 < G < 1)\), and a state \(s\)

returns the values

\[ q_n(s, \text{exercise}) \quad \text{and} \quad q_n(s, \text{relax}) \]

for \(\gamma = G\). You may use any of the following programming languages

Prolog, Java, Python

but be prepared to demonstrate your program on Tue, March 6 (noon-1, LG 12, O’Reilly) or Wed, March 7 (10-11, LB04; on your machine).

\[ ^1 \text{It may help to read } \text{Poole & Mackworth, 9.5 Decision Processes} \]