This assignment asks you to apply the A* search algorithm to graphs over the set of nodes \{1, 2, 3, \ldots\}, with arcs N, M and costs Cost induced by a positive integer Seed as follows

\[ \text{arc}(N, M, \text{Seed}, \text{Cost}) :- M \text{ is } N \times \text{Seed}, \text{Cost}=1. \]
\[ \text{arc}(N, M, \text{Seed}, \text{Cost}) :- M \text{ is } N \times \text{Seed} + 1, \text{Cost}=2. \]

(E.g. Seed = 3 yields arc 1, 3 with cost 1 and 1, 4 with cost 2.) Let us agree also that the goal nodes are given by a positive integer Target as those nodes divisible by Target — i.e. Target, 2\times Target, 3\times Target, \ldots

\[ \text{goal}(N, \text{Target}) :- 0 \text{ is } N \text{ mod Target}. \]

Given Target, let us set the heuristic function to 0 on goal nodes, and to the reciprocal elsewhere.

\[ \text{h}(N, \text{Hvalue}, \text{Target}) :- \text{goal}(N, \text{Target}), !, \text{Hvalue} = 0 \]
\[ \text{Hvalue} = 1/N. \]

Your task is to define a predicate

\[ \text{a-star}(+\text{Start}, +\text{Seed}, +\text{Target}, ?\text{Found}) \]

that given positive integers Start, Seed and Target returns the lowest cost goal node Found calculated by A*. The idea is to modify the skeletal search algorithm

\[ \text{search}([\text{Node}|\text{FRest}]) :- \text{goal}(\text{Node}). \]
\[ \text{search}([\text{Node}|\text{FRest}]) :- \text{setof}(X, \text{arc}(\text{Node}, X), F\text{Node}), \text{add-to-frontier}(F\text{Node}, F\text{Rest}, F\text{New}), \text{search}(F\text{New}). \]

so that the list FNew obtained in add-to-frontier is (as prescribed by A*) sorted in order of increasing \(f\)-values, where \(f(\text{node}) = \text{cost}(\text{node}) + \text{h(\text{node})}\).

Hint. Let the frontier be a list of node-cost pairs (instead of just nodes), being careful to add the cost of the parent to its children, and to bring in the heuristic function in ordering the frontier FNew.

\[ \text{less-than}([\text{Node1}, \text{Cost1}], [\text{Node2}, \text{Cost2}], \text{Target}) :- \]
\[ \text{h}(\text{Node1}, \text{Hvalue1}, \text{Target}), \text{h}(\text{Node2}, \text{Hvalue2}, \text{Target}), \]
\[ \text{F1 is Cost1+Hvalue1, F2 is Cost2+Hvalue2, F1 }<\text{ F2}. \]

Test your definitions with queries such as

\[ ?- \text{a-star}(1, 3, 6, F). \]

\[ ^1\text{For any extensions beyond that date, email your demonstrator/marker, David Woods (dwoods@tcd.ie).} \]