Faculty of Engineering, Mathematics and Science  
School of Computer Science & Statistics

Integrated Computer Science Programme  
B.A. (Mod.) Computer Science & Business  
B.A. (Mod.) Computer Science & Language  
Year 3 Annual Examinations

Artificial Intelligence I

Mon, 14 May 2018  
SPORTS CENTRE  
14:00 – 16:00

Dr Tim Fernando

Instructions to Candidates:
Attempt two questions. All questions carry equal marks. Each question is scored out of a total of 50 marks.

You may not start this examination until you are instructed to do so by the Invigilator.

Exam paper is not to be removed from the venue.

Materials permitted for this examination:
Non-programmable calculators are permitted for this examination — please indicate the make and model of your calculator on each answer book used.
1. Recall that a goal node connected by arc to Node can be searched by calling frontierSearch([Node]), with the following Prolog clauses.

frontierSearch([Node|_]) :- goal(Node).
frontierSearch([Node|Rest]) :-
    findall(Next, arc(Node,Next), Children),
    add2frontier(Children, Rest, NewFrontier),
    frontierSearch(NewFrontier).

(a) Define add2frontier(Children, Rest, NewFrontier) so that frontierSearch([Node]) searches depth-first for a goal connected to Node by arc.  

[5 marks]

(b) Define add2frontier(Children, Rest, NewFrontier) so that frontierSearch([Node]) searches breadth-first for a goal connected to Node by arc.

[5 marks]

(c) What modifications to add2frontier(Children, Rest, NewFrontier) are required for A-star?

[10 marks]

(d) What does it mean for A-star to be admissible?

[5 marks]

(e) Give three conditions sufficient for A-star to be admissible. Do these conditions guarantee that A-star will terminate? Justify your answer.

[10 marks]

(f) True or false: breadth-first is admissible. Justify your answer.

[5 marks]

(g) Suppose we were to apply A-star to the graph shown below

\[ S \xrightarrow{\frac{1}{2}} q_1 \xrightarrow{\frac{1}{4}} q_2 \xrightarrow{\frac{1}{6}} q_3 \xrightarrow{\frac{1}{8}} q_4 \xrightarrow{\frac{1}{10}} \ldots \]

\[ \downarrow^{1} \]

\[ g \]

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with arcs \((s, g), (s, q_1)\) and \((q_n, q_{n+1})\) costing 1, \(\frac{1}{2}\) and \(\frac{1}{2^{n+1}}\) respectively.

\[
\begin{align*}
\text{cost}(s, g) &= 1 \\
\text{cost}(s, q_1) &= \frac{1}{2} \\
\text{cost}(q_n, q_{n+1}) &= \frac{1}{2^{n+1}} \quad \text{for } n \geq 1
\end{align*}
\]

and heuristics \(h(s) = 1, h(g) = h(q_n) = 0\). Assuming \(g\) is the only goal node, is A-star admissible under this set-up? Justify your answer.

[5 marks]

(h) Suppose we were to change the cost of an arc \((q_n, q_{n+1})\) to 

\[
\text{cost}(q_n, q_{n+1}) = \frac{1}{n + 2} \quad \text{for } n \geq 1.
\]

Would A-star be admissible if all other details of the set-up in part (g) were preserved. Justify your answer.

[5 marks]
2. Let $\langle S, A, p, r, \gamma \rangle$ be a Markov decision process.

(a) What is a policy? Supposing $S$ consists of three states, and $A$ of two actions, how many possible policies are there?

[5 marks]

(b) What is a $\gamma$-optimal policy and how is it computed from the $\gamma$-discounted value of a pair $(s, a)$ of a state $s \in S$ and action $a \in A$? How are $\gamma$-discounted values computed by value iteration $q_0, q_1, q_2, q_3 \ldots$?

Compute $q_2(s_3, a_2)$ for

\[
S = \{s_1, s_2, s_3\}
\]
\[
A = \{a_1, a_2\}
\]
\[
\gamma = .1
\]

and probabilities and immediate rewards given by Table $a_1$ and Table $a_2$ as follows: the entry of Table $a_i$ at row $s$, column $s'$ is the pair $p(s, a_i, s')$, $r(s, a_i, s')$.

<table>
<thead>
<tr>
<th></th>
<th>$s_1$</th>
<th>$s_2$</th>
<th>$s_3$</th>
<th>$s_1$</th>
<th>$s_2$</th>
<th>$s_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s_1$</td>
<td>.5 , .3 , 0</td>
<td>.2 , -2</td>
<td>( s_2 )</td>
<td>.2 , 4 .2 , 2 .6 , -3</td>
<td>( s_3 )</td>
<td>.1 , 1 , 0 , 0 .9 , -2</td>
</tr>
<tr>
<td>$s_2$</td>
<td>.3 , 0 .5 , 1</td>
<td>.2 , 2</td>
<td>( s_2 )</td>
<td>.1 , 1</td>
<td>.0 , 0 .9 , -2</td>
<td></td>
</tr>
<tr>
<td>$s_3$</td>
<td>0 , 0 , 0</td>
<td>1 , 1</td>
<td>( s_3 )</td>
<td>0 , 0</td>
<td>0 , 0</td>
<td>1 , 0</td>
</tr>
</tbody>
</table>

Table $a_1$ Table $a_2$

[25 marks]

(c) How can we learn $\gamma$-discounted values when we do not know the probabilities $p$ and immediate rewards $r$?

[10 marks]

(d) What is the exploration-exploitation tradeoff in (c), and how can we adjust the notion of a policy (discussed in part (a)) to accommodate the trade-off?

[10 marks]
3. (a) (i) What is a **definite clause**? [5 marks]

(ii) What is a **Horn clause**? [5 marks]

(iii) True or false: every set of definite clauses is satisfiable. Justify your answer. [5 marks]

(iv) Outline an efficient algorithm to determine whether a set of Horn clauses is satisfiable. [10 marks]

(b) True or false: a set KB of clauses is satisfiable if and only if the atom false is a logical consequence of KB. Justify your answer, stating what it means for a clause to be a logical consequence of KB. [10 marks]

(c) Given the Bayes net and probabilities below for the Boolean variables X₁, X₂, X₃, X₄ (with negations X₁, X₂, X₃, X₄), calculate the probabilities in (i), (ii) and (iii).

\[
\begin{align*}
P(X₁) &= 0.3 \\
P(X₂|X₁) &= 0.7 \\
P(X₂|\overline{X₁}) &= 0.5 \\
P(X₃|X₁) &= 0.2 \\
P(X₃|\overline{X₁}) &= 0.6 \\
P(X₄|X₃) &= 0.6 \\
P(X₄|\overline{X₃}) &= 0.6
\end{align*}
\]

(i) \(P(X₁|X₂)\) [5 marks]

(ii) \(P(X₃|X₂)\) [5 marks]

(iii) \(P(X₃|X₄)\) [5 marks]