Frontier search (manage choices)

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frontierSearch([Node|Rest]) :- goal(Node);
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Breadth-first: `append(Rest, Children, NewFrontier)`

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Depth first: `append(Children, Rest, NewFrontier)`

Breadth-first: `append(Rest, Children, NewFrontier)`

For `add2frontier(Children, Rest, NewFrontier)`, require

`NewFrontier` merges `Children` and `Rest`

where a list `L` is defined to *merge* lists `L1` *and* `L2` if

(a) every member of `L` is a member of `L1` or `L2`

(b) every member of `L1` or of `L2` is a member of `L`.

Exercise (Prolog)

Suppose a positive integer `Seed` links nodes `1,2,...` in two ways

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arc(N,M,Seed) :- M is N*Seed.
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Goal nodes are multiples of a positive integer `Target`

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Modify frontier search to define predicates

```
breadth1st(+Start, ?Found, +Seed, +Target)
depth1st(+Start, ?Found, +Seed, +Target)
```

that search breadth-first and depth-first respectively for a `Target`-goal node `Found` linked to `Start` by `Seed`-arcs.

Refining frontier search

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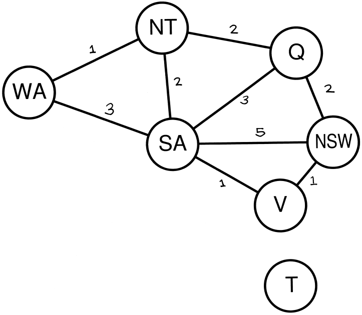
$\leadsto$ A-star

Arc costs (space, time, money, ...)

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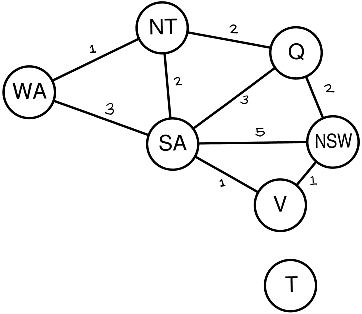


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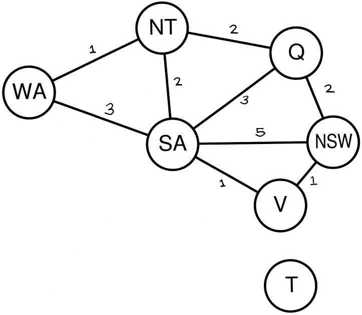


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$$\text{cost}(\texttt{wa,sa,nsw}) \;=\; 3+5 \;=\; 8$$

Heuristics

$$h(\texttt{Node}) = \text{estimate the minimum cost of}$$
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- estimate assuming lots of arcs (simplifying the problem)

Best-first search

Form `NewFrontier = [Head|Tail]` such that

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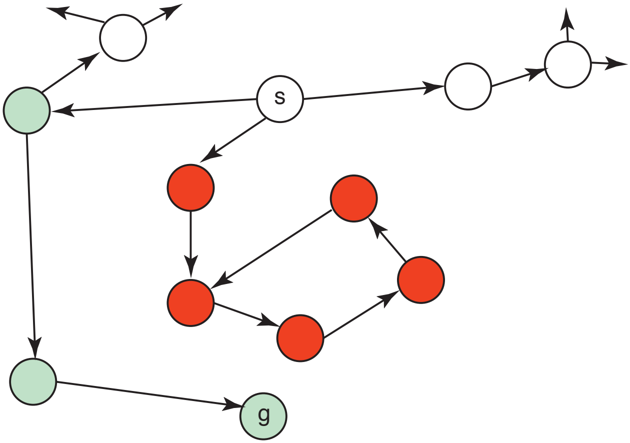
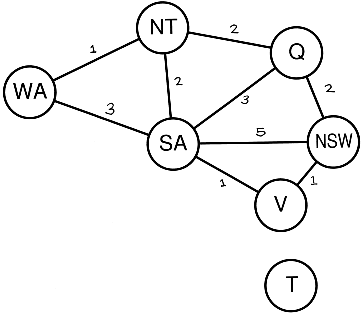
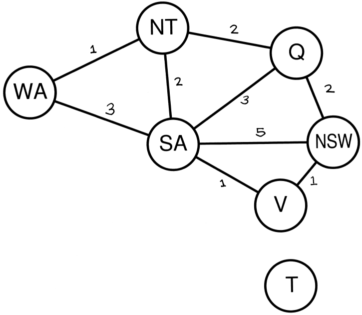


Poole & Mackworth

Min-cost



Min-cost $\neq$ breadth-first



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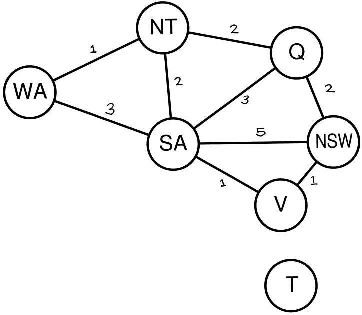

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`add2frontier(Children, Rest, [Head|Tail])`

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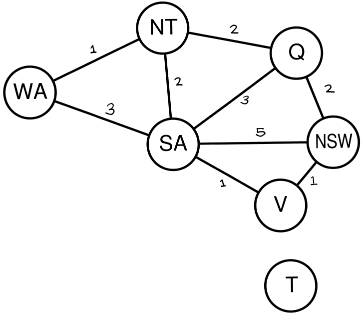

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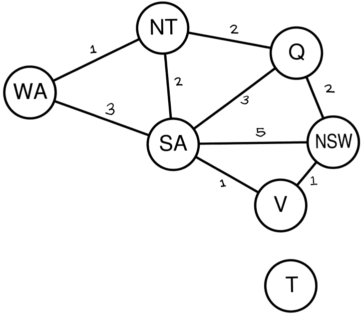

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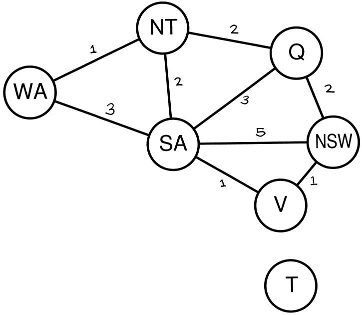

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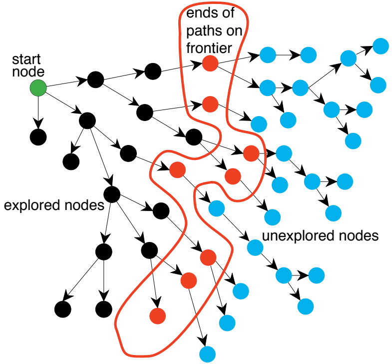
- node $\rightsquigarrow$ path or pair $(n,\text{cost}(\texttt{Start}\cdots n))$
- what about proximity to goal?

$$h(n) = \text{estimate of min cost path } n \cdots \text{goal}$$

A*



$$\text{solution} = \underbrace{\text{start} \cdots n}_{\text{explored}} \cdots \text{goal}$$

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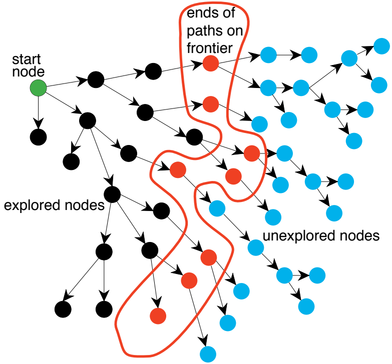

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Ensure `Frontier = [Head|Tail]` where `Head` has minimal f

- $h(n) = 0$ for every n $\leadsto$ min-cost
- $\text{cost}(\text{start} \cdots n) = 0$ for every n $\leadsto$ best-first (disregarding the past)

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(ii) $c = c_n$ for some n s.t. the head of F_n is a solution.