Solution to Homework 2

I. a.

\[ \text{Cat} \sqsubseteq [\text{AND } [\text{FILLS :Loves percy}][\text{EXISTS 1 :Tail}]] \]

\[ [\text{FILLS :Loves percy}] \sqsubseteq [\text{ALL :Eats Chicken}] \]

cosmic \rightarrow [\text{AND Cat [FILLS :OwnedBy thomas]}]

b.

\[ (\forall x) \quad \text{Cat}(x) \supset (\text{Loves}(x, \text{percy}) \land \exists y : \text{Tail}(x, y)) \]

\[ (\forall x) \quad \text{Loves}(x, \text{percy}) \supset (\forall y) (\text{Eats}(x, y) \supset \text{Chicken}(y)) \]

\[ \text{Cat(cosmic)} \land \text{:OwnedBy(cosmic,thomas)} \]

c. OUTLINE: Given (3), it is enough to prove the subsumption

\[ [\text{AND Cat [FILLS :OwnedBy thomas]}] \sqsubseteq [\text{ALL :Eats Chicken}]. \]

But this follows structurally from

\[ \text{Cat} \sqsubseteq [\text{ALL :Eats Chicken}] \]

which, in turn, follows from (i) and (ii) after the obvious normalization

\[ \text{Cat} \triangleq [\text{AND [FILLS :Loves percy}]
\[ \quad [\text{EXISTS 1 Tail}]
\[ \quad [\text{ALL :Eats Chicken}]
\[ \quad a_{Cat}] \]

II. a. A sentence is *satisfiable* if it has an interpretation/model. A subsumption \( C \subseteq D \) is reduced to checking if \( C(x) \land \neg D(x) \) is satisfiable. Tableaux are a constraint system for checking if a sentence is satisfiable. A model is built incrementally as completion rules are applied to a formula, decomposing it top-down while considering all possibilities exhaustively. Completion rules are applied until either a clash is generated on every branch or there is a completed branch where no further rule is applicable (from which a model can be extracted).
b. Form the negation of

\((\exists \text{know})(\text{student} \sqcap (\exists \text{admire})\neg \text{student})\) (pat)

that is,

\((\forall \text{know})(\neg \text{student} \sqcup (\forall \text{admire}) \text{student})\) (pat) . \quad (1)

Use the \(\forall R\)-rule and the fact know(pat,ann) to get

\((\neg \text{student} \sqcup ((\forall \text{admire}) \text{student}))\) (ann)

cancelling the first disjunct by the fact student(ann), and applying the \(\forall R\)-rule and the fact admire(ann,bob) to get

\(\text{student(bob)}\).

Next, apply the \(\forall R\)-rule to (1) and know(pat,bob) to get

\((\neg \text{student} \sqcup ((\forall \text{admire}) \text{student}))\) (bob)

cancelling the first disjunct by the fact (2), and applying the \(\forall R\)-rule and the fact admire(bob,chris) to get

\(\text{student(chris)}\)

which clashes with the fact \(\neg \text{student(chris)}\).

**Solution to Homework 3**

1. The MSO\(_{\{a,b\}}\)-models corresponding to \(a^*b\) are \(\langle D_n, S_n, \{[P_a]_n, [P_b]_n\} \rangle\) for integers \(n \geq 1\), where

- \(D_n := \{1, 2, \ldots, n\}\)
- \(S_n := \{(1, 2), \ldots, (n-1, n)\} \) (\(\emptyset\) for \(n = 1\))
- \([P_a]_n := D_n - \{n\}\)
- \([P_b]_n := \{n\}\)

2. \(a^*b\) is equivalent to the MSO-sentence

\(\exists y \ (P_b(y) \land \forall x (\neg S(y, x) \land (x = y \lor P_b(x))))\)

3. As \(<\) is a total order, \(x < y\) is equivalent to

\(\neg(y < x) \land \neg x = y\)

i.e., \(\exists Z \ \psi(x, y)\) for \(\psi(x, y)\) equal to

\(\text{closed}_S(Z) \land Z(y) \land \neg Z(x)\)

where \(\text{closed}_S(Z)\) abbreviates

\(\forall u \forall v \ (Z(u) \land S(u, v) \supset Z(v))\)