Proofs

➤ A proof is a mechanically derivable demonstration that a formula logically follows from a knowledge base.

➤ Given a proof procedure, \( KB \models g \) means \( g \) can be derived from knowledge base \( KB \).

➤ Recall \( KB \models g \) means \( g \) is true in all models of \( KB \).

➤ A proof procedure is sound if \( KB \models g \) implies \( KB \models g \).

➤ A proof procedure is complete if \( KB \models g \) implies \( KB \models g \).
One rule of derivation, a generalized form of modus ponens:

If “$h \leftarrow b_1 \land \ldots \land b_m$” is a clause in the knowledge base, and each $b_i$ has been derived, then $h$ can be derived.

You are forward chaining on this clause.

(This rule also covers the case when $m = 0$.)
Bottom-up proof procedure

$KB \vdash g$ if $g \in C$ at the end of this procedure:

$C := \{\};$

repeat

 select clause “$h \leftarrow b_1 \land \ldots \land b_m$” in $KB$ such that

 $b_i \in C$ for all $i$, and

 $h \notin C$;

 $C := C \cup \{h\}$

 until no more clauses can be selected.
Example

\[
a \leftarrow b \land c.
\]

\[
a \leftarrow e \land f.
\]

\[
b \leftarrow f \land k.
\]

\[
c \leftarrow e.
\]

\[
d \leftarrow k.
\]

\[
e.
\]

\[
f \leftarrow j \land e.
\]

\[
f \leftarrow c.
\]

\[
j \leftarrow c.
\]
Soundness of bottom-up proof procedure

If $KB \vdash g$ then $KB \models g$.

Suppose there is a $g$ such that $KB \vdash g$ and $KB \not\models g$.

Let $h$ be the first atom added to $C$ that’s not true in every model of $KB$. Suppose $h$ isn’t true in model $I$ of $KB$. There must be a clause in $KB$ of form

$$h \leftarrow b_1 \land \ldots \land b_m$$

Each $b_i$ is true in $I$. $h$ is false in $I$. So this clause is false in $I$. Therefore $I$ isn’t a model of $KB$.

Contradiction: thus no such $g$ exists.
Fixed Point

The $C$ generated at the end of the bottom-up algorithm is called a fixed point.

Let $I$ be the interpretation in which every element of the fixed point is true and every other atom is false.

$I$ is a model of $KB$.

Proof: suppose $h \leftarrow b_1 \land \ldots \land b_m$ in $KB$ is false in $I$. Then $h$ is false and each $b_i$ is true in $I$. Thus $h$ can be added to $C$. Contradiction to $C$ being the fixed point.

$I$ is called a Minimal Model.
Completeness

If $KB \models g$ then $KB \vdash g$.

Suppose $KB \models g$. Then $g$ is true in all models of $KB$.

Thus $g$ is true in the minimal model.

Thus $g$ is generated by the bottom up algorithm.

Thus $KB \vdash g$. 

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