This assignment asks you to apply the A* search algorithm to graphs over the set of nodes \( \{1, 2, 3, \ldots \} \), with arcs \( N,M \) and costs \( \text{Cost} \) induced by a positive integer \( \text{Seed} \) as follows

\[
\begin{align*}
\text{arc}(N,M,\text{Seed},\text{Cost}) & : - M \text{ is } N\times\text{Seed}, \text{Cost}=1. \\
\text{arc}(N,M,\text{Seed},\text{Cost}) & : - M \text{ is } N\times\text{Seed} + 1, \text{Cost}=2.
\end{align*}
\]

(E.g. \( \text{Seed} = 3 \) yields arc \( 1,3 \) with cost 1 and \( 1,4 \) with cost 2.) Let us agree also that the goal nodes are given by a positive integer \( \text{Target} \) as those nodes divisible by \( \text{Target} \) — i.e. \( \text{Target}, 2\times\text{Target}, 3\times\text{Target}, \ldots \)

\[
\text{goal}(N,\text{Target}) : - 0 \text{ is } N \bmod \text{Target}.
\]

Given \( \text{Target} \), let us set the heuristic function to 0 on goal nodes, and to the reciprocal elsewhere.

\[
\text{h}(N,H\text{value},\text{Target}) : - \text{goal}(N,\text{Target}), !, H\text{value} = 0 \\
& ;
\text{Hvalue} = 1/N.
\]

Your task is to define a predicate

\[
\text{a-star}(+\text{Start},+\text{Seed},+\text{Target},?\text{Found})
\]
that given positive integers \( \text{Start}, \text{Seed} \) and \( \text{Target} \) returns the lowest cost goal node \( \text{Found} \) calculated by A*.

The idea is to modify the skeletal search algorithm

\[
\text{search([Node|FRest])} : - \text{goal}(\text{Node}).
\]

\[
\text{search([Node|FRest])} : - \text{setof}(X,\text{arc}(\text{Node},X), \text{FNode}), \\
\text{add-to-frontier}(\text{FNode},\text{FRest},\text{FNew}), \\
\text{search}(\text{FNew}).
\]

so that the list \( \text{FNew} \) obtained in \text{add-to-frontier} is (as prescribed by A*) sorted in order of increasing \( f \)-values, where \( f(\text{node}) = \text{cost}(\text{node}) + \text{h}(\text{node}). \)

\textbf{Hint.} Let the frontier be a list of node-cost pairs (instead of just nodes), being careful to add the cost of the parent to its children, and to bring in the heuristic function in ordering the frontier \( \text{FNew} \).

\[
\text{less-than([Node1,\text{Cost1}],[Node2,\text{Cost2}],\text{Target}) : -} \\
\text{h(\text{Node1},Hvalue1,\text{Target}), h(\text{Node2},Hvalue2,\text{Target}),} \\
\text{F1 is Cost1+Hvalue1, F2 is Cost2+Hvalue2,} \\
\text{F1 =< F2}.
\]

Test your definitions with queries such as

\[
?- \text{a-star}(1,3,6,F).
\]

\footnote{Submit to Blackboard by Mon, Feb 20. For any extensions beyond that date, email your demonstrator/marker, David Woods (dwoods@tcd.ie).}