This assignment asks you to apply the A* search algorithm to graphs over the set of nodes \{1, 2, 3, \ldots\}, with arcs N, M and costs Cost induced by a positive integer Seed as follows

\[
\text{arc}(N, M, \text{Seed}, \text{Cost}) :- M \text{ is } N \times \text{Seed}, \text{Cost}=1. \\
\text{arc}(N, M, \text{Seed}, \text{Cost}) :- M \text{ is } N \times \text{Seed} + 1, \text{Cost}=2.
\]

(E.g. Seed = 3 yields arc 1, 3 with cost 1 and 1, 4 with cost 2.) Let us agree also that the goal nodes are given by a positive integer Target as those nodes divisible by Target — i.e. Target, 2*Target, 3*Target, \ldots

\[
\text{goal}(N, \text{Target}) :- 0 \text{ is } N \mod \text{Target}.
\]

Given Target, let us set the heuristic function to 0 on goal nodes, and to the reciprocal elsewhere.

\[
\text{h}(N, \text{Hvalue}, \text{Target}) :- \text{goal}(N, \text{Target}), !, \text{Hvalue is 0} \\
\quad ; \\
\quad \text{Hvalue is } 1/N.
\]

Your task is to define a predicate

\[
\text{a-star}(+\text{Start}, +\text{Seed}, +\text{Target}, ?\text{Found})
\]

that given positive integers Start, Seed and Target returns the lowest cost goal node Found calculated by A*.

The idea is to modify the skeletal search algorithm

\[
\text{search}([\text{Node}|\text{FRest}]) :- \text{goal}((\text{Node}). \\
\text{search}([\text{Node}|\text{FRest}]) :- \text{setof}(X, \text{arc}(\text{Node}, X), \text{FNode}), \\
\quad \text{add-to-frontier}(\text{FNode}, \text{FRest}, \text{FNew}), \\
\quad \text{search}(\text{FNew}).
\]

so that the list FNew obtained in add-to-frontier is (as prescribed by A*) sorted in order of increasing f-values, where \( f(\text{node}) = \text{cost}(\text{node}) + \text{h}(\text{node}) \).

\textbf{Hint}. Let the frontier be a list of node-cost pairs (instead of just nodes), being careful to add the cost of the parent to its children, and to bring in the heuristic function in ordering the frontier FNew.

\[
\text{less-than}([\text{Node1}, \text{Cost1}], [\text{Node2}, \text{Cost2}], \text{Target}) :- \\
\quad \text{h}(\text{Node1}, \text{Hvalue1}, \text{Target}), \text{h}(\text{Node2}, \text{Hvalue2}, \text{Target}), \\
\quad \text{F1 is Cost1+Hvalue1, F2 is Cost2+Hvalue2,} \\
\quad \text{F1 =< F2}.
\]

Test your definitions with queries such as

\[
?- \text{a-star}(1, 3, 6, F).
\]

\footnote{Submit to Blackboard by Tue, Feb 16. For any extensions beyond that date, email your demonstrator/marker, Derek Kelleher (kellehdt@scss.tcd.ie).}