A.W. Faber Model 366 - System Schumacher A Very Unusual Slide Rule

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1. The Schumacher Slide Rule and Its Inventor

Introduction

The Schumacher slide rule has unusual scale divisions and configuration. This is because the slide rule is not based on logarithms but instead is based on a totally different mathematical concept.

Some History

This slide rule was the invention of Dr. Johannes Schumacher (1858-1930). When he invented this slide rule, he was professor at the School for the Royal Bavarian Cadet Corps in Munich.

Figure 1. Dr. Johannes Schumacher.

Dr. Schumacher is shown in the above picture that appeared in the 1956 Festschrift titled *Das königliche Bayerische Kadettenkorps, ein Rückblick auf die einstige Erziehungsstätte 200 Jahre nach ihrer Gründung* (The Royal Bavarian Cadet Corps - a look back at the former educational site 200 years after its founding). The picture's caption reads "Assistant Headmaster, Major d. R. a. D. Dr. Johannes Schumacher, Cadet Corps 1897-1920. Greatly honored for his teaching and, beyond that, especially esteemed during 1918-1920." It appears from this caption that, during these very difficult early post-war years, Schumacher personally attended to the welfare of former students in distress.

The Schumacher slide rule was manufactured by A.W. Faber. Faber acquired DRGM 344576 for this "slide rule with divisions at equal intervals and indices based on number theory". A corresponding circular slide rule (which was never made) was also covered by DRGM 344576. The linear version was offered (as Model 366) from 1909 through 1929.

The origins of the Schumacher-Faber collaboration are not known to us. However, some speculation about this connection seems appropriate here. In 1909 the head of A.W. Faber was Count Alexander von Faber-Castell (1866-1928). In accordance with tradition and his station as third in line for succession in the Castell-Rüdenhausen family, he received his education in the military. During the period 1878-1885 he was a member of the Cadet Corps in Munich. In 1898 he married the sole Faber heir, Baroness Ottilie von Faber, and took charge of the firm. Although Dr. Johannes Schumacher began to teach at the Cadet Academy in 1897 (too late to be one of Count Alexander's instructors), perhaps Schumacher met Count Alexander at one of the Academy's annual reunions and took the opportunity to interest the Count in his invention.

Construction

The first version of the Schumacher slide rule was designed and produced in a limited series intended for the inventor to use during instruction. The design already incorporated the features of DRP 206428 (1907), i.e. "metal bands inserted in the edges of the stator and slide". This slide rule also incorporated (but did not mention) DRGM 296340 (1908), i.e., "wooden pegs to secure the celluloid outer layer". Thus this slide rule was a very high-quality product for its time.

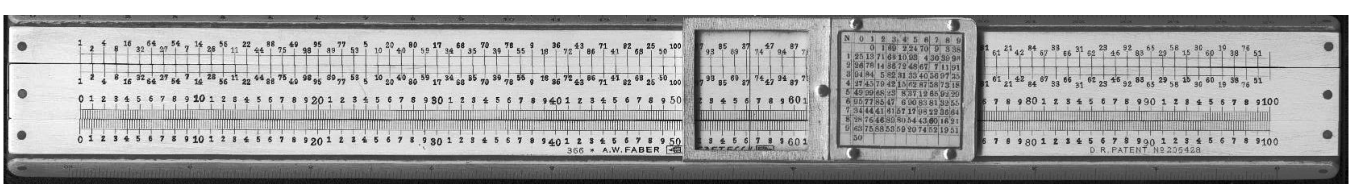

Figure 2. 1909 Schumacher slide rule. (From the collection of D. v J.)

The slide rule was first made of boxwood and later of pearwood. The four 250 mm-long non-logarithmic scales are printed on celluloid strips attached to the slide rule with glue and wooden pins. The floor of the stator is split and contains springs. There is a centimeter scale on the upper slanted edge, as well as an inch scale on the lower slanted edge. The cursor is framed in aluminum and has two windows. The left window has a hairline on glass; the right window has a "read-off" table. The slide rule is 280 mm long, 35 mm wide, and 10 mm thick.

Description of the Scales

There are two pairs of scales, both arranged along the slide-stator interfaces. The scales are very precisely embossed in black.

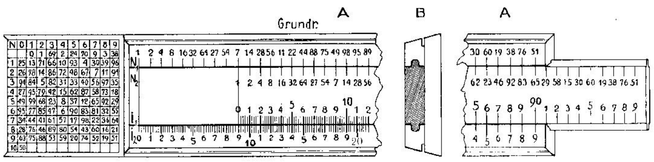

Figure 3. Structural details. (From the 1909 instruction manual by Dr. Schumacher)

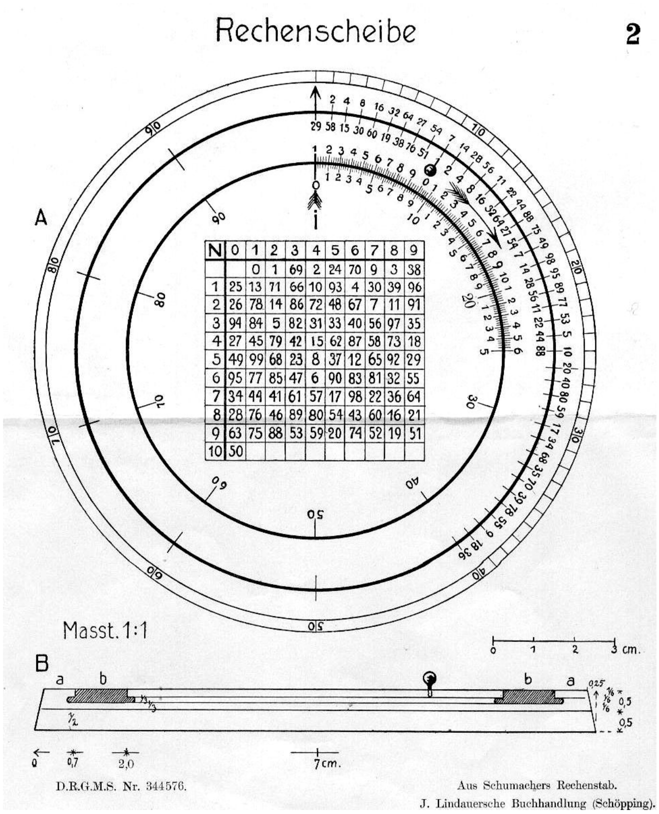

Figure 4. The planned circular version of the Schumacher slide rule (Never manufactured.)

The two scales in the upper pair are identical and are designated N1/N2 (Numerus 1/Numerus 2). The markings are equally spaced and are numbered as follows: 1 2 4 8 16 32 64 27 54 7 14 28 56 ... 15 30 60 19 38 76 51. The two scales in the lower pair are identical and are designated I1/I2 (Index 1/Index 2). The markings are equally spaced and are numbered 0 through 100. As will be shown later in this article, the upper pair of scales correspond in a limited way to the usual slide rule in the sense that these scales are used for multiplication and division of whole numbers.

The Index Table

The index table (which appears in Figure 2 to the right of the cursor window) can be viewed as a separate and distinct part of the slide rule. The table's use will be explained later. Schumacher had originally envisioned the table at the left end of the slide rule itself. (See Figure 3.) However, it would have been technically difficult to produce such an unusual configuration. (It would have required making and processing a blank that was beyond the capabilities of Faber's production line at that time.) Therefore the index table was printed on a custom-designed cursor. In the case of the circular version of the slide rule (discussed below) the index table is very advantageously located in the center of the rings of scales.

The Proposed Circular Slide Rule

As already mentioned, patent protection exists (in the form of DRGM 344576) for a circular version of the Schumacher slide rule. It even appears that Dr. Schumacher would have preferred this form of the slide rule, since it would have been easier to understand and to handle. However, at that time, Faber did not have the capacity to produce circular slide rules. In Schumacher's instruction manual [1] his planned circular slide rule was depicted in a drawing, and its application was discussed only briefly.

2. The Mathematical Foundations of System Schumacher

Calculating with real numbers and within prime fields

If one wants to understand the theory of Johannes Schumacher's slide rule, it is necessary to make a short journey into discrete mathematics. For this reason we compare how to calculate with real numbers (using slide rules) and how to calculate using modulo arithmetic, or more correctly within prime fields.

First we take a look at calculating with real numbers. Each real number can be regarded as a single point on a number line. (See Figure 5.)

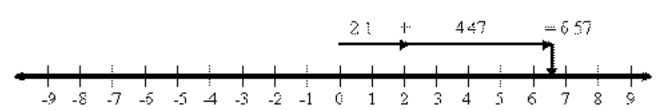

Figure 5. Calculating $2.1 + 4.47 = 6.57$ on a number line.

The sum of two real numbers is always a real number (e.g., $2.1 + 4.47 = 6.57$). The same is valid for subtraction, multiplication, division, exponentiation, and root extraction. All results are real numbers. The only exceptions to this rule are exponentiation and root extraction of real numbers less than 0, which may lead to imaginary numbers, which will not be considered here.

An infinite number of real numbers exist, which seems to be necessary for completeness of these operations. But for finite sets of numbers similar mathematical operations can be defined, too. Schumacher's slide rule uses this kind of operations. Examples for corresponding finite sets of numbers are so-called prime fields, a concept introduced below.

Throughout the following sections p represents an arbitrary prime number. A prime number p is an integer greater than 1, which only can be divided by 1 and p itself without remainder. Examples of prime numbers are 2 (the only even prime), 3, 5, 7, 11,...., 101, ...

The prime field of prime number p consists of all integers 0 to $p-1$, together with specifically defined mathematical operations.

Consider the prime field with $p = 11$. This prime field uses the numbers 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10. In the following subsections the corresponding mathematical operations will be defined.

Addition in prime fields

Because any sum of two numbers of the given set should lie in the same set of numbers, the following rule is defined. Any two numbers a and b are added as follows. Calculate the sum $a + b$ as usual. Then divide this sum by the prime p and take the remainder. By taking this remainder c as the final result of the addition, it is guaranteed that this result c is an integer lying within 0 to p-1.

We will write this operation as $a + b \equiv c \mod 11$. (The spoken version of this is: a plus b is congruent c modulo 11).

For example: Let $p = 11, a = 7, b = 9$. It follows that a + b = 16. Now dividing 16 by 11 leads to $16/11 = 1$ remainder 5. So the final result (c) is 5. In mathematical notation we write $7 + 9 \equiv 5 \mod 11$.

Note that when using this rule for addition the two numbers, a and $a + p$ cannot be distinguished any more. This is because dividing both numbers by p leads to the same remainder.

Subtraction in prime fields

A number b is subtracted from the number a as follows: Calculate $a - b$ as usual. If the result is less than 0, add multiples of p until the result is greater or equal to 0. Then determine the remainder of the division by p. This remainder is the final result.

For example: Let $p = 11, a = 7$, and $b = 9$. It holds $a - b = -2$. Then calculate $-2 + 11 = 9$. Dividing the result by 11 and keeping the remainder leads to $9/11 = 0$ remainder 9. Therefore the final result is 9. In mathematical notation we write $7 - 9 \equiv 9 \mod 11$.

For a more intuitive understanding see the graphical representation of the operations defined above (Fig. 6). We write the numbers $0, ..., p-1$ on a number-ring (similar to a clock with p hours). Addition (or subtraction) can be illustrated by going clockwise (or counter-clockwise) the corresponding number of units around the ring.

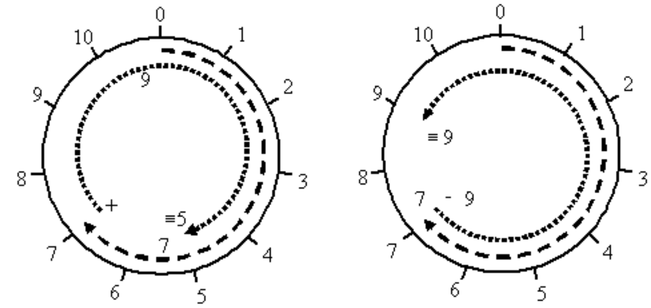

Figure 6. Addition $7 + 9 \equiv 5 \mod 11$ (left) and subtraction $7-9 \equiv 9 \mod 11$ (right) illustrated on number-rings.

Multiplication in prime fields

Any two numbers a and b are multiplied as follows. Calculate $a * b$ as usual. Then divide this intermediate result by the prime p and take the remainder c as the final result.

For example: Let $p = 11, a = 7, b = 9$. It holds that $a * b = 63$. Now dividing 63 by 11 leads to $63/11 = 5$ remainder 8. So the final result is 8. In mathematical notation $7 * 9 \equiv 8 \mod 11$.

Division in prime fields

Dividing in prime fields is a bit more difficult to understand. One reason is that the set of numbers $0, ..., p - 1$ does not contain any fraction comparable to fractions within the set of real numbers. Therefore we first have to take a detailed look at the division in the set of real numbers.

If we want to divide a real number a by a real number b (i.e. a/b), this operation can be rewritten in a different form. Instead of directly calculating a/b, it is also possible to calculate the multiplication $a*b'$ when the equation $b * b' = 1$ holds.

Perhaps you will say, "This is banal. Let $a = 3$ and $b = 4$. There is no difference in calculating $a/b = 3/4 = 0.75$ or $a*b' = 3*0.25 = 0.75$ (note $b*b' = 4*0.25 = 1$). The result is the same." Yes, this is true for real numbers. But this is not true in prime fields! Because in prime fields no fraction exists, one first has to determine a proper b'. Then multiplication in the prime field with b' is performed. This procedure assures that the final result is always within the set of numbers 0 to $p - 1$.[1]

For example: Let $p = 11, a = 7$, and $b = 9$. According to the division rule, first determine b'. For $b = 9$ it holds that $b' = 5$ because $b * b' = 9 * 5 = 45 \equiv 1 \mod 11$. Getting the result for the division a/b we calculate $a * b' = 7 * 5 = 35 \equiv 2 \mod 11$. So the final result of a/b modulo 11 is 2.

You may ask, "Why do we need all these mathematical rules in an article about Schumacher's slide rule?" The answer is in the next subsection, where we introduce a special kind of logarithms used in prime fields. These logarithms are the bases of the operations of Schumacher's slide rule.

[1] There are efficient algorithms for determining b' for any number. We will not go into detail about these algorithms because they are not important to the understanding of how Schumacher's slide rule works.

Primitive roots in prime fields

Consider any arbitrary real number a greater than 0 and a given base, e.g. 3. The number a can be uniquely represented by an exponent x such that $a = 3^x$. The exponent x is called the logarithm of the number a for the base 3. For real numbers this relationship holds for every base that is a real and positive number.

However, in prime fields one cannot use an arbitrary number as a base; one can only use so called "primitive roots". A number w is a primitive root of a given prime field if any number in the range 1 to $p-1$ of this prime field can be generated by multiplying w multiple times by itself. This means that any number a (unequal to 0) of the prime field can be written in the form w^i. The exponent i is called the index or "discrete logarithm" of the number a.

Example for $p = 11$: A primitive root of $p = 11$ is $w = 2$, since

$$\begin{array}{ccccccccl}
w^0 & = & 2^0 & = & 1 & \equiv & 1 & \bmod 11 \\
w^1 & = & 2^1 & = & 2 & \equiv & 2 & \bmod 11 \\
w^2 & = & 2^2 & = & 4 & \equiv & 4 & \bmod 11 \\
w^3 & = & 2^3 & = & 8 & \equiv & 8 & \bmod 11 \\
w^4 & = & 2^4 & = & 16 & \equiv & 5 & \bmod 11 \\
w^5 & = & 2^5 & = & 32 & \equiv & 10 & \bmod 11 \\
w^6 & = & 2^6 & = & 64 & \equiv & 9 & \bmod 11 \\
w^7 & = & 2^7 & = & 128 & \equiv & 7 & \bmod 11 \\
w^8 & = & 2^8 & = & 256 & \equiv & 3 & \bmod 11 \\
w^9 & = & 2^9 & = & 512 & \equiv & 6 & \bmod 11
\end{array}$$

Note that all numbers 1 to 10 occur as a result on the right side. Additionally see that performing further multiplication by $w = 2$ all results repeat in an eternal cycle (e.g. $w^{10} = 2^{10} = 1024 \equiv 1 \mod 11$ and $w^{11} = 2^{11} = 2048 \equiv 2 \mod 11$ and so on).

As already mentioned, there is a little pitfall. In contrast to real numbers, one may not use any arbitrary number as a base. For example, 4 is not a primitive root for $p = 11$, since

$$\begin{array}{cccccll}
4^0 & = & 1 & \equiv & 1 & \bmod 11 & \\
4^1 & = & 4 & \equiv & 4 & \bmod 11 & \\
4^2 & = & 16 & \equiv & 5 & \bmod 11 & \\
4^3 & = & 64 & \equiv & 9 & \bmod 11 & \\
4^4 & = & 256 & \equiv & 3 & \bmod 11 & \\
4^5 & = & 1024 & \equiv & 1 & \bmod 11 & (\text{i.e., } 4^5 \equiv 4^0 \mod 11)
\end{array}$$

and cyclic repetitions from then on. Because certain numbers, for example the number 7, cannot be expressed in the form 4^i, the number 4 is not a primitive root.

Now let us go back to the example $w = 2$. Since $w = 2$ is a primitive root as shown by the above overview, for each number 1 to $p-1$ discrete logarithms exist and can be looked up in the table given above. For example, the discrete logarithm of 10 is 5, since $2^5 \equiv 10 \mod 11$. Please note that only the discrete logarithms 0 to $p-2$ exist, since the number 0 cannot be expressed by an expression w^i. Therefore calculations with exponents always have to be performed modulo $p-1$, not modulo p.

If you take a look at Schumacher's slide rule, you will find a table of discrete logarithms (also called indices) of the prime field for $p = 101$ and primitive root 2.

The scales I_1 and I_2 show the numbers $0, ..., p-2$ which will be used for the operations with discrete logarithms.

The scales N_1 and N_2 are labelled with corresponding values $w^0, ..., w^{p-2}$, all calculated modulo p. Therefore above any number i on scales I_1 or I_2 the corresponding value w^i can be read on scales N_2 or N_1 respectively.

With this knowledge one can use the slide rule for multiplication (or division) and exponentiation in a way very similar to ordinary slide rules. Multiplication (or division) can be realized by addition (or subtraction) of the discrete logarithms (also called indices). Exponentiation can be realized by multiplication of the discrete logarithms. However, note that all calculations with the discrete logarithms have to be done modulo p, not modulo $p-1$.

3. Decimal representations of integers

To widen the range of the slide rule usage, Schumacher offers several mathematical tricks for calculation with numbers larger than 101. For these the properties of prime fields are not used. Instead, specific properties of the number 101 and the notation of numbers in the decimal system are used.

The way we write numbers can be viewed as shorthand notation. Remember primary school, when you might have learned that large numbers consist of "ones", "tens", "hundreds", "thousands", etc. For example, the number 7489 can be written in the form

7489 = 7 thousands + 4 hundreds + 8 tens + 9 ones

The units "ones", "tens", "hundreds", "thousands", etc. can be written as 10^0, 10^1 10^2, 10^3, etc. or, more formally, x^i with $x = 10$ (starting with $i = 0$). Using this so-called polynomial representation, we can write the number 7489 in the following form:

$$\begin{aligned}
7489 &= 7 * 10^3 + 4 * 10^2 + 8 * 10^1 + 9 * 10^0 \\
&= 7 * x^3 + 4 * x^2 + 8 * x^1 + 9 * x^0 \text{ with } x = 10
\end{aligned}$$

Instead of $x = 10$ (decimal representation), we can chose $x' = 100$ (i.e., a "hundreds" representation). In this case,

$$\begin{aligned}
7489 &= 74 * 100^1 + 89 * 10^0 \\
&= 74 * x'^1 + 89 * x'^0 \text{ where } x' = 100
\end{aligned}$$

Since Schumacher choose $p = 101$ for his slide rule, it can easily be used for calculating remainders modulo 101. During addition everyone can easily determine remainders modulo 100 by heart. Combining these different remainders in ingenious ways, Schumacher's slide rule can be used for calculating with large numbers.

For this reason Schumacher provides several rules about how to get the coefficients of polynomials in the hundreds representations. For most non-mathematicians the polynomial representation of numbers seems rather exotic, but in the area of mathematics and computer science it is very common. Here it should be noted that Schumacher's PhD thesis deals with polynomial equations and polynomial arithmetic. [2,3] Schumacher knew that calculating with polynomials can be interpreted as calculating with large numbers (in polynomial representation) and vice versa. This might be the reason why Schumacher was convinced that his rules of calculations should be simple and easy to understand.

4. Operating Instructions for the Schumacher Slide Rule

The following explanation is based on the instruction manual written by Dr. Schumacher himself. The focus here is on his recommendations regarding practical application—not on number theory. It must be recognized that the slide rule is suited only to calculations involving whole numbers. Interpolation between whole numbers is not possible.[2]

To multiply two numbers one uses the upper two scales on the Schumacher (N_1/N_2) and proceeds in a way similar to that used in the conventional slide rule. The slide is positioned so that number 1 on scale N_2 is below the first factor on scale N_1. The second factor is found on scale N_2, and the product is read above on scale N_1.
Here are two examples:

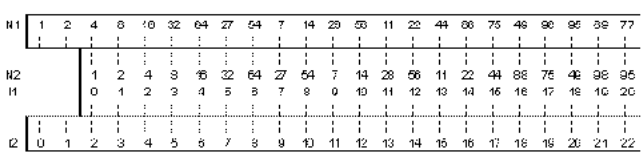

Figure 7. Example: $4 \times 8 = 32$

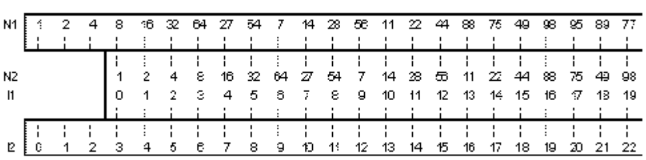

Figure 8. Example: $8 \times 11 = 88$

In the second example one can already see that it can be somewhat difficult to find the second factor (i.e., 11) on N_2. In order to facilitate locating numbers on the N scales, every Schumacher slide rule is equipped with an "Index Table". The numbers in the first column indicate the "tens" rows. The numbers across the top indicate the "ones" columns. The number in each remaining cell refers to the position to be found on the I scales. For example, the table indicates that the location of the number 8 (on an N scale) is to be found opposite position 3 on the corresponding I scale.

	0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
0		0	1	69	2	24	70	9	3	38
1	25	13	71	66	10	93	4	30	39	96
2	26	78	14	86	72	48	67	7	11	91
3	94	84	5	82	31	33	40	56	97	35
4	27	45	79	42	15	62	87	58	73	18
5	49	99	68	23	8	37	12	65	92	29
6	95	77	85	47	6	90	83	81	32	55
7	34	44	41	61	57	17	98	22	36	64
8	28	76	46	89	80	54	43	60	16	21
9	63	75	88	53	59	20	74	52	19	51
10	50									

Figure 9. Index Table.

The table also indicates that the number 11 (on the N scale) is to be found opposite position 13 on the corresponding I scale.

The instructions offered so far permit calculation of products up to 101. Such results can be read directly off the slide rule. Since the slide rule provides only these first 101 remainders (see Section 2 above), larger products must be obtained through the addition of multiples of 101. For example, if one uses the method described above to solve 4×32, one obtains the result 27. This is obviously wrong since $4 \times 7 = 28$. So that last digit of the product should be 8, i.e., one unit larger than 7. That indicates that one must add one multiple of 101 to the product, i.e., $27 + 1 \times 101 = 128$. This difference between 27 and 28 is called the "Completion Number".

Here is another example of this process. Consider 4×64. The preliminary result obtained with the Schumacher slide rule is 54. However, $4 \times 4 = 16$ so one can see that the correct answer must end in 6. Thus one can mentally calculate that the Completion Number is $6 - 4 = 2$. Therefore the correct product is $54 + 2 \times 101 = 256$.

If the Completion Number is less than 1, then it must be increased by 10. For example, the preliminary calculation of 15×61 with the Schumacher slide rule yields the answer 6. Since $5 \times 1 = 5$ and $5 - 6 = -1$, the Completion Number must be $-1 + 10 = 9$. Therefore the correct product is $6 + 9 \times 101 = 915$.

In addition to use of the Completion Number, one should also evaluate whether the order of magnitude of the result is plausible. For example, if the Completion Number is 0, then the correct answer may be less than 101 or greater that 1010. Thus, when the slide rule indicates that $16 \times 64 = 14$ (which is obviously too small), and the Completion Number is 0, then one should add 1010, i.e., $14 + 1010 = 1024$.

[2]It is interesting to note that the I1 and I2 scales on the production model had fine markings suggesting that interpolation between integers was possible. There are at least three possible explanations for this elaborate design. First, the draftsman who executed the final design may have misunderstood the limitation of the Schumacher slide rule to whole numbers. Second, it is also possible that, by using some existing slide rule scale (e.g., an L scale), set up costs were reduced. Finally, the design may have been intentional. As Schumacher himself noted, the scales are linear and could also be used for addition and subtraction. In that case, the ability to interpolate might have been useful.

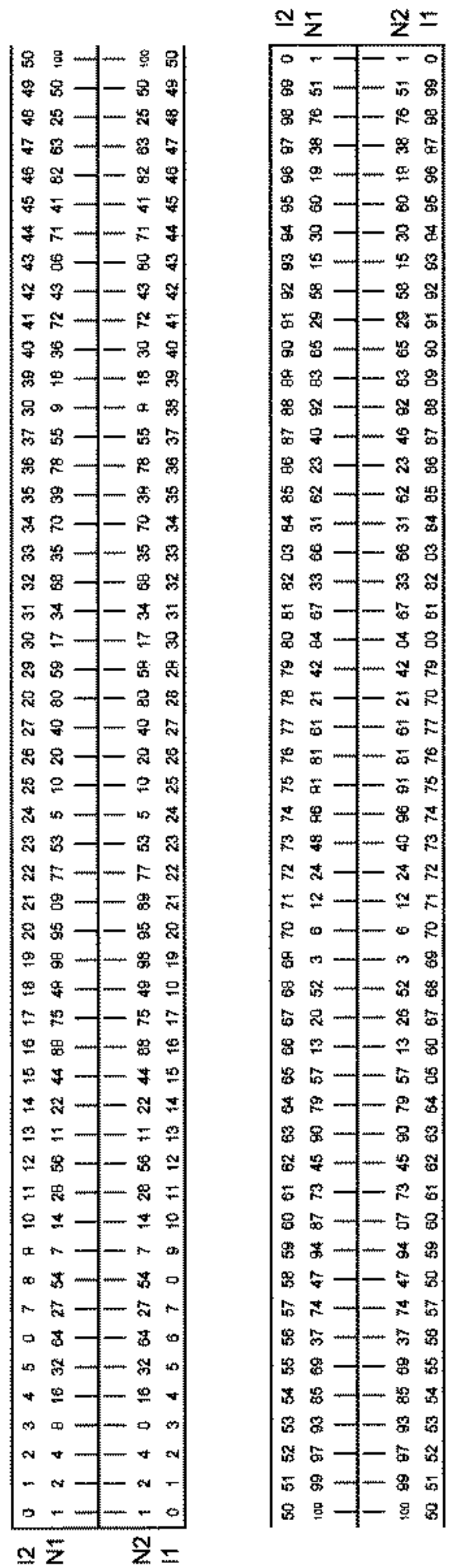

Figure 10. Two-part working model based on System Schumacher (See text for setup instructions.)

If the Completion Number is greater than 0 and the estimated result is greater than 1000, then one should break the multiplication into several simple steps. For example, according to the Schumacher slide rule, $23\times 49 = 16$. This is obviously wrong since $3\times 9 = 27$, so the last digit in the answer should be 7. Thus the Completion Number is $7 - 6 = 1$. However, $16 + 1\times 101 = 117$ is obviously too small. Since it can be seen that the correct result will be a large number, it is recommended that the calculation be broken down into the following steps: $2\times 49\times 10 = 980$ and $3\times 49 + 147$ Therefore $23\times 49 = 980 + 147 = 1127$. The Schumacher slide rule can be used to carry out the two steps involving multiplication, but the addition must be carried out in one's head or on paper.

The scales shown in Figure 10 can be used to explore the use of the slide rule as described above. First, photocopy the scales on a piece of white paper. Then cut out the two parts. Glue the two parts together so that the "open" ends overlap and $N = 50$ and $I = 100$ on the two pieces are correctly aligned. Finally, separate the scales along the centerline. The upper strip will correspond the stator, and the lower strip will correspond to the slide.

5. Assessments of System Schumacher

Schumacher's Own Assessment

In Schumacher's 1909 instruction book (page 1), he wrote: "The layout of the most commonly used slide rules is based on the theory of logarithms. One can raise the following questions: Are there other quantities which are subject to analogous rules? If there are such quantities, in what range would they permit application to the development of a device like the slide rule? What would be the advantages of such an application? The answer to the first question is affirmative. In number theory the indices play a role analogous to logarithms in arithmetic. The second question lends itself to assessment through the theory of indices."

Earlier in Schumacher's foreword, he had accordingly conceded that he had weighed the advantages and disadvantages of his scales against logarithmic scales very carefully. His design "would prove to be useful for instruction in elementary and middle schools and as an introduction into the realm of advanced algebra and number theory in higher education." However, he recognized that his slide rule would not replace the logarithmic slide rule.

The Assessment of a Contemporary

In a 1909 critique, Heinrich Wieleitner wrote,[3] "With regards to the practical applicability of this slide rule, the inventor himself is of the opinion that it will not displace the logarithmic slide rule in the hands of engineers and technicians. The main obstacle certainly lies in the fact that the reversed calculations are not practicable, except when they leave no remainder. Even in the case of multiplication of two- and three-digit numbers one must carry out in one's head a remarkable series of operations and

[3] Heinrich Wieleitner (1874 - 1931) was a teacher and mathematics historian. He authored numerous publications on the history and the teaching of mathematics.

deliberations. This is because the slide rule only yields the remainder resulting from division by 101. One can obtain the final result much faster in other ways." [4]

An evaluation from today's point of view

You may ask whether the mathematical operations described above in Section 3 can be used in daily life. To give an answer to this question one should not take into account only the requirements from common engineering disciplines. Dr. Schumacher was not an engineer but a mathematician. In 1884 he finished his PhD dissertation, *Zur Theorie der biquadratischen Gleichungen* ("On the theory of biquadratic equations"), at the University of Erlangen, Germany. This dissertation deals with biquadratic equations with special attention to the theory of calculating with such equations. In this class of equations only integer coefficients are considered. Approximated results do not make sense in this class of problems.

For this kind of problem common slide rules are not adequate. Because of imprecise reading of values one cannot determine whether obtained results are exact integers or possibly include a small fraction besides an integer. Therefore Schumacher's slide rule was intended for users (i.e., mathematicians dealing with mathematical problems on integers) different from users of common slide rules. So, it was not Schumacher's intention to replace common slide rules. This was explicitly mentioned by both Schumacher and Wieleitner.

The mathematical operations described in the previous sections seem to be very theoretic and not practicable. From today's point of view this is not true. Nowadays, calculations in prime fields are basic lessons in the study of mathematics and computer science. Corresponding concepts are even introduced in some special courses in secondary school (the German Abitur).

These theoretical concepts are the basis for many everyday applications. Examples of such applications are data transfer (e.g., by satellite and telephone), digital storage (especially for error detection and correction on CD and DVD), and great parts of cryptography. Even the security of cryptographic algorithms is based on the difficulty of computing discrete logarithms in very large prime fields (i.e., for prime numbers several hundred digits long). The basics of the corresponding theories had been developed by mathematicians like Galois and Jacobi. The latter was explicitly mentioned in Schumacher's book. For mathematicians working in this theoretical area, Schumacher's slide rule might be a helpful tool. However, from a practical point of view, we should mention that Schumacher's slide rule might not be usable even for the above-mentioned practicable applications, because of the restriction on one specific and small prime number $p = 101$. Nevertheless, for educational purposes the slide rule might be suitable for showing number theoretical concepts, like the ones given above.

One main disadvantage of Schumacher's slide rule is its restriction to a specific prime field. Additionally, its usage is restricted by the choice of the primitive root $w = 2$, which cannot be changed. If one wants to use a different primitive root, e.g., $w' = 3$ the labelling of the slide rules must be modified. The resulting table of discrete logarithms would look different, too. The scales N1 and N2 would need complete modification, such that the values $w'^i = 3^i$ modulo 101 for $i = 0, ..., p-2$ would be written from left to right.

These disadvantages might explain why Schumacher's slide rule was not a success story. In our opinion, even at Faber-Castell nobody expected to sell a large number of this type of slide rule. From the manufacturing point of view the labelling should not have been very challenging, because any calculation result is of an integer type. Therefore no special accuracy requirements were needed for the labelling.

6. Conclusion

It must be acknowledged that Schumacher achieved his bold goal of designing a slide rule that was not based on logarithms. However, even from the beginning it was clear that the Schumacher slide rule was a curiosity—a kind of "anti-slide rule"—if one judged it in terms of ease of application, understandability, speed of calculations, and usefulness in practice (e.g., in engineering).

Model 366 appeared for the last time in the 1929 catalog, and this slide rule probably attracted very few buyers during its 20-year life span. Because the slide rule is so unusual and so few were sold, it is now a rare and a much sought-after collector's item. The authors know of only five surviving specimens: three in the hands of private collectors, one in the Faber-Castell archives, and one in the Technisches Museum in Vienna.

References

[1] Schumacher, Johannes, *Ein Rechenschieber mit Teilung in gleiche Intervalle auf der Grundlage der zahlentheoretishen Indices,* München 1909 , J. Lindauersche Buchhandlung (Schöpping), 48 pages, 2 tables. (In the collection of D.vJ.)

[2] Schumacher, Johannes, *Zur Theorie der biquadratischen Gleichung,* Universität Erlangen, Dissertation, 1884, 49 pages.

[3] Schumacher, Johannes, *Zur Theorie der biquadratischen Gleichung,* Programm der k. bayer. Studienanstalt (Schweinfurt), Fortsetzung der gleichnamigen Dissertation, 1887, 57 pages.

[4] Wieleitner, Heinrich, *Rezension in Zeitschrift für den Mathematischen und Naturwissenschaftlichen Unterricht,* Nr. 40, 1909.

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The co-authors and I will be delighted if our article on this odd slide rule causes readers to give it more consideration and to carry out further research on it. *D.vJ.*