Ludgate's analytical machine of 1909

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This paper discusses the little known analytical machine, or program-controlled mechanical calculator, designed by Percy E. Ludgate in Ireland during the years 1903 to 1909, and documents the results of a search for information about his life and work.

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1. Introduction

Whilst seeking information about Charles Babbage and Ada Augusta, Countess of Lovelace, for use as introductory material for a lecture on the evolution of programming, I came across a reference to a paper entitled 'Automatic Calculating Engines' by one Percy E. Ludgate (1914). The paper proved to contain a brief but competent account of Babbage's Analytical Engine, but, to my great surprise, it ended with the following comments:

> 'I have myself designed an analytical machine, on different lines from Babbage's, to work with 192 variables of 20 figures each. A short account of it appeared in the *Scientific Proceedings, Royal Dublin Society*, April 1909. Complete descriptive drawings of the machine exist, as well as a description in manuscript, but I have not been able to take any steps to have it constructed.'

I immediately sought out a copy of this 1909 paper and found that it consisted of a fascinating account of a machine which was indeed quite comparable to, yet quite different from, Babbage's famous Analytical Engine. At the time I had never questioned the more or less standard account, found in the introductions to many textbooks on computing, which jump straight from the work of Babbage, started in 1834, to that of Aiken (and sometimes Stibitz and Zuse) just over 100 years later.* The purpose of the present paper is to put Ludgate's work into perspective, and to document the results of an extensive search for further information about his life and work. His 1909 paper and a review of it by Professor C. V. Boys that appeared shortly afterwards are reprinted in full as Appendices 1 and 2.

2. Background

Charles Babbage died in 1872, having failed to complete either the full scale version of his difference engine, or his analytical engine (a large amount of original material on Babbage's engines has been reprinted by Morrison and Morrison, 1961). Shortly before his death part of the analytical engine (the arithmetic mill, together with a printing device) was put together and is now in the Science Museum, London (see Baxandall, 1926). In 1878, a committee of the British Association for the Advancement of Science wrote a report (Merrifield, 1878) which praised the basic conception of the analytical engine 'as a marvel of mechanical ingenuity and resource', but concluded that 'in the present state of the design it is not more than a theoretical possibility; that is to say, we do not consider it a certainty that it could be constructed and put together so as to run smoothly and correctly, and to do the work expected of it. . . . we have come, not without reluctance, to the conclusion, that we cannot advise the British Association to take any steps, either by way of recommendation or otherwise, to procure the construction of Mr. Babbage's Analytical Engine'. However, according to Babbage's son (H. P. Babbage, 1888) an arithmetic unit (presumably the one referred to above), was used to give a successful demonstration to the committee of the operation of addition, with parallel assimilation of the carry digits.

Shortly afterwards Babbage's son put together another arithmetic unit and printing device, using pieces of mechanism and designs left by his father. This was completed and in 1910 by way of a demonstration was used to calculate and print successive multiples of π (Baxandall, 1926). This machine is now also in the Science Museum. In contrast, it would seem that the other major components of the engine, namely the store (originally planned to have a storage capacity for 1,000 numbers, each of 50 decimal digits, later reduced to 200 numbers, each of 25 digits) and the control mechanism were never constructed.

It is, in retrospect, clear that the complete analytical engine was far ahead of the technology of the time—indeed it has been claimed that Babbage's efforts were worthwhile merely for the benefits that they brought to mechanical engineering.

The boldness of Babbage's plans becomes clear when one realises that it was only during the mid-ninteenth century that a calculating machine (the arithometer of Thomas de Colmar, the first version of which was invented in 1820) achieved commercial success. In fact it was only towards the end of the century that mechanical calculating machines received widespread use (see Wilkes, 1956). On the other hand, Jacquard looms, whose technique of punched card control Babbage intended to use, were well established quite early in the nineteenth century. However, the use of punched cards for recording logical and numerical data had to await the work of Hollerith in the 1880s. His system, which was electro-mechanical, was used with great success in the 1890 US Census and within a few years had spread to several European countries, although for several years card handling was manual, rather than mechanised.

One can, therefore, with hindsight, claim that by the turn of the century the time had become much more propitious for the development of an analytical engine, or as we would now term it, a program-controlled computer. However, Ludgate's contribution was not that of making a second attempt to implement Babbage's machine, taking advantage of the improved technological capabilities of the day. Rather, he claims that

*I have since found just two accounts of the history of computers which even mention Ludgate's description of his analytical machine, namely Hoffman (1962) and Wilkes (1956). The latter contains the most detailed modern appraisal of Babbage's analytical engine that I have encountered.

until the later stages of his efforts, he had been in ignorance of Babbage's work, and his design is sufficiently novel for this to be accepted. Indeed all three main components of his analytical machine, the store, the arithmetic unit and the sequencing mechanism show evidence of considerable ingenuity and originality.

3. The store

The method of data storage that Ludgate designed used a 'shuttle' for each variable. Each shuttle acted as a carrier for a set of protruding metal rods, there being one rod for the sign, and for each of the 20 decimal digits comprising a number. The current value of each digit of the number currently stored in the shuttle was represented by the lateral position of the corresponding rod, i.e. by the length of rod protruding from the shuttle. The shuttles were to be held in 'two co-axial cylindrical shuttle-boxes'. A particular number could be brought to the arithmetic unit by rotating the appropriate shuttle box through an appropriate angle. There was also to be provision for tables of constants, represented by sets of holes, of depth from one to nine units, drilled into the surface of one or more special cylinders.

Assuming that there are appropriate means of transferring data between this type of representation and that used in the arithmetic unit (a topic on which Ludgate's paper is rather obscure), this method of storage would appear to have considerable advantages over that used by Babbage, i.e. columns of toothed discs, each capable of being connected by a train of gear wheels to the arithmetic unit. Certainly it is very convenient to access a number merely by an appropriate rotation of a cylindrical shuttle box. Ludgate mentions a further advantage, i.e. 'that the shuttles are quite independent of the machine, so that new shuttles, representing new variables can be introduced at any time'—one could perhaps claim that this was the forerunner of the modern replaceable disk!

4. The arithmetic unit

It is in the arithmetic unit that Ludgate's machine differs most markedly from that of Babbage, and indeed, as far as I can prove, from all other mechanical calculating machines. The unit is a 'direct' or 'partial product' multiplying machine, rather than one in which multiplication is performed by repeated addition. Direct multiplying machines already existed by the time of Ludgate. The first successful one was that of Bollée invented in 1889, although patents had earlier been granted to Barbour in 1872 and to Verea in 1878 for machines of this type. Indeed by the turn of the century a direct multiplying machine, known as the 'Millionaire', was starting to achieve wide distribution in Europe and America (see Chase 1952).

Each of these machines performed their multiplication of individual digits from the two different operands by what was in essence a table-look-up on a complete multiplication table. (Bollée represented the table by an array of 100 pairs of rods, each rod being one to nine units long—there is no means of knowing whether this was the inspiration for Ludgate's method of number storage.)

In Ludgate's machine what is essentially a logarithmic method of multiplication is used. Each digit of one operand is translated into the corresponding 'index number' (or 'Irish logarithm', as Boys so delightfully terms it). This set of index numbers is then added to the index number form of one of the digits of the other operand. The additions are performed concurrently by simple concatenation of lateral displacements. Then a reverse translation is performed to obtain the set of two-digit partial products. (The description of the mechanism for doing all this is somewhat obscure, and gives one a clearer impression of its ingenuity than its practicality.) The set of partial products so obtained for each digit in the second operand are then accumulated using a 'mill', which is presumably a fairly conventional set of co-axial toothed wheels incorporating a carrying mechanism. Ludgate claims that he designed his own version of Babbage's 'anticipating carriage', i.e. mechanism for assimilation, in a single step, of all the carry digits produced during the addition of two numbers (described in Babbage, *1851*), but gives no details of his design.

Ludgate was equally unconventional in his scheme for division, which instead of using repeated subtractions was based upon a table of reciprocals of the integers 100 to 999, and a rapidly convergent series for $(1 + x)^{-1}$, where $|x| < 10^{-3}$. The calculation of the series was controlled by what we would now call a built-in subroutine.

5. The sequencing mechanism

The sequencing mechanism that Ludgate describes has more in common with that used on the Harvard Mk. 1 (Aiken and Hopper, 1946) nearly 40 years later, than that designed by Babbage for his analytical engine.* Ludgate's machine was to be controlled by a perforated paper tape, termed a 'formula paper', on which each row of perforations defined a complete instruction. Each instruction specified two operands, the type of arithmetic operation to be performed, and the location (or pair of locations) which was to receive the result. Babbage on the other hand, for some unknown reason, intended to use two distinct sets of Jacquard cards, one for specifying which variables were to provide the operands for and receive the result from, each operation (the so-called 'variable cards'), the other for specifying the sequence of types of operations ('operation cards'). Furthermore, there were to be means for economising on operation cards (but not apparently variable cards) by indicating the number of times that there were to be repeated applications of the same type of arithmetic operation, rather than supply a sequence of identical operation cards (see Lady Lovelace's translation of Menabrea's article (Menabrea, 1843 note D)). It is not clear how Babbage intended to use the specification, on a variable card, of a particular variable, to access the column of disks representing that variable; in Ludgate's machine, as mentioned earlier, all that was necessary was to arrange for the appropriate angle of rotation of the shuttle-box containing the shuttle representing the required variable.

Ludgate clearly agreed with Babbage as to the fundamental importance of conditional branching, although he does not indicate how it was to be done—presumably, following Babbage, he intended that the mechanism that read the formula paper could be directed to skip a specified number of rows, either forwards or backwards. (It is interesting to note that the original Harvard Mk. 1 had only a very limited form of conditional branching.)

A third feature of the sequencing mechanism was the provision of built-in subroutines. The operation code for division, for example, caused control to pass temporarily to a sequence of instructions represented by rows of perforations on a permanent 'dividing cylinder'. Another cylinder provided a logarithm subroutine, and Ludgate mentions the possibility of indefinite expansion of the set of such auxiliary cylinders.

*Interestingly enough, a memorandum written by Aiken (1937) outlining his plans for an automatic calculating machine, mentions Ludgate in addition to describing Babbage's work on difference and analytical engines. However, the reference, whose wording closely follows that used earlier by Baxandall (1926), merely lists Ludgate amongst the designers of difference engines, so there is little reason to suppose that Aiken was familiar with Ludgate's plans.

6. Percy E. Ludgate

The two papers by Ludgate and the review of the first of these by C. V. Boys, gave only a few meagre starting points for a search for further information about his life and work. Baxandall (1926) had indicated that Ludgate was Irish but it was not known whether this was based merely on the fact that his first paper had appeared in the Scientific Proceedings of the Royal Dublin Society.

A search of standard reference works proved fruitless, and no further papers by Ludgate were traced, leading to the surmise (later proved correct) that he had died at a fairly early age. Inquiries of academic institutions and societies, mainly in Ireland, were similarly unsuccessful. Eventually his niece, Miss Violet Ludgate, who luckily is still living in Dublin, was traced through the heroic efforts of Mr. Desmond Clarke, Librarian and Secretary of the Royal Dublin Society, who contacted each of the Ludgates listed in the Dublin telephone directory. The following details of Percy Ludgate's life were obtained either directly from Miss Ludgate, or by following up leads that she furnished.

Percy Edwin Ludgate was born on 2 August 1883, at the house of his parents Michael and Mary Ludgate in Townsend Street, Skibbereen, County Cork, Ireland. He was the youngest of four children, all boys, his brothers being named Thomas, Frederick and Alfred. His father, Michael Ludgate, was born at Mallow, County Cork, and was married whilst serving in the army. He and his wife spent a part of their married life in India, where their first child, Thomas, was born. The second child, Frederick, was born in Winchester in 1879. Later the family moved to Ireland, first to Skibbereen, and later to Dublin, where Percy was brought up. It is believed that Percy Ludgate attended North Strand Parish School, and that he studied accountancy at Rathmines College of Commerce, Dublin, and was awarded a gold medal by the Corporation of Accountants on the occasion of his final examination, which he passed with distinction. (Efforts to confirm these details of his education have not so far met with any success.) He attended St. George's Church, Temple Street, Dublin.

Percy Ludgate worked as an auditor until his death, with the firm of Kevans and Son, 31 Dame Street, Dublin, which later transferred to Westmoreland Street, and is now part of the firm of Cooper Brothers. It seems almost certain that his work on the analytical machine was a private hobby which, according to his niece, 'he used to work at nightly, until the small hours of the morning'. He never married. Quoting from another letter that I received from Miss Ludgate: 'Percy liked walking; he took long solitary walks. I do not think he had many other interests. He attended his parish church services regularly. He was very gentle, a modest simple man. I never heard him make a condemning remark about anyone. I would say he was a really good man, highly thought of by anyone who knew him. Always appeared to be thinking deeply.' The photograph (**Fig. 1**), is believed to have been taken a few years before his death.

The one other person I have traced who has recollections of Percy Ludgate is Mr. E. Dunne, of Cooper Brothers, who joined the firm of Kevans and Son early in 1921. According to Mr. Dunne, 'My association with Mr. Ludgate was quite brief, but I had known him by repute for some time . . . As a person he possessed the characteristics one usually associates with genius, and he was so regarded by his colleagues on the staff . . . Like all men of his stature he was humble, courteous, patient and popular, and his early death closed a career that was full of promise for the future . . . The books and other memoranda of his disappeared and whether they were taken away by Percy before he became ill, or treated as part of the flotsam when Kevans and Son moved to Westmoreland Street, I cannot unfortunately say.' During the 1914-18 war he worked for a committee, set up by the War Office, headed by Mr. T. Condren-Flinn, senior partner of Kevans and Son. The task of this committee was to control the production and sale of oats, over a wide area of the country, in order to maintain a supply for the cavalry divisions of the army. This involved planning and organisation on a vast scale and Ludgate was much praised for the major role that he played. It is interesting to note that this provides a further parallel to the work of Charles Babbage who, because of his book *On the Economy of Machinery and Manufactures*, has often been called one of the originators of what is now known as 'Operational Research'.

Fig. 1. Percy E. Ludgate

It has not proved possible to obtain any information about his contacts with Professor A. W. Conway, of University College Dublin, who communicated Ludgate's paper to the Royal Dublin Society. Similarly unsuccessful have been efforts to trace the present whereabouts of the papers of Professor C. V. Boys, in the hope of finding his correspondence with Ludgate. (The obituary notice for Professor Boys which appeared in the *Proceedings of the Physical Society* in November 1944 stated that his papers 'were found well preserved and in meticulous order at his death'.) Furthermore, the records of the committee set up by the Royal Society of Edinburgh to organise the Napier Tercentenary Celebration, for whose handbook Ludgate contributed the article entitled 'Automatic Calculating Engines', have apparently not been preserved. (For some unknown reason Ludgate is not included in the listing of names and affiliations of contributors given at the back of the handbook.) Finally, no records have been found of any attempts to patent the analytical machine, or to obtain financial backing for its construction from the government.

At his death, on 16 October 1922, which occurred shortly after his return from a holiday in Lucerne, and which was announced in a brief obituary notice in the *Irish Times* two days later, Percy Ludgate was living with his widowed mother and his brother Alfred, at 30 Dargle Road, Drumcondra, Dublin. He had developed pneumonia, and his brother Frederick's wife (or rather, widow, since Frederick had died nine months earlier) who had helped to nurse Percy during his fatal illness, contracted pneumonia herself and died six days after Percy, leaving a daughter, Violet, who is now the sole surviving descendant of Michael and Mary Ludgate.

In his will, drawn up some five years before his death, Percy Ludgate had appointed his brother Alfred as his executor, and had willed the residue of his estate to his mother. His assets, mostly government stocks, amounted to somewhat over £800, and included a mere £10 for his personal effects. There is no means of knowing whether his drawings and manuscripts relating to the analytical engine were amongst these personal effects. His mother died in 1946, aged 97, and his brother Thomas, who had lived most of his life in Peacehaven, Sussex, in 1951. If any drawings or manuscripts had remained in the family they would presumably have passed into the possession of Percy's brother Alfred. However, there is no indication that this happened, and at Alfred's death in 1953 no such papers were found amongst his effects.

7. Concluding remarks

It seems unlikely that Ludgate ever attempted to construct the machine described in his 1909 paper. In fact in the 1914 paper he implies that he had discarded the plans, in favour of a second design:

> 'The most pleasing characteristic of a difference engine made on Babbage's principle is the simplicity of its action, the difference being added together in unvarying sequence; but notwithstanding its simple action, its structure is complicated by a large amount of adding mechanism—a complete set of adding wheels with carrying gear being required for the tabular number, and every order of difference except the highest order. On the other hand, while the best feature of the analytical engine or machine is the Jacquard apparatus (which, without being itself complicated, may be made a powerful instrument for interpreting mathematical formula), its weakness lies in the diversity of movements the Jacquard apparatus must control. Impressed by these facts, and with the desirability of reducing the expense of construction, I designed a second machine in which are combined the best principles of both the analytical and difference types, and from which are excluded their more expensive characteristics. By using a Jacquard I found it possible to eliminate the redundancy of parts hitherto found in difference-engines, while retaining the native symmetry of structure and harmony of action of machines of that class. My second machine, of which the design is on the point of completion, will contain but *one* set of adding wheels, and its movements will have a rhythm resembling that of the Jacquard loom itself. It is primarily intended to be used as a difference-machine, the number of orders of differences being sixteen. Moreover, the machine will also have the power of automatically evaluating a wide range of miscellaneous formulae.'

Excepting the possibility that further searches, perhaps stimulated by this paper, succeed in locating Ludgate's designs or correspondence, or trace some hitherto unsuspected collaborator, our appraisal of him will have to remain based on the fragmentary evidence afforded by his two published papers. One must, however, wonder just how much more he might have achieved if he had had but a modest fraction of the resources available to Babbage (to say nothing of Aiken!), and had not succumbed to pneumonia at such a tragically early age.

8. Acknowledgements

I have been aided in the search for information on the life and work of Ludgate by many people, only a few of whom is it possible to mention here. First and foremost I am indebted to Miss Violet Ludgate, who has provided much information about her uncle, and who has made extensive efforts herself to trace further possible sources of information. Other individuals to whom I wish to express my gratitude include Mr. Desmond Clarke, of the Royal Dublin Society, Mr. E. Dunne of Cooper Brothers, Wilton Place, Dublin, Miss M. C. Griffith, of the Public Record Office of Ireland, Mr. P. Henchy, Director of the National Library of Ireland, Mr. B. J. Lynch, of the Institute of Chartered Accountants in Ireland, Mrs. Ann MacDonald, until recently librarian at the Computing Laboratory of the University of Newcastle upon Tyne, Professor T. Murphy and Professor T. E. Nevin of University College Dublin, Mr. M. Woodger of the National Physical Laboratory, and Mr. H. Woolfe, of the Science Museum Library, London.

Ludgate's 1909 paper is reproduced by the kind permission of the Royal Dublin Society, and the review by C. V. Boys by kind permission of the Editor of *Nature*. The quotations from Ludgate's paper of 1914 are reproduced by kind permission of the Royal Society of Edinburgh.

Appendix 1

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ON A PROPOSED ANALYTICAL MACHINE

by

Percy E. Ludgate

(Communicated by Professor A. W. Conway, M.A.)

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I purpose to give in this paper a short account of the result of about six years' work, undertaken by me with the object of designing machinery capable of performing calculations, however intricate or laborious, without the immediate guidance of the human intellect.

In the first place I desire to record my indebtedness to Professor C. V. Boys, F.R.S., for the assistance which I owe to his kindness in entering into correspondence with me on the matter to which this paper is devoted.

It would be difficult and very inadvisable to write on the present subject without referring to the remarkable work of Charles Babbage, who, having first invented two Difference Engines, subsequently (about eighty years ago) designed an Analytical Engine, which was shown to be at least a theoretical possibility; but unfortunately its construction had not proceeded far when its inventor died. Since Babbage's time his Analytical Engine seems to have been almost forgotten; and it is probable that no living person understands the details of its projected mechanism. My own knowledge of Babbage's Engines is slight, and for the most part limited to that of their mathematical principles.

The following definitions of an Analytical Engine, written by Babbage's contemporaries, describe its essential functions as viewed from different standpoints:

> 'A machine to give us the same control over the executive which we have hitherto only possessed over the legislative department of mathematics.'*
>
> 'The material expression of any indefinite function of any degree of generality and complexity, such as, for instance: $F(x, y, z, \log x, \sin y,$ &c.), which is, it will be observed, a function of all other possible functions of any number of quantities.'†

*C. Babbage: 'Passages from the Life of a Philosopher', p. 129.
†R. Taylor's 'Scientific Memoirs', 1843, vol. iii., p. 691.

'An embodying of the science of operations constructed with peculiar reference to abstract number as the subject of those operations.'*

'A machine for weaving algebraical patterns.'†

These four statements show clearly that an Analytical Machine 'does not occupy common ground with mere "calculating machines". It holds a position wholly its own'.

In order to prevent misconception, I must state that my work was not based on Babbage's results—indeed, until after the completion of the first design of my machine, I had no knowledge of his prior efforts in the same direction. On the other hand, I have since been greatly assisted in the more advanced stages of the problem by, and have received valuable suggestions from, the writings of that accomplished scholar. There is in some respects a great resemblance between Babbage's Analytical Engine and the machine which I have designed—a resemblance which is not, in my opinion, due wholly to chance, but in a great measure to the nature of the investigations, which tend to lead to those conclusions on which the resemblance depends. This resemblance is almost entirely confined to the more general, abstract, or mathematical side of the question; while the contrast between the proposed structure of the two projected machines could scarcley be more marked.

It is unnecessary for me to prove the possibility of designing a machine capable of automatically solving all problems which can be solved by numbers. The principles on which an Analytical Machine may rest 'have been examined, admitted, recorded, and demonstrated'.‡ I would refer those who desire information thereon to the Countess of Lovelace's translation of an article on Babbage's Engine, which, together with copious notes by the translator, appears in R. Taylor's 'Scientific Memoirs', vol. iii.; to Babbage's own work, 'Passages from the Life of a Philosopher'; and to the Report of the British Association for the year 1878, p. 92. These papers furnish a complete demonstration that the whole of the developments and operations of analysis are capable of being executed by machinery.

Notwithstanding the complete and masterly treatment of the question to be found in the papers mentioned, it will be necessary for me briefly to outline the principles on which an Analytical Machine is based, in order that my subsequent remarks may be understood.

An Analytical Machine must have some means of storing the numerical data of the problem to be solved, and the figures produced at each successive step of the work (together with the proper algebraical signs); and, lastly, a means of recording the result or results. It must be capable of submitting any two of the numbers stored to the arithmetical operation of addition, subtraction, multiplication, or division. It must also be able to select from the numbers it contains the proper numbers to be operated on; to determine the nature of the operation to which they are to be submitted; and to dispose of the result of the operation, so that such result can be recalled by the machine and further operated on, should the terms of the problem require it. The sequence of operations, the numbers (considered as abstract quantities only) submitted to those operations, and the disposition of the result of each operation, depend upon the algebraical statement of the calculation on which the machine is engaged; while the magnitude of the numbers involved in the work varies with the numerical data of that particular case of the general formula which is in process of solution. The question therefore naturally arises as to how a machine can be made to follow a particular law of development as expressed by an algebraic formula. An eminently satisfactory answer to that question (and one utilised by both Babbage and myself) is suggested by the Jacquard loom, in which interesting invention a system of perforated cards is used to direct the movements of the warp and weft threads, so as to produce in the woven material the pattern intended by the designer. It is not difficult to imagine that a similar arrangement of cards could be used in a mathematical machine to direct the weaving of numbers, as it were, into algebraic patterns, in which case the cards in question would constitute a kind of mathematical notation. It must be distinctly understood that, if a set of such cards were once prepared in accordance with a specified formula, it would possess all the generality of algebra, and include an infinite number of particular cases.

I have prepared many drawings of the machine and its parts; but it is not possible in a short paper to go into any detail as to the mechanism by means of which elaborate formulae can be evaluated, as the subject is necessarily extensive and somewhat complicated; and I must, therefore, confine myself to a superficial description, touching only points of particular interest or importance.

Babbage's Jacquard-system and mine differ considerably; for, while Babbage designed two sets of cards—one set to govern the operations, and the other set to select the numbers to be operated on—I use one sheet or roll of perforated paper (which, in principle, exactly corresponds to a set of Jacquard-cards) to perform both these functions in the order and manner necessary to solve the formula to which the particular paper is assigned. To such a paper I apply the term formula-paper. Each row of perforations across the formula-paper directs the machine in some definite step in the process of calculation—such as, for instance, a complete multiplication, including the selection of the numbers to be multiplied together. Of course a single formula-paper can be used for an indefinite number of calculations, provided that they are all of one type or kind (i.e. algebraically identical).

In referring to the numbers stored in the machine, the difficulty arises as to whether we refer to them as mere numbers in the restricted arithmetical sense, or as quantities, which, though always expressed in numerals, are capable of practically infinite variation. In the latter case they may be regarded as true mathematical variables. It was Babbage's custom (and one which I shall adopt) when referring to them in this sense to use the term 'Variable' (spelt with capital V), while applying the usual meanings to the words 'number' and 'variable'.

In my machine each Variable is stored in a separate shuttle, the individual figures of the Variable being represented by the relative positions of protruding metal rods or 'type', which each shuttle carries. There is one of these rods for every figure of the Variable, and one to indicate the sign of the Variable. Each rod protrudes a distance of from 1 to 10 units, according to the figure or sign which it is at the time representing. The shuttles are stored in two co-axial cylindrical shuttle-boxes, which are divided for the purpose into compartments parallel to their axis. The present design of the machine provides for the storage of 192 Variables of twenty figures each; but both the number of Variables and the number of figures in each Variable may, if desired, be greatly increased. It may be observed, too, that the shuttles are quite independent of the machine, so that new shuttles, representing new Variables, can be introduced at any time.

When two Variables are to be multiplied together, the corresponding shuttles are brought to a certain system of slides called the *index*, by means of which the machine computes the product. It is impossible precisely to describe the mechanism of the index without drawings; but it may be compared to a slide-rule on which the usual markings are replaced by moveable blades. The index is arranged so as to give several readings simultaneously. The numerical values of the readings are indi-

*loc. cit., p. 694.
†loc. cit., p. 696.
‡C. Babbage: 'Passages from the Life of a Philosopher', p. 450.

cated by periodic displacements of the blades mentioned, the duration of which displacements are recorded in units measured by the driving shaft on a train of wheels called the *mill*, which performs the carrying of tens, and indicates the final product. The product can be transferred from thence to any shuttle, or to two shuttles simultaneously, provided that they do not belong to the same shuttle-box. The act of inscribing a new value in a shuttle automatically cancels any previous value that the shuttle may have contained. The fundamental action of the machine may be said to be the multiplying together of the numbers contained in any two shuttles, and the inscribing of the product in one or two shuttles. It may be mentioned here that the fundamental process of Babbage's Engine was not multiplication but addition.

Though the index is analogous to the slide-rule, it is not divided logarithmically, but in accordance with certain *index numbers*, which, after some difficulty, I have arranged for the purpose. I originally intended to use the logarithmic method, but found that some of the resulting intervals were too large; while the fact that a logarithm of zero does not exist is, for my purpose, an additional disadvantage. The index numbers (which I believe to be the smallest whole numbers that will give the required results) are contained in the following tables:

Column 1 of **Table 1** contains zero and the nine digits, and column 2 of the same Table the corresponding *simple index numbers*. Column 1 of **Table 2** sets forth all *partial products* (a term applied to the product of any two units), while column 2 contains the corresponding *compound index numbers*. The relation between the index numbers is such that the sum of the simple index numbers of any two units is equal to the compound index number of their product. **Table 3** is really a re-arrangement of Table 2, the numbers 0 to 66 (representing 67 divisions on the index) being placed in column 1, and in column 2, opposite to each number in column 1 which is a compound index number, is placed the corresponding simple product.

Now, to take a very simple example, suppose the machine is supplied with a formula-paper designed to cause it to evaluate x for given values of a, b, c, and d, in the equation $ab + cd = x$, and suppose we wish to find the value of x in the particular case where $a = 9247$, $b = 8132$, $c = 21893$, and $d = 823$.

The four given numbers are first transferred to the machine by the key-board hereafter mentioned; and the formula-paper causes them to be inscribed in four shuttles. As the shuttles of the inner and outer co-axial shuttle-boxes are numbered consecutively, we may suppose the given values of a and c to be inscribed in the first and second shuttles respectively of the inner box, and of b and d in the first and second shuttles respectively of the outer box; but it is important to remember that it is a function of the formula-paper to select the shuttles to receive the Variables, as well as the shuttles to be operated on, so that (except under certain special circumstances, which

Table 1

UNIT	SIMPLE INDEX NO.	ORDINAL
0	50	9
1	0	0
2	1	1
3	7	4
4	2	2
5	23	7
6	8	5
7	33	8
8	3	3
9	14	6

Table 2

PARTIAL PRODUCT	COMP. INDEX NO.	PARTIAL PRODUCT	COMP. INDEX NO.	PARTIAL PRODUCT	COMP. INDEX NO.
1	0	15	30	36	16
2	1	16	4	40	26
3	7	18	15	42	41
4	2	20	25	45	37
5	23	21	40	48	11
6	8	24	10	49	66
7	33	25	46	54	22
8	3	27	21	56	36
9	14	28	35	63	47
10	24	30	31	64	6
12	9	32	5	72	17
14	34	35	56	81	28

Comp. index numbers of zero: 50, 51, 52, 53, 57, 58, 64, 73, 83, 100.

Table 3

COMP. INDEX NO.	PARTIAL PRODUCT	COMP. INDEX NO.	PARTIAL PRODUCT
0	1	34	14
1	2	35	28
2	4	36	56
3	8	37	45
4	16	38	—
5	32	39	—
6	64	40	21
7	3	41	42
8	6	42	—
9	12	43	—
10	24	44	—
11	48	45	—
12	—	46	25
13	—	47	63
14	9	48	—
15	18	49	—
16	36	50	0
17	72	51	0
18	—	52	0
19	—	53	0
20	—	54	—
21	27	55	—
22	54	56	35
23	5	57	0
24	10	58	0
25	20	59	—
26	40	60	—
27	—	61	—
28	81	62	—
29	—	63	—
30	15	64	0
31	30	65	—
32	—	66	49
33	7		

arise only in more complicated formulae) any given formula-paper always selects the same shuttles in the same sequence and manner, whatever be the values of the Variables. The magnitude of a Variable only effects the type carried by its shuttle, and in no way influences the movements of the shuttle as a whole

The machine, guided by the formula-paper, now causes the shuttle-boxes to rotate until the first shuttles of both inner and outer boxes come opposite to a shuttle-race. The two shuttles are then drawn along the race to a position near the index; and certain slides are released, which move forward until stopped by striking the type carried by the outer shuttle. The slides in question will then have moved distances corresponding to the simple index numbers of the corresponding digits of the Variables b. In the particular case under consideration, the first four slides will therefore move 3, 0, 7, and 1 units respectively, the remainder of the slides indicating zero by moving 50 units (see Table 1). Another slide moves *in the opposite direction* until stopped by the first type of the inner shuttle, making a movement proportional to the simple index number of the first digit of the multiplier a—in this case 14. As the index is attached to the last-mentioned slide, and partakes of its motion, the *relative* displacements of the index and each of the four slides are respectively $3 + 14$, $0 + 14$, $7 + 14$, and $1 + 14$ units (that is, 17, 14, 21, and 15 units), so that pointers attached to the four slides, which normally point to zero on the index, will now point respectively to the 17th, 14th, 21st and 15th divisions of the index. Consulting Table 3, we find that these divisions correspond to the partial products 72, 9, 27, and 18. In the index the partial products are expressed mechanically by movable blades placed at the intervals shown in column 2 of the third table. Now, the duration of the first movement of any blade is as the unit figure of the partial product which it represents, so that the movements of the blades concerned in the present case will be as the numbers 2, 9, 7, and 8, which movements are conveyed by the pointers to the mill, causing it to register the number 2978. A carriage near the index now moves one step to effect multiplication by 10, and then the blades partake of a second movement, this time transferring the tens' figures of the partial products (i.e. 7, 0, 2, and 1) to the mill, which completes the addition of the units' and tens' figures thus:

$$\begin{array}{r} 2978 \\ 7021 \\ \hline 73188 \end{array}$$

the result being the product of the multiplicand b by the first digit of the multiplier a. After this the index makes a rapid reciprocating movement, bringing its slide into contact with the second type of the inner shuttle (which represents the figure 2 in the quantity a), and the process just described is repeated for this and the subsequent figures of the multiplier a until the whole product ab is found. The shuttles are afterwards replaced in the shuttle-boxes, the latter being then rotated until the second shuttles of both boxes are opposite to the shuttle-race. These shuttles are brought to the index, as in the former case, and the product of their Variables (21893×823) is obtained, which, being added to the previous product (that product having been purposely retained in the mill), gives the required value of x. It may be mentioned that the position of the decimal point in a product is determined by special mechanism which is independent of both mill and index.

Most of the movements mentioned above, as well as many others, are derived from a set of cams placed on a common shaft parallel to the driving-shaft; and all movements so derived are under the control of the formula-paper.

The ordinals in Table 1 are not mathematically important, but refer to special mechanism which cannot be described in this paper, and are included in the tables merely to render them complete.

The sum of two products is obtained by retaining the first product in the mill until the second product is found—the mill will then indicate their sum. By reversing the direction of rotation of the mill before the second product is obtained, the difference of the products results. Consequently, by making the multiplier unity in each case, simple addition and subtraction may be performed.

In designing a calculating machine it is a matter of peculiar difficulty and of great importance to provide for the expeditious carrying of tens. In most machines the carryings are performed in rapid succession; but Babbage invented an apparatus (of which I have been unable to ascertain the details) by means of which the machine could 'foresee' the carryings and act on the foresight. After several years' work on the problem, I have devised a method in which the carrying is practically in complete mechanical independence of the adding process, so that the two movements proceed simultaneously. By my method the sum of m numbers of n figures would take $9m + n$ units of time. In finding the product of two numbers of twenty figures each, forty additions are required (the units' and tens' figures of the partial products being added separately). Substituting the values 40 and 20 for m and n, we get $9 \times 40 + 20 = 380$, or $9\frac{1}{2}$ time-units for each addition—the time-unit being the period required to move a figure-wheel through $\frac{1}{10}$ revolution. With Variables of 20 figures each the quantity n has a constant value of 20, which is the number of units of time required by the machine to execute any carrying which has not been performed at the conclusion of an indefinite number of additions. Now, if the carryings were performed in succession, the time required could not be less than $9 + n$, or 29 units for each addition, and is, in practice, considerably greater.*

In ordinary calculating machines division is accomplished by repeated subtractions of the divisor from the dividend. The divisor is subtracted from the figures of the dividend representing the higher powers of ten until the remainder is less than the divisor. The divisor is then moved one place to the right, and the subtraction proceeds as before. The number of subtractions performed in each case denotes the corresponding figure of the quotient. This is a very simple and convenient method for ordinary calculating machines; but it scarcely meets the requirements of an Analytical Machine. At the same time, it must be observed that Babbage used this method, but found it gave rise to many mechanical complications.

My method of dividing is based on quite different principles, and to explain it I must assume that the machine can multiply, add, or subtract any of its Variables; or, in other words, that a formula-paper can be prepared which could direct the machine to evaluate any specified function (which does not contain the sign of division or its equivalent) for given values of its variables.

Suppose, then, we wish to find the value of p/q for particular values of p and q which have been communicated to the machine. Let the first three figures of q be represented by f, and let A be the reciprocal of f, where A is expressed as a decimal of 20 figures. Multiplying the numerator and denominator of the fraction by A, we have $(Ap)/(Aq)$, where Aq must give a number of the form 100 . . . because $Aq = q/f$. On placing the decimal point after the unit, we have unity plus a small decimal. Represent this decimal by x: then

$$\frac{p}{q} = \frac{Ap}{1 + x} \text{ or } Ap\ (1 + x)^{-1}$$

Expanding by the binomial theorem

*For further notes on the problem of the carrying of tens, see C. Babbage: 'Passages from the Life of a Philosopher', p. 114, etc.

$$(1) \qquad \frac{p}{q} = Ap(1 - x + x^2 - x^3 + x^4 - x^5 + \text{etc.}),$$

or

$$(2) \qquad \frac{p}{q} = Ap(1 - x)(1 + x^2)(1 + x^4)(1 + x^8), \text{etc.}$$

The series (1) converges rapidly, and by finding the sum as far as x^{10} we obtain the correct result to at least twenty figures; whilst the expression (2) gives the result correctly to at least thirty figures. The position of the decimal point in the quotient is determined independently of these formulae. As the quantity A must be the reciprocal of one of the numbers 100 to 999, it has 900 possible values. The machine must, therefore, have the power of selecting the proper value for the quantity A, and of applying that value in accordance with the formula. For this purpose the 900 values of A are stored in a cylinder—the individual figures being indicated by holes of from one to nine units deep in its periphery. When division is to be performed, this cylinder is rotated, by a simple device, until the number A (represented on the cylinder by a row of holes), which is the reciprocal of the first three figures of the divisor, comes opposite to a set of rods. These rods then transfer that number to the proper shuttle, whence it becomes an ordinary Variable, and is used in accordance with the formula. It is not necessary that every time the process of division is required the dividing formula should be worked out in detail in the formula-paper. To obviate the necessity of so doing the machine is provided with a special permanent *dividing cylinder*, on which this formula is represented in the proper notation of perforations. When the arrangement of perforations on the formula-paper indicates that division is to be performed, and the Variables which are to constitute divisor and dividend, the formula-paper then allows the dividing cylinder to usurp its functions until that cylinder has caused the machine to complete the division.

It will be observed that, in order to carry out the process of division, the machine is provided with a small table of numbers (the numbers A) which it is able to consult and apply in the proper way. I have extended this system to the logarithmic series, in order to give to that series a considerable convergency; and I have also introduced a *logarithmic cylinder* which has the power of working out the logarithmic formula, just as the dividing cylinder directs the dividing process. This system of auxiliary cylinders and tables for special formulae may be indefinitely extended.

The machine prints all results, and, if required, the data, and any noteworthy values which may transpire during the calculation. It may be mentioned, too, that the machine may be caused to calculate and print, quite automatically, a table of values—such, for instance, as a table of logs, sines, squares, etc. It has also the power of recording its results by a system of perforations on a sheet of paper, so that when such a *number-paper* (as it may be called) is replaced in the machine, the latter can 'read' the numbers indicated thereon, and inscribe them in the shuttles reserved for the purpose.

Among other powers with which the machine is endowed is that of changing from one formula to another as desired, or in accordance with a given mathematical law. It follows that the machine need never be idle; for it can be set to tabulate successive values of any function, while the work of the tabulation can be suspended at any time to allow of the determination by it of one or more results of greater importance or urgency. It can also 'feel' for particular events in the progress of its work—such, for instance, as a change of sign in the value of a function, or its approach to zero or infinity; and it can make any pre-arranged change in its procedure, when any such event occurs. Babbage dwells on these and similar points, and explains their bearing on the automatic solution (by approximation) of an equation of the nth degree;* but I have not been able to ascertain whether his way of attaining these results has or has not any resemblance to my method of so doing.

The Analytical Machine is under the control of two key-boards, and in this respect differs from Babbage's Engine. The upper key-board has ten keys (numbered 0 to 9), and is a means by which numbers are communicated to the machine. It can, therefore, undertake the work of the number-paper previously mentioned. The lower key-board can be used to control the working of the machine, in which case it performs the work of a formula-paper. The key-boards are intended for use when the nature of the calculation does not warrant the preparation of a formula-paper or a number-paper, or when their use is not convenient. An interesting illustration of the use of the lower key-board is furnished by a case in which a person is desirous of solving a number of triangles (say) of which he knows the dimensions of the sides, but has not the requisite formula-paper for the purpose. His best plan is to put a plain sheet of paper in the controlling apparatus, and on communicating to the machine the known dimensions of one of the triangles by means of the upper key-board, to guide the machine by means of the lower key-board to solve the triangle in accordance with the usual rule. The manipulations of the lower key-board will be recorded on the paper, which can then be used as a formula-paper to cause the machine automatically to solve the remaining triangles. He can communicate to the machine the dimensions of these triangles individually by means of the upper key-board; or he may, if he prefers so doing, tabulate the dimensions in a number-paper, from which the machine will read them of its own accord. The machine is, therefore, able to 'remember', as it were, a mathematical rule; and having once been shown how to perform a certain calculation, it can perform any similar calculation automatically so long as the same paper remains in the machine.

It must be clearly understood that the machine is designed to be quite automatic in its action, so that a person almost entirely ignorant of mathematics could use it, in some respects, as successfully as the ablest mathematician. Suppose such a person desired to calculate the cosine of an angle, he obtains the correct result by inserting the formula-paper bearing the correct label, depressing the proper number-keys in succession to indicate the magnitude of the angle, and starting the machine, though he may be quite unaware of the definition, nature, or properties of a cosine.

While the machine is in use its central shaft must be maintained at an approximately uniform rate of rotation—a small motor might be used for this purpose. It is calculated that a velocity of three revolutions per second would be safe; and such a velocity would ensure the multiplication of any two Variables of twenty figures each in about 10 seconds, and their addition or subtraction in about three seconds. The time taken to divide one Variable by another depends on the degree of convergency of the series derived from the divisor, but $1\frac{1}{2}$ minutes may be taken as the probable maximum. When constructing a formula-paper, due regard should therefore be had to the relatively long time required to accomplish the routine of division; and it will, no doubt, be found advisable to use this process as sparingly as possible. The determination of the logarithm of any number would take two minutes, while the evaluation of a^n (for any value of n) by the expotential theorem, should not require more than $1\frac{1}{2}$ minutes longer—all results being of twenty figures.†

The machine, as at present designed, would be about 26 inches long, 24 inches broad, and 20 inches high; and it would therefore be of a portable size. Of the exact dimensions of Babbage's

*C. Babbage: 'Passages from the Life of a Philosopher', p. 131.

†The times given include that required for the selection of the Variables to be operated on.

Engine I have no information; but evidently it was to have been a ponderous piece of machinery, measuring many feet in each direction. The relatively large size of this engine is doubtless due partly to its being designed to accommodate the large number of one thousand Variables of fifty figures each, but more especially to the fact that the Variables were to have been stored on columns of wheels, which, while of considerable bulk in themselves, necessitated somewhat intricate gearing arrangements to control their movements. Again, Babbage's method of multiplying by repeated additions, and of dividing by repeated subtractions, though from a mathematical point of view very simple, gave rise to very many mechanical complications.*

To explain the power and scope of an Analytical Machine or Engine, I cannot do better than quote the words of the Countess of Lovelace: 'There is no finite line of demarcation which limits the powers of the Analytical Engine. These powers are coextensive with the knowledge of the laws of analysis itself, and need be bounded only by our acquaintance with the latter. Indeed, we may consider the engine as the material and mechanical representative of analysis, and that our actual working powers in this department of human study will be enabled more effectually than heretofore to keep pace with our theoretical knowledge of its principles and laws, through the complete control which the engine gives us over the executive manipulations of algebraical and numerical symbols.'†

A Committee of the British Association which was appointed to report on Babbage's Engine stated that, 'apart from the question of its saving labour in operations now possible, we think the existence of such an instrument would place within reach much which, if not actually impossible, has been too close to the limits of human skill and endurance to be practically available'.‡

In conclusion, I would observe that of the very numerous branches of pure and applied science which are dependent for their development, record, or application on the dominant science of mathematics, there is not one of which the progress would not be accelerated, and the pursuit would not be facilitated, by the complete command over the numerical interpretation of abstract mathematical expressions, and the relief from the time-consuming drudgery of computation, which the scientist would secure through the existence of machinery capable of performing the most tedious and complex calculations with expedition, automatism, and precision.

Appendix 2

(Reprinted from *Nature* 81, 2070 (1 July 1909) pp. 14-15)

A NEW ANALYTICAL ENGINE

The April number of the Scientific Proceedings of the Royal Dublin Society contains an interesting and very original paper by Mr. Percy E. Ludgate on a proposed analytical machine. Of all calculating machines, the analytical machine or engine is the most comprehensive in its powers. Cash till reckoners and adding machines merely add or add and print results. Arithmometers are used for multiplying and dividing, which they really only accomplish by rapidly repeated addition or subtraction, with the exception alone, perhaps, of the arithmometer of Bollée, which, in a way, works by means of a mechanical multiplication table. Difference engines originated by Babbage produce and print tables of figures of almost any variety, but the process is one of addition of successive differences. The analytical engine proposed by Babbage was intended to have powers of calculation so extensive as to seem a long way outside the capacity of mere mechanism, but this was to be brought about by the use of operation cards supplied by the director or user, which, like the cards determining the pattern in a Jacquard loom, should direct the successive operations of the machine, much as the timing cam of an automatic lathe directs the successive movements of the different tools and feeding and chucking devices. However elaborate the mechanism of Babbage, if completed, might have been, the individual elements of operation would, so far as the writer has been able to understand it, have been actually operations of addition or subtraction only, and, with the exception of the method of multiplication created by Bollée, the writer does not recall any case in which mechanism has been used to compute numerical results except by the use of the processes of addition or subtraction, simple or cumulative. Of course, harmonic analysers and other instruments depending on geometry are not included in the category of machines which operate on numbers.

The simplicity of the logarithmic method of multiplying must have made many inventors regret the inherent incommensurability of the function to any simple base, or, if commensurability is attained for any particular number and its powers by the use of an incommensurable base, the incommensurability of the corresponding logarithms of numbers prime to those first selected. On this account the writer has always imagined that the logarithmic method was unsuited to mechanism, or, if applied at all, could only be so applied at the expense of complication, which would more than compensate for the directness of the process of logarithmic multiplication.

Mr. Ludgate, however, in effect, uses for each of the prime numbers below ten a logarithmic system with a different incommensurable base, which as a fact never appears, and is able to take advantage of the additive principle, or, rather, it is so applied that the machine may use it. These mixed or Irish logarithms, or index numbers, as the author calls them, are very surprising at first, but, if the index numbers of zero be excepted, it is not difficult to follow the mode by which they have been selected. The index numbers of the ten digits are as follows:

Digit	0	1	2	3	4	5	6	7	8	9
Index number	50	0	1	7	2	23	8	33	3	14

When two numbers are to be multiplied, the index numbers of the several digits are mechanically added to the index numbers of each of the digits of the other, and, the process of carrying the tens being carried on simultaneously, the time required is very small. For instance, the author gives as an example the multiplication of two numbers of 20 digits each, which will require 40 of these additions, which he shows will require $9\frac{1}{2}$ time units if a time unit is one-tenth of the time of revolution of a figure wheel.

Unfortunately, while the principle on which the proposed machine is to work is described, only the barest idea of the mechanical construction is given, so that it is difficult to judge of the practicability of the intended construction. Whatever this may be, the originality of the method of mixed commensurable logarithms to incommensurable bases seems to the writer so great and the conception so bold as to be worthy of special attention.

Division has hitherto always been effected by the process of rapid but repeated subtraction, following in this respect the method practised with pencil and paper. Having discovered how to harness the logarithm to mechanism, Mr. Ludgate would, it would be expected, have managed to effect division by a logarithmic method, and possibly he could have done so, but here again he has left the beaten track, and by his ingenuity has made division a direct, and not, as hitherto, an indirect or trial-and-error process. Starting with a table of reciprocals of all numbers from 100 to 999, which in a mechanical form is

*See Report Brit. Assoc., 1878, p. 100.
†R. Taylor's 'Scientific Memoirs', 1843, vol. iii., p. 696.
‡Report Brit. Assoc., 1878, p. 101.

intended to be stored in the machine, he imagines both numerator and denominator of the required fraction p/q to be multiplied by the reciprocal A of the first three digits of q so as to become $(Ap)/(Aq)$. Aq must, then, in every case begin with the digits 100, and it may be written $1 + x$, where x is a small fraction. Then

$$\frac{p}{q} = Ap(1 - x)(1 + x^2)(1 + x^6)(1 + x^8)\ldots$$

a highly convergent series of which five terms will give a result correct to twenty figures at least, and so division is intended to be effected by a process of direct multiplication.

Until more detail as to the proposed construction and drawings are available it is not possible to form any opinion as to the practicability or utility of the machine as a whole, but it is to be hoped that if the author receives, as he deserves, encouragement to proceed with his task, he will not allow himself to become swamped in the complexity which must be necessary if he aims at the wide generality of a complete analytical engine. If he will, in the first instance, produce his design for a machine of restricted capacity, even if it does no more than an arithmometer, he will, by demonstrating its practicability and advantages be more likely to be enabled to proceed step by step to the more perfect instrument than he will if, as Babbage did, he imagines his whole machine at once. In the writer's opinion, the ingenuity required to arrange a complete analytical engine is really in great part misplaced. Such a machine can only be used and kept in order by someone who really understands it, and it would seem to the writer of this notice more practicable to allow the user's attention to replace the action of operation cards, and leave to the machine the more direct numerical evaluations.

C. V. Boys

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