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Planimeters are instruments for the determination by mechanical means of the area of any figure. A pointer, generally called the [Sidenote: Planimeters.] "tracer," is guided round the boundary of the figure, and then the area is read off on the recording apparatus of the instrument. The simplest and most useful is Amsler's (fig. 5). It consists of two bars of metal OQ and QT, [v.04 p.0976] which are hinged together at Q. At O is a needle-point which is driven into the drawing-board, and at T is the tracer. As this is guided round the boundary of the figure a wheel W mounted on QT rolls on the paper, and the turning of this wheel measures, to some known scale, the area. We shall give the theory of this instrument fully in an elementary manner by aid of geometry. The theory of other planimeters can then be easily understood.

Consider the rod QT with the wheel W, without the arm OQ. Let it be placed with the wheel on the paper, and now moved perpendicular to itself from AC to BD (fig. 6). The rod sweeps over, or generates, the area of the rectangle ACDB = lp, where l denotes the length of the rod and p the distance AB through which it has been moved. This distance, as measured by the rolling of the wheel, which acts as a curvometer, will be called the "roll" of the wheel and be denoted by w. In this case p = w, and the area P is given by P = wl. Let the circumference of the wheel be divided into say a hundred equal parts u; then w registers the number of u's rolled over, and w therefore gives the number of areas lu contained in the rectangle. By suitably selecting the radius of the wheel and the length l, this area lu may be any convenient unit, say a square inch or square centimetre. By changing l the unit will be changed.

Again, suppose the rod to turn (fig. 7) about the end Q, then it will describe an arc of a circle, and the rod will generate an area 1/2l squared[theta], where [theta] is the angle AQB through which the rod has turned. The wheel will roll over an arc c[theta], where c is the distance of the wheel from Q. The "roll" is now w = c[theta]; hence the area generated is

P = 1/2 l squared/c w,

and is again determined by w.

Next let the rod be moved parallel to itself, but in a direction not perpendicular to itself (fig. 8). The wheel will now not simply roll. Consider a *small* motion of the rod from QT to Q'T'. This may be resolved into the motion to RR' perpendicular to the rod, whereby the rectangle QTR'R is generated, and the sliding of the rod along itself from RR' to Q'T'. During this second step no area will be generated. During the first step the roll of the wheel will be QR, whilst during the second step there will be no roll at all. The roll of the wheel will therefore measure the area of the rectangle which equals the parallelogram QTT'Q'. If the whole motion of the rod be considered as made up of a very great number of small steps, each resolved as stated, it will be seen that the roll again measures the area generated. But it has to be noticed that now the wheel does not only roll, but also slips, over the paper. This, as will be pointed out later, may introduce an error in the reading.

We can now investigate the most general motion of the rod. We again resolve the motion into a number of small steps. Let (fig. 9) AB be one position, CD the next after a step so small that the arcs AC and BD over which the ends have passed may be considered as straight lines. The area generated is ABDC. This motion we resolve into a step from AB to CB', parallel to AB and a turning about C from CB' to CD, steps such as have been investigated. During the first, the "roll" will be p the altitude of the parallelogram; during the second will be c[theta]. Therefore

w = p + c[theta].

The area generated is lp + 1/2 l^2[theta], or, expressing p in terms of w, lw + (1/2l^2 - lc)[theta]. For a finite motion we get the area equal to the sum of the areas generated during the different steps. But the wheel will continue rolling, and give the whole roll as the sum of the rolls for the successive steps. Let then w denote the whole roll (in fig. 10), and let [alpha] denote the sum of all the small turnings [theta]; then the area is

P = lw + (1/2l^2 - lc)[alpha] . . . (1)

Here [alpha] is the angle which the last position of the rod makes with the first. In all applications of the planimeter the rod is brought back to its original position. Then the angle [alpha] is either zero, or it is 2[pi] if the rod has been once turned quite round.

Hence in the first case we have

P = lw . . . (2a)

and w gives the area as in case of a rectangle.

In the other case

P = lw + lC . . . (2b)

where C = (1/2l-c)2[pi], if the rod has once turned round. The number C will be seen to be always the same, as it depends only on the dimensions of the instrument. Hence now again the area is determined by w if C is known.

Thus it is seen that the area generated by the motion of the rod can be measured by the roll of the wheel; it remains to show how any given area can be generated by the rod. Let the rod move in any manner but return to its original position. Q and T then describe closed curves. Such motion may be called cyclical. Here the theorem holds:—*If a rod QT performs a cyclical motion, then the area generated equals the difference of the areas enclosed by the paths of T and Q respectively.* The truth of this proposition will be seen from a figure. In fig. 11 different positions of the moving rod QT have been marked, and its motion can be easily followed. It will be seen that every part of the area TT'BB' will be passed over once and always by a *forward motion* of the rod, whereby the wheel will *increase* its roll. The area AA'QQ' will also be swept over once, but with a *backward* roll; it must therefore be counted as negative. The area between the curves is passed over twice, once with a forward and once with a backward roll; it therefore counts once positive and once negative; hence not at all. In more complicated figures it may happen that the area within one of the curves, say TT'BB', is passed over several times, but then it will be passed over once more in the forward direction than in the backward one, and thus the theorem will still hold.

To use Amsler's planimeter, place the pole O on the paper *outside* the figure to be measured. Then the area generated by QT is that of the figure, because the point Q moves on an arc of a circle to and fro enclosing no area. At the same time the rod comes back without making a complete rotation. We have therefore in formula (1), [alpha] = 0; and hence

P = lw,

[v.04 p.0977] which is read off. But if the area is too large the pole O may be placed within the area. The rod describes the area between the boundary of the figure and the circle with radius r = OQ, whilst the rod turns once completely round, making [alpha] = 2[pi]. The area measured by the wheel is by formula (1), lw + (1/2l squared-lc) 2[pi].

To this the area of the circle [pi]r squared must be added, so that now

P = lw + (1/2l squared-lc)2[pi] + [pi]r squared,

or

P = lw + C,

where

C = (1/2l squared-lc)2[pi] + [pi]r squared,

is a constant, as it depends on the dimensions of the instrument alone. This constant is given with each instrument.

Amsler's planimeters are made either with a rod QT of fixed length, which gives the area therefore in terms of a fixed unit, say in square inches, or else the rod can be moved in a sleeve to which the arm OQ is hinged (fig. 13). This makes it possible to change the unit lu, which is proportional to l.

In the planimeters described the recording or integrating apparatus is a smooth wheel rolling on the paper or on some other surface. Amsler has described another recorder, viz. a wheel with a sharp edge. This will roll on the paper but not slip. Let the rod QT carry with it an arm CD perpendicular to it. Let there be mounted on it a wheel W, which can slip along and turn about it. If now QT is moved parallel to itself to Q'T', then W will roll without slipping parallel to QT, and slip along CD. This amount of slipping will equal the perpendicular distance between QT and Q'T', and therefore serve to measure the area swept over like the wheel in the machine already described. The turning of the rod will also produce slipping of the wheel, but it will be seen without difficulty that this will cancel during a cyclical motion of the rod, provided the rod does not perform a whole rotation.

The first planimeter was made on the following principles:—A frame FF (fig. 15) can move parallel to OX. It carries a rod TT [Sidenote: Early forms.] movable along its own length, hence the tracer T can be guided along any curve ATB. When the rod has been pushed back to Q'Q, the tracer moves along the axis OX. On the frame a cone VCC' is mounted with its axis sloping so that its top edge is horizontal and parallel to TT', whilst its vertex V is opposite Q'. As the frame moves it turns the cone. A wheel W is mounted on the rod at T', or on an axis parallel to and rigidly connected with it. This wheel rests on the top edge of the cone. If now the tracer T, when pulled out through a distance y above Q, be moved parallel to OX through a distance dx, the frame moves through an equal distance, and the cone turns through an angle d[theta] proportional to dx. The wheel W rolls on the cone to an amount again proportional to dx, and also proportional to y, its distance from V. Hence the roll of the wheel is proportional to the area ydx described by the rod QT. As T is moved from A to B along the curve the roll of the wheel will therefore be proportional to the area AA'B'B. If the curve is closed, and the tracer moved round it, the roll will measure the area independent of the position of the axis OX, as will be seen by drawing a figure. The cone may with advantage be replaced by a horizontal disk, with its centre at V; this allows of y being negative. It may be noticed at once that the roll of the wheel gives at every moment the area A'ATQ. It will therefore allow of registering a set of values of [Integral,a:x] ydx for any values of x, and thus of tabulating the values of any indefinite integral. In this it differs from Amsler's planimeter. Planimeters of this type were first invented in 1814 by the Bavarian engineer Hermann, who, however, published nothing. They were reinvented by Prof. Tito Gonnella of Florence in 1824, and by the Swiss engineer Oppikofer, and improved by Ernst in Paris, the astronomer Hansen in Gotha, and others (see Henrici, *British Association Report*, 1894). But all were driven out of the field by Amsler's simpler planimeter.

Altogether different from the planimeters described is the hatchet planimeter, invented by Captain Prytz, a Dane, and made by Herr [Sidenote: Hatchet planimeters.] Cornelius Knudson in Copenhagen. It consists of a single rigid piece like fig. 16. The one end T is the tracer, the other Q has a sharp hatchet-like edge. If this is placed with QT on the paper and T is moved along any curve, Q will follow, describing a "curve of pursuit." In consequence of the sharp edge, Q can only move in the direction of QT, but the whole can turn about Q. Any small step forward can therefore be considered as made up of a motion along QT, together with a turning about Q. The latter motion alone generates an area. If therefore a line OA = QT is turning about a fixed point O, always keeping parallel to QT, it will sweep over an area equal to that generated by the more general motion of QT. Let now (fig. 17) QT be placed on OA, and T be guided round the closed curve in the sense of the arrow. Q will describe a curve OSB. It may be made visible by putting a piece of "copying paper" under the hatchet. When T has returned to A the hatchet has the position BA. A line turning from OA about O kept parallel to QT will describe the circular sector OAC, which is equal in magnitude and sense to AOB. This therefore measures the area generated by the motion of QT. To make this motion cyclical, suppose the hatchet turned about A till Q comes from B to O. Hereby the sector AOB is again described, and again in the positive sense, if it is remembered that it turns about the tracer T fixed at A. The whole area now generated is therefore twice the area of this sector, or equal to OA. OB, where OB is measured along the arc. According to the theorem given above, this area also equals the area of the given curve less the area OSBO. To make this area disappear, a slight modification of the motion of QT is required. Let the tracer T be moved, both from the first position OA and the last BA of the rod, along some straight line AX. Q describes curves OF and BH respectively. Now begin the motion with T at some point R on AX, and move it along this line to A, round the curve and back to R. Q will describe the curve DOSBED, if the motion is again made cyclical by turning QT with T fixed at A. If R is properly selected, the path of Q will cut itself, and parts of the area will be positive, parts negative, as marked in the figure, and may therefore be made to vanish. When this is done the area of the curve will equal twice the area of the sector RDE. It is therefore equal to the arc DE multiplied by the length QT; if the latter equals 10 in., then 10 times the number of inches contained in the arc DE gives the number of square inches contained within the given figure. If the area is not too large, the arc DE may be replaced by the straight line DE.

To use this simple instrument as a planimeter requires the possibility of selecting the point R. The geometrical theory here given has so far failed to give any rule. In fact, every line through any point in the curve contains such a point. The analytical theory of the inventor, which is very similar to that given by F.W. Hill (*Phil. Mag.* 1894), is too complicated to repeat here. The integrals expressing the area generated by QT have to be expanded in a series. By retaining only the most important terms a result is obtained which comes to this, that if the mass-centre of the area be taken as R, then A may be any point on the curve. This is only approximate. Captain Prytz gives the following instructions:—Take a point R as near as you can guess to the mass-centre, put the tracer T on it, the knife-edge Q outside; make a mark on the paper by pressing the knife-edge into it; guide the tracer from R along a straight line to a point A on the boundary, round the boundary, [v.04 p.0978] and back from A to R; lastly, make again a mark with the knife-edge, and measure the distance c between the marks; then the area is nearly cl, where l = QT. A nearer approximation is obtained by repeating the operation after turning QT through 180 deg. from the original position, and using the mean of the two values of c thus obtained. The greatest dimension of the area should not exceed 1/2l, otherwise the area must be divided into parts which are determined separately. This condition being fulfilled, the instrument gives very satisfactory results, especially if the figures to be measured, as in the case of indicator diagrams, are much of the same shape, for in this case the operator soon learns where to put the point R.

Integrators serve to evaluate a definite integral [Integral,a:b] f(x)dx. If we plot out [Sidenote: Integrators.] the curve whose equation is y = f(x), the integral [Integral]ydx between the proper limits represents the area of a figure bounded by the curve, the axis of x, and the ordinates at x=a, x=b. Hence if the curve is drawn, any planimeter may be used for finding the value of the integral. In this sense planimeters are integrators. In fact, a planimeter may often be used with advantage to solve problems more complicated than the determination of a mere area, by converting the one problem graphically into the other. We give an example:—

Let the problem be to determine for the figure ABG (fig. 18), not only the area, but also the first and second moment with regard to the axis XX. At a distance a draw a line, C'D', parallel to XX. In the figure draw a number of lines parallel to AB. Let CD be one of them. Draw C and D vertically upwards to C'D', join these points to some point O in XX, and mark the points C_1D_1 where OC' and OD' cut CD. Do this for a sufficient number of lines, and join the points C_1D_1 thus obtained. This gives a new curve, which may be called the first derived curve. By the same process get a new curve from this, the second derived curve. By aid of a planimeter determine the areas P, P_1, P_2, of these three curves. Then, if x is the distance of the mass-centre of the given area from XX; x_1 the same quantity for the first derived figure, and I = Ak squared the moment of inertia of the first figure, k its radius of gyration, with regard to XX as axis, the following relations are easily proved:—

Px = aP_1; P_1x_1 = aP_2; I = aP_1x_1 = a squaredP_1P_2; k squared = xx_1,

which determine P, x and I or k. Amsler has constructed an integrator which serves to determine these quantities by guiding a tracer once round the boundary of the given figure (see below). Again, it may be required to find the value of an integral [Integral]y[phi](x)dx between given limits where [phi](x) is a simple function like sin nx, and where y is given as the ordinate of a curve. The harmonic analysers described below are examples of instruments for evaluating such integrals.

Amsler has modified his planimeter in such a manner that instead of the area it gives the first or second moment of a figure about an axis in its plane. An instrument giving all three quantities simultaneously is known as Amsler's integrator or moment-planimeter. It has one tracer, but three recording wheels. It is mounted on a [Sidenote: Amsler's Integrator.] carriage which runs on a straight rail (fig. 19). This carries a horizontal disk A, movable about a vertical axis Q. Slightly more than half the circumference is circular with radius 2a, the other part with radius 3a. Against these gear two disks, B and C, with radii a; their axes are fixed in the carriage. From the disk A extends to the left a rod OT of length l, on which a recording wheel W is mounted. The disks B and C have also recording wheels, W*1 and W*2, the axis of W*1 being perpendicular, that of W*2 parallel to OT. If now T is guided round a figure F, O will move to and fro in a straight line. This part is therefore a simple planimeter, in which the one end of the arm moves in a straight line instead of in a circular arc. Consequently, the "roll" of W will record the area of the figure. Imagine now that the disks B and C also receive arms of length l from the centres of the disks to points T*1 and T*2, and in the direction of the axes of the wheels. Then these arms with their wheels will again be planimeters. As T is guided round the given figure F, these points T*1 and T*2 will describe closed curves, F*1 and F*2, and the "rolls" of W*1 and W*2 will give their areas A*1 and A*2. Let XX (fig. 20) denote the line, parallel to the rail, on which O moves; then when T lies on this line, the arm BT*1 is perpendicular to XX, and CT*2 parallel to it. If OT is turned through an angle [theta], clockwise, BT*1 will turn counter-clockwise through an angle 2[theta], and CT*2 through an angle 3[theta], also counter-clockwise. If in this position T is moved through a distance x parallel to the axis XX, the points T*1 and T*2 will move parallel to it through an equal distance. If now the first arm is turned through a small angle d[theta], moved back through a distance x, and lastly turned back through the angle d[theta], the tracer T will have described the boundary of a small strip of area. We divide the given figure into [v.04 p.0979] such strips. Then to every such strip will correspond a strip of equal length x of the figures described by T*1 and T*2.

The distances of the points, T, T*1, T*2, from the axis XX may be called y, y*1, y*2. They have the values

y = l sin [theta], y*1 = l cos 2[theta], y*2 = -l sin 3[theta],

from which

dy = l cos [theta].d[theta], dy*1 = - 2l sin 2[theta].d[theta], dy*2 = - 3l cos 3[theta].d[theta].

The areas of the three strips are respectively

dA = xdy, dA*1 = xdy*1, dA*2 = xdy*2.

Now dy*1 can be written dy*1 = - 4l sin [theta] cos [theta]d[theta] = - 4 sin [theta]dy; therefore

dA_1 = - 4 sin [theta].dA = - (4/l) ydA;

whence

A_1 = - 4/l [Integral]ydA = - 4/l Aȳ,

where A is the area of the given figure, and ȳ the distance of its mass-centre from the axis XX. But A_1 is the area of the second figure F_1, which is proportional to the reading of W_1. Hence we may say

Aȳ = C*1w*1,

where C_1 is a constant depending on the dimensions of the instrument. The negative sign in the expression for A_1 is got rid of by numbering the wheel W_1 the other way round.

Again

dy_2 = - 3l cos [theta] {4 cos squared [theta] - 3} d[theta] = - 3 {4 cos squared [theta] - 3} dy

= - 3 {(4/l squared) y squared - 3} dy,

which gives

dA_2 = - (12/l squared)y squareddA + 9dA,

and

A_2 = - (12/l squared) [Integral]y squareddA + 9A.

But the integral gives the moment of inertia I of the area A about the axis XX. As A*2 is proportional to the roll of W*2, A to that of W, we can write

I = Cw - C_2 w_2, Aȳ = C_1 w_1, A = C_c w.

If a line be drawn parallel to the axis XX at the distance ȳ, it will pass through the mass-centre of the given figure. If this represents the section of a beam subject to bending, this line gives for a proper choice of XX the neutral fibre. The moment of inertia for it will be I + Aȳ squared. Thus the instrument gives at once all those quantities which are required for calculating the strength of the beam under bending. One chief use of this integrator is for the calculation of the displacement and stability of a ship from the drawings of a number of sections. It will be noticed that the length of the figure in the direction of XX is only limited by the length of the rail.

This integrator is also made in a simplified form without the wheel W_2. It then gives the area and first moment of any figure.

While an integrator determines the value of a definite integral, hence a [Sidenote: Integraphs.] mere constant, an integraph gives the value of an indefinite integral, which is a function of x. Analytically if y is a given function f(x) of x and

Y = [Integral,c:x]ydx or Y = [Integral]ydx + const.

the function Y has to be determined from the condition

dY/dx = y.

Graphically y = f(x) is either given by a curve, or the graph of the equation is drawn: y, therefore, and similarly Y, is a length. But dY/dx is in this case a mere number, and cannot equal a length y. Hence we introduce an arbitrary constant length a, the unit to which the integraph draws the curve, and write

dY/dx = y/a and aY = [Integral]ydx.

Now for the Y-curve dY/dx = tan [phi], where [phi] is the angle between the tangent to the curve, and the axis of x. Our condition therefore becomes

tan [phi] = y / a.

This [phi] is easily constructed for any given point on the y-curve:—From the foot B' (fig. 21) of the ordinate y = B'B set off, as in the figure, B'D = a, then angle BDB' = [phi]. Let now DB' with a perpendicular B'B move along the axis of x, whilst B follows the y-curve, then a pen P on B'B will describe the Y-curve provided it moves at every moment in a direction parallel to BD. The object of the integraph is to draw this new curve when the tracer of the instrument is guided along the y-curve.

The first to describe such instruments was Abdank-Abakanowicz, who in 1889 published a book in which a variety of mechanisms to obtain the object in question are described. Some years later G. Coradi, in Zuerich, carried out his ideas. Before this was done, C.V. Boys, without knowing of Abdank-Abakanowicz's work, actually made an integraph which was exhibited at the Physical Society in 1881. Both make use of a sharp edge wheel. Such a wheel will not slip sideways; it will roll forwards along the line in which its plane intersects the plane of the paper, and while rolling will be able to turn gradually about its point of contact. If then the angle between its direction of rolling and the x-axis be always equal to [phi], the wheel will roll along the Y-curve required. The axis of x is fixed only in direction; shifting it parallel to itself adds a constant to Y, and this gives the arbitrary constant of integration.

In fact, if Y shall vanish for x = c, or if

Y = [Integral,c:x]ydx,

then the axis of x has to be drawn through that point on the y-curve which corresponds to x = c.

In Coradi's integraph a rectangular frame F*1F*2F*3F*4 (fig. 22) rests with four rollers R on the drawing board, and can roll freely in the direction OX, which will be called the axis of the instrument. On the front edge F*1F*2 travels a carriage AA' supported at A' on another rail. A bar DB can turn about D, fixed to the frame in its axis, and slide through a point B fixed in the carriage AA'. Along it a block K can slide. On the back edge F*3F*4 of the frame another carriage C travels. It holds a vertical spindle with the knife-edge wheel at the bottom. At right angles to the plane of the wheel, the spindle has an arm GH, which is kept parallel to a [v.04 p.0980] similar arm attached to K perpendicular to DB. The plane of the knife-edge wheel r is therefore always parallel to DB. If now the point B is made to follow a curve whose y is measured from OX, we have in the triangle BDB', with the angle [phi] at D,

tan [phi] = y/a,

where a = DB' is the constant base to which the instrument works. The point of contact of the wheel r or any point of the carriage C will therefore always move in a direction making an angle [phi] with the axis of x, whilst it moves in the x-direction through the same distance as the point B on the y-curve—that is to say, it will trace out the integral curve required, and so will any point rigidly connected with the carriage C. A pen P attached to this carriage will therefore draw the integral curve. Instead of moving B along the y-curve, a tracer T fixed to the carriage A is guided along it. For using the instrument the carriage is placed on the drawing-board with the front edge parallel to the axis of y, the carriage A being clamped in the central position with A at E and B at B' on the axis of x. The tracer is then placed on the x-axis of the y-curve and clamped to the carriage, and the instrument is ready for use. As it is convenient to have the integral curve placed directly opposite to the y-curve so that corresponding values of y or Y are drawn on the same line, a pen P' is fixed to C in a line with the tracer.

Boys' integraph was invented during a sleepless night, and during the following days carried out as a working model, which gives highly satisfactory results. It is ingenious in its simplicity, and a direct realization as a mechanism of the principles explained in connexion with fig. 21. The line B'B is represented by the edge of an ordinary T-square sliding against the edge of a drawing-board. The points B and P are connected by two rods BE and EP, jointed at E. At B, E and P are small pulleys of equal diameters. Over these an endless string runs, ensuring that the pulleys at B and P always turn through equal angles. The pulley at B is fixed to a rod which passes through the point D, which itself is fixed in the T-square. The pulley at P carries the knife-edge wheel. If then B and P are kept on the edge of the T-square, and B is guided along the curve, the wheel at P will roll along the Y-curve, it having been originally set parallel to BD. To give the wheel at P sufficient grip on the paper, a small loaded three-wheeled carriage, the knife-edge wheel P being one of its wheels, is added. If a piece of copying paper is inserted between the wheel P and the drawing paper the Y-curve is drawn very sharply.

Integraphs have also been constructed, by aid of which ordinary differential equations, especially linear ones, can be solved, the solution being given as a curve. The first suggestion in this direction was made by Lord Kelvin. So far no really useful instrument has been made, although the ideas seem sufficiently developed to enable a skilful instrument-maker to produce one should there be sufficient demand for it. Sometimes a combination of graphical work with an integraph will serve the purpose. This is the case if the variables are separated, hence if the equation

Xdx + Ydy = 0

has to be integrated where X = p(x), Y = [phi](y) are given as curves. If we write

au = [Integral]Xdx, av = [Integral]Ydy,

then u as a function of x, and v as a function of y can be graphically found by the integraph. The general solution is then

u + v = c

with the condition, for the determination for c, that y = y*0, for x = x*0. This determines c = u*0 + v*0, where u*0 and v*0 are known from the graphs of u and v. From this the solution as a curve giving y a function of x can be drawn:—For any x take u from its graph, and find the y for which v = c - u, plotting these y against their x gives the curve required.

If a periodic function y of x is given by its graph for one period c, it can, according to the theory of Fourier's Series, be [Sidenote: Harmonic analysers.] expanded in a series.

$$y = A_0 + A_1 \cos\theta + A_2 \cos 2\theta + \dots + A_n \cos n\theta + \dots + B_1 \sin\theta + B_2 \sin 2\theta + \dots + B_n \sin n\theta + \dots$$

where $\theta = 2\pi x / c$.

The absolute term A_0 equals the mean ordinate of the curve, and can therefore be determined by any planimeter. The other co-efficients are

$$A_n = \frac{1}{\pi}\int_0^{2\pi} y \cos n\theta \, d\theta; \quad B_n = \frac{1}{\pi}\int_0^{2\pi} y \sin n\theta \, d\theta.$$

A harmonic analyser is an instrument which determines these integrals, and is therefore an integrator. The first instrument of this kind is due to Lord Kelvin (*Proc. Roy Soc.*, vol xxiv., 1876). Since then several others have been invented (see Dyck's *Catalogue*; Henrici, *Phil. Mag.*, July 1894; *Phys. Soc.*, 9th March; Sharp, *Phil. Mag.*, July 1894; *Phys. Soc.*, 13th April). In Lord Kelvin's instrument the curve to be analysed is drawn on a cylinder whose circumference equals the period *c*, and the sine and cosine terms of the integral are introduced by aid of simple harmonic motion. Sommerfeld and Wiechert, of Konigsberg, avoid this motion by turning the cylinder about an axis perpendicular to that of the cylinder. Both these machines are large, and practically fixtures in the room where they are used. The first has done good work in the Meteorological Office in London in the analysis of meteorological curves. Quite different and simpler constructions can be used, if the integrals determining A_n and B_n be integrated by parts. This gives

$$nA_n = -\frac{1}{\pi}\int_0^{2\pi} \sin n\theta \, dy; \quad nB_n = \frac{1}{\pi}\int_0^{2\pi} \cos n\theta \, dy.$$

An analyser presently to be described, based on these forms, has been constructed by Coradi in Zurich (1894). Lastly, a most powerful analyser has been invented by Michelson and Stratton (U.S.A.) (*Phil Mag.*, 1898), which will also be described.

The *Henrici-Coradi* analyser has to add up the values of $dy.\sin n\theta$ and $dy.\cos n\theta$. But these are the components of dy in two directions perpendicular to each other, of which one makes an angle $n\theta$ with the axis of x or of θ. This decomposition can be performed by Amsler's registering wheels. Let two of these be mounted, perpendicular to each other, in one horizontal frame which can be turned about a vertical axis, the wheels resting on the paper on which the curve is drawn. When the tracer is placed on the curve at the point $\theta = 0$ the one axis is parallel to the axis of θ. As the tracer follows the curve the frame is made to turn through an angle $n\theta$. At the same time the frame moves with the tracer in the direction of y. For a small motion the two wheels will then register just the components required, and during the continued motion of the tracer along the curve the wheels will add these components, and thus give the values of nA_n and nB_n. The factors $1/\pi$ and $-1/\pi$ are taken account of in the graduation of the wheels. The readings have then to be divided by n to give the coefficients required. Coradi's realization of this idea will be understood from fig. 23. The frame PP' of the instrument rests on three rollers E, E', and D. The first two drive an axis with a disk C on it. It is placed parallel to the axis of x of the curve. The tracer is attached to a carriage WW which runs on the rail P. As it follows the curve this carriage moves through a distance x whilst the whole instrument runs forward through a distance y. The wheel C turns through an angle proportional, during each small motion, to dy. On it rests a glass sphere which will therefore also turn about its horizontal axis proportionally, to dy. The registering frame is suspended by aid of a spindle S, having a disk H. It is turned by aid of a wire connected with the carriage WW, and turns n times round as the tracer describes the whole length of the curve. The registering wheels R, R' rest against the glass sphere and give the values nA_n and nB_n. The value of n can be altered by changing the disk H into one of different diameter. It is also possible to mount on the same frame a number of spindles with registering wheels and glass spheres, each of the latter resting on a separate disk C. As many as five have been introduced. One guiding of the tracer over the curve gives then at once the ten coefficients A_n and B_n for n = 1 to 5.

All the calculating machines and integrators considered so far have been kinematic. We have now to describe a most remarkable instrument based on the equilibrium of a rigid body under the action of springs. The body itself for rigidity's sake is made a hollow [v.04 p.0981] [Sidenote: Michelson and Stratton analyzer] cylinder H, shown in fig. 24 in end view. It can turn about its axis, being supported on knife-edges O. To it springs are attached at the prolongation of a horizontal diameter; to the left a series of n small springs s, all alike, side by side at equal intervals at a distance a from the axis of the knife-edges; to the right a single spring S at distance b. These springs are supposed to follow Hooke's law. If the elongation beyond the natural length of a spring is [lambda], the force asserted by it is p = k[lambda]. Let for the position of equilibrium l, L be respectively the elongation of a small and the large spring, k, K their constants, then

nkla = KLb.

The position now obtained will be called the _normal_ one. Now let the top ends C of the small springs be raised through distances y_1, y_2, ... y_n. Then the body H will turn; B will move down through a distance z and A up through a distance (a/b)z. The new forces thus introduced will be in equilibrium if

ak([Sigma]y - n (a/b) z) = bKz.

Or

z = [Sigma]y / (n a/b + b/a K/k) = [Sigma]y / (n (a/b + l/L)).

This shows that the displacement z of B is proportional to the sum of the displacements y of the tops of the small springs. The arrangement can therefore be used for the addition of a number of displacements. The instrument made has eighty small springs, and the authors state that from the experience gained there is no impossibility of increasing their number even to a thousand. The displacement z, which necessarily must be small, can be enlarged by aid of a lever OT'. To regulate the displacements y of the points C (fig. 24) each spring is attached to a lever EC, fulcrum E. To this again a long rod FG is fixed by aid of a joint at F. The lower end of this rod rests on another lever GP, fulcrum N, at a changeable distance y" = NG from N. The elongation y of any spring s can thus be produced by a motion of P. If P be raised through a distance y', then the displacement y of C will be proportional to y'y"; it is, say, equal to [mu] y'y" where [mu] is the same for all springs. Now let the points C, and with it the springs s, the levers, &c., be numbered C_0, C_1, C_2 ... There will be a zero-position for the points P all in a straight horizontal line. When in this position the points C will also be in a line, and this we take as axis of x. On it the points C_0, C_1, C_2 ... follow at equal distances, say each equal to h. The point C_k lies at the distance kh which gives the x of this point. Suppose now that the rods FG are all set at unit distance NG from N, and that the points P be raised so as to form points in a continuous curve y' = [phi](x), then the points C will lie in a curve y = [mu] [phi](x). The area of this curve is

[mu] [Integral,0:c][phi](x)dx.

Approximately this equals [Sigma]hy = h[Sigma]y. Hence we have

[Integral,0:c][phi](x)dx = h/[mu] [Sigma]y = ([lambda]h/[mu])z,

where z is the displacement of the point B which can be measured. The curve y' = [phi](x) may be supposed cut out as a templet. By putting this under the points P the area of the curve is thus determined—the instrument is a simple integrator.

The integral can be made more general by varying the distances NG = y". These can be set to form another curve y" = f(x). We have now y = [mu]y'y" = [mu] f(x) [phi](x), and get as before

[Integral,0:c]f(x) [phi](x)dx = ([lambda]h/[mu])z.

These integrals are obtained by the addition of ordinates, and therefore by an approximate method. But the ordinates are numerous, there being 79 of them, and the results are in consequence very accurate. The displacement z of B is small, but it can be magnified by taking the reading of a point T' on the lever AB. The actual reading is done at point T connected with T' by a long vertical rod. At T either a scale can be placed or a drawing-board, on which a pen at T marks the displacement.

If the points G are set so that the distances NG on the different levers are proportional to the terms of a numerical series

$u_0 + u_1 + u_2 + ...$

and if all P be moved through the same distance, then z will be proportional to the sum of this series up to 80 terms. We get an *Addition Machine*.

The use of the machine can, however, be still further extended. Let a templet with a curve $y' = \phi(\xi)$ be set under each point P at right angles to the axis of x hence parallel to the plane of the figure. Let these templets form sections of a continuous surface, then each section parallel to the axis of x will form a curve like the old $y' = \phi(x)$, but with a variable parameter ξ, or $y' = \phi(\xi, x)$. For each value of ξ the displacement of T will give the integral

$$Y = \int_0^c f(x)\, \phi(\xi x)\, dx = F(\xi), \ . \ . \ . \ (1)$$

where Y equals the displacement of T to some scale dependent on the constants of the instrument.

If the whole block of templets be now pushed under the points P and if the drawing-board be moved at the same rate, then the pen T will draw the curve $Y = F(\xi)$. The instrument now is an *integraph* giving the value of a definite integral as function of a *variable parameter*.

Having thus shown how the lever with its springs can be made to serve a variety of purposes, we return to the description of the actual instrument constructed. The machine serves first of all to sum up a series of harmonic motions or to draw the curve

$$Y = a_1 \cos x + a_2 \cos 2x + a_3 \cos 3x + \ . \ . \ . \ (2)$$

The motion of the points $P_1 P_2$ ... is here made harmonic by aid of a series of excentric disks arranged so that for one revolution of the first the other disks complete 2, 3, ... revolutions. They are all driven by one handle. These disks take the place of the templets described before. The distances NG are made equal to the amplitudes a_1, a_2, a_3, ... The drawing-board, moved forward by the turning of the handle, now receives a curve of which (2) is the equation. If all excentrics are turned through a right angle a sine-series can be added up.

It is a remarkable fact that the same machine can be used as a harmonic analyser of a given curve. Let the curve to be analysed be set off along the levers NG so that in the old notation it is

$$y'' = f(x),$$

whilst the curves $y' = \phi(x\xi)$ are replaced by the excentrics, hence ξ by the angle θ through which the first excentric is turned, so that $y'_k = \cos k\theta$. But $kh = x$ and $nh = \pi$, n being the number of springs s, and π taking the place of c. This makes

$$k\theta = (n/\pi)\theta . x.$$

Hence our instrument draws a curve which gives the integral (1) in the form

$$y = \frac{2}{\pi} \int_0^{\pi} f(x) \cos((n/\pi)\theta x)\, dx$$

as a function of [theta]. But this integral becomes the coefficient a_m in the cosine expansion if we make

[theta]n/[pi] = m or [theta] = m[pi]/n.

The ordinates of the curve at the values [theta] = [pi]/n, 2[pi]/n, ... give therefore all coefficients up to m = 80. The curve shows at a glance which and how many of the coefficients are of importance.

The instrument is described in _Phil. Mag._, vol. xlv., 1898. A number of curves drawn by it are given, and also examples of the analysis of curves for which the coefficients a_m are known. These indicate that a remarkable accuracy is obtained.

(O. H.)

[1] For a fuller description of the manner in which a mere addition machine can be used for multiplication and division, and even for the extraction of square roots, see an article by C.V. Boys in *Nature*, 11th July 1901.

CALCUTTA, the capital of British India and also of the province of Bengal. It is situated in 22 deg. 34' N. and 88 deg. 24' E., on the left or east bank of the Hugli, about 80 m. from the sea. Including its suburbs it covers an area of 27,267 acres, and contains a population (1901) of 949,144. Calcutta and Bombay have long contested the position of the premier city of India in population and trade; but during the decade 1891-1901 the prevalence of plague in Bombay gave a considerable advantage to Calcutta, which was comparatively free from that disease. Calcutta lies only some 20 ft. above sea-level, and extends about 6 m. along the Hugli, and is bounded elsewhere by the Circular Canal and the Salt Lakes, and by suburbs which form separate municipalities. Fort William stands in its centre.

Public Buildings.—Though Calcutta was called by Macaulay "the city of palaces," its modern public buildings cannot compare with those of Bombay. Its chief glory is the Maidan or park, which is large enough to embrace the area of Fort William and a racecourse. Many monuments find a place on the Maidan, among them being modern equestrian statues of Lord Roberts and Lord Lansdowne, which face one another on each side of the Red Road, where the rank and [v.04 p.0982] fashion of Calcutta take their evening drive. In the north-eastern corner of the Maidan the Indian memorial to Queen Victoria, consisting of a marble hall, with a statue and historical relics, was opened by the prince of Wales in January 1906. The government acquired Metcalfe Hall, in order to convert it into a public library and reading-room worthy of the capital of India; and also the country-house of Warren Hastings at Alipur, for the entertainment of Indian princes. Lord Curzon restored, at his own cost, the monument which formerly commemorated the massacre of the Black Hole, and a tablet let into the wall of the general post office indicates the position of the Black Hole in the north-east bastion of Fort William, now occupied by the roadway. Government House, which is situated near the Maidan and Eden Gardens, is the residence of the viceroy; it was built by Lord Wellesley in 1799, and is a fine pile situated in grounds covering six acres, and modelled upon Kedleston Hall in Derbyshire, one of the Adam buildings. Belvedere House, the official residence of the lieutenant-governor of Bengal, is situated close to the botanical gardens in Alipur, the southern suburb of Calcutta. Facing the Maidan for a couple of miles is the Chowringhee, one of the famous streets of the world, once a row of palatial residences, but now given up almost entirely to hotels, clubs and shops.

Commerce.—Calcutta owes its commercial prosperity to the fact that it is situated near the mouth of the two great river systems of the Ganges and Brahmaputra. It thus receives the produce of these fertile river valleys, while the rivers afford a cheaper mode of conveyance than any railway. In addition Calcutta is situated midway between Europe and the Far East and thus forms a meeting-place for the commerce and peoples of the Eastern and Western worlds. The port of Calcutta is one of the busiest in the world, and the banks of the Hugli rival the port of London in their show of shipping. The total number of arrivals and departures during 1904-1905 was 3027 vessels with an average tonnage of 3734. But though the city is such a busy commercial centre, most of its industries are carried on outside municipal limits. Howrah, on the opposite side of the Hugli, is the terminus of three great railway systems, and also the headquarters of the jute industry and other large factories. It is connected with

Calcutta by an immense floating bridge, 1530 ft. in length, which was constructed in 1874. Other railways have their terminus at Sealdah, an eastern suburb. The docks lie outside Calcutta, at Kidderpur, on the south; and at Alipur are the zoological gardens, the residence of the lieutenant-governor of Bengal, cantonments for a native infantry regiment, the central gaol and a government reformatory. The port of Calcutta stretches about 10 m. along the river. It is under the control of a port trust, whose jurisdiction extends to the mouth of the Hugli and also over the floating bridge. New docks were opened in 1892, which cost upwards of two millions sterling. The figures for the sea-borne trade of Calcutta are included in those of Bengal. Its inland trade is carried on by country boat, inland steamer, rail and road, and amounted in 1904-1905 to about four and three quarter millions sterling. More than half the total is carried by the East Indian railway, which serves the United Provinces. Country boats hold their own against inland steamers, especially in imports.

Municipality.—The municipal government of Calcutta was reconstituted by an act of the Bengal legislature, passed in 1899. Previously, the governing body consisted of seventy-five commissioners, of whom fifty were elected. Under the new system modelled upon that of the Bombay municipality, this body, styled the corporation, remains comparatively unaltered; but a large portion of their powers is transferred to a general committee, composed of twelve members, of whom one-third are elected by the corporation, one-third by certain public bodies and one-third are nominated by the government. At the same time, the authority of the chairman, as supreme executive officer, is considerably strengthened. The two most important works undertaken by the old municipality were the provision of a supply of filtered water and the construction of a main drainage system. The water-supply is derived from the river Hugli, about 16 m. above Calcutta, where there are large pumping-stations and settling-tanks. The drainage-system consists of underground sewers, which are discharged by a pumping-station into a natural depression to the eastward, called the Salt Lake. Refuse is also removed to the Salt Lake by means of a municipal railway.

Education.—The Calcutta University was constituted in 1857, as an examining body, on the model of the university of London. The chief educational institutions are the Government Presidency College; three aided missionary colleges, and four unaided native colleges; the Sanskrit College and the Mahommedan Madrasah; the government medical college, the government engineering college at Sibpur, on the opposite bank of the Hugli, the government school of art, high schools for boys, the Bethune College and high schools for girls.

Population.—The population of Calcutta in 1710 was estimated at 12,000, from which figure it rose to about 117,000 in 1752. In the census of 1831 it was 187,000, in 1839 it had become 229,000 and in 1901, 949,144. Thus in the century between 1801 and 1901 it increased sixfold, while during the same period London only increased fivefold. Out of the total population of town and suburbs in 1901, 615,000 were Hindus, 286,000 Mahommedans and 38,000 Christians.

Climate and Health.—The climate of the city was originally very unhealthy, but it has improved greatly of recent years with modern sanitation and drainage. The climate is hot and damp, but has a pleasant cold season from November to March. April, May and June are hot; and the monsoon months from June to October are distinguished by damp heat and malaria. The mean annual temperature is 79 deg. F., with a range from 85 deg. in the hot season and 83 deg. in the rains to 72 deg. in the cool season, a mean maximum of 102 deg. in May and a mean minimum of 48 deg. in January. Calcutta has been comparatively fortunate in escaping the plague. The disease manifested itself in a sporadic form in April 1898, but disappeared by September of that year. Many of the Marwari traders fled the city, and some trouble was experienced in shortage of labour in the factories and at the docks. The plague returned in 1899 and caused a heavy mortality during the early months of the following year; but the population was not demoralized, nor was trade interfered with. A yet more serious outbreak occurred in the early months of 1901, the number of deaths being 7884. For three following years the totals were (1902-1903) 7284; (1903-1904) 8223; and (1904-1905) 4689; but these numbers compared very favourably with the condition of Bombay at the same time.

History.—The history of Calcutta practically dates from the 24th of August 1690, when it was founded by Job Charnock (*q.v.*) of the English East India Company. In 1596 it had obtained a brief entry as a rent-paying village in the survey of Bengal executed by command of the emperor Akbar. But it was not till ninety years later that it emerged into history. In 1686 the

English merchants at Hugli under Charnock's leadership, finding themselves compelled to quit their factory in consequence of a rupture with the Mogul authorities, retreated about 26 m. down the river to Sutanati, a village on the banks of the Hugli, now within the boundaries of Calcutta. They occupied Sutanati temporarily in December 1686, again in November 1687 and permanently on the 24th of August 1690. It was thus only at the third attempt that Charnock was able to obtain the future capital of India for his centre and the subsequent prosperity of Calcutta is due entirely to his tenacity of purpose. The new settlement soon extended itself along the river bank to the then village of Kalikata, and by degrees the cluster of neighbouring hamlets grew into the present town. In 1696 the English built the original Fort William by permission of the nawab, and in 1698 they formally purchased the three villages of Sutanati, Kalikata and Govindpur from Prince Azim, son of the emperor Aurangzeb.

The site thus chosen had an excellent anchorage and was defended by the river from the Mahrattas, who harried the districts on the other side. The fort, subsequently rebuilt on the Vauban principle, and a moat, designed to form a semicircle [v.04 p.0983] round the town, and to be connected at both ends with the river, but never completed, combined with the natural position of Calcutta to render it one of the safest places for trade in India during the expiring struggles of the Mogul empire. It grew up without any fixed plan, and with little regard to the sanitary arrangements required for a town. Some parts of it lay below high-water mark on the Hugli, and its low level throughout rendered its drainage a most difficult problem. Until far on in the 18th century the malarial jungle and paddy fields closely hemmed in the European mansions; the vast plain (*maidan*), now covered with gardens and promenades, was then a swamp during three months of each year; the spacious quadrangle known as Wellington Square was built upon a filthy creek. A legend relates how one-fourth of the European inhabitants perished in twelve months, and during seventy years the mortality was so great that the name of Calcutta, derived from the village of Kalikata, was identified by mariners with Golgotha, the place of a skull.

The chief event in the history of Calcutta is the sack of the town, and the capture of Fort William in 1756, by Suraj-ud-Dowlah, the nawab of Bengal. The majority of the English officials took ship and fled to the mouth of the Hugli river. The Europeans, under John Zephaniah Holwell, who remained were compelled, after a short resistance, to surrender themselves to the mercies of the young prince. The prisoners, numbering 146 persons, were forced into the guard-room, a chamber measuring only 18 ft. by 14 ft. 10 in., with but two small windows, where they were left for the night. It was the 20th of June; the heat was intense; and next morning only 23 were taken out alive, among them Holwell, who left an account of the awful sufferings endured in the "Black Hole." The site of the Black Hole is now covered with a black marble slab, and the incident is commemorated by a monument erected by Lord Curzon in 1902. The Mahommedans retained possession of Calcutta for about seven months, and during this brief period the name of the town was changed in official documents to Alinagar. In January 1757 the expedition despatched from Madras, under the command of Admiral Watson and Colonel Clive, regained possession of the city. They found many of the houses of the English residents demolished and others damaged by fire. The old church of St John lay in ruins. The native portion of the town had also suffered much. Everything of value had been swept away, except the merchandise of the Company within the fort, which had been reserved for the nawab. The battle of Plassey was fought on the 23rd of June 1757, exactly twelve months after the capture of Calcutta. Mir Jafar, the nominee of the English, was created nawab of Bengal, and by the treaty which raised him to this position he agreed to make restitution to the Calcutta merchants for their losses. The English received L500,000, the Hindus and Mahommedans L200,000, and the Armenians L70,000. By another clause in this treaty the Company was permitted to establish a mint, the visible sign in India of territorial sovereignty, and the first coin, still bearing the name of the Delhi emperor, was issued on the 19th of August 1757. The restitution money was divided among the sufferers by a committee of the most respectable inhabitants. Commerce rapidly revived and the ruined city was rebuilt. Modern Calcutta dates from 1757. The old fort was abandoned, and its site devoted to the custom-house and other government offices. A new fort, the present Fort William, was begun by Clive a short distance lower down the river, and is thus the second of that name. It was not finished till 1773, and is said to have cost two millions sterling. At this time also the *maidan*, the park of Calcutta, was formed; and the healthiness of its position induced the European inhabitants gradually to shift their dwellings eastward, and to occupy what is now the Chowringhee quarter.

Up to 1707, when Calcutta was first declared a presidency, it had been dependent upon the older English settlement at Madras. From 1707 to 1773 the presidencies were maintained on a footing of equality; but in the latter year the act of parliament was passed, which provided that the presidency of Bengal should exercise a control over the other possessions of the Company; that the chief of that presidency should be styled governor-general; and that a supreme court of judicature should be established at Calcutta. In the previous year, 1772, Warren Hastings had taken under the immediate management of the Company's servants the general administration of Bengal, which had hitherto been left in the hands of the old Mahommedan officials, and had removed the treasury from Murshidabad to Calcutta. The latter town thus became the capital of Bengal and the seat of the supreme government in India. In 1834 the governor-general of Bengal was created governor-general of India, and was permitted to appoint a deputy-governor to manage the affairs of Lower Bengal during his occasional absence. It was not until 1854 that a separate head was appointed for Bengal, who, under the style of lieutenant-governor, exercises the same powers in civil matters as those vested in the governors in council of Madras or Bombay, although subject to closer supervision by the supreme government. Calcutta is thus at present the seat both of the supreme and the local government, each with an independent set of offices. (See BENGAL.)

See A.K. Ray, *A Short History of Calcutta* (Indian Census, 1901); H.B. Hyde, *Parochial Annals of Bengal* (1901); K. Blechynden, *Calcutta, Past and Present* (1905); H.E. Busteed, *Echoes from Old Calcutta* (1897); G.W. Forrest, *Cities of India* (1903); C.R. Wilson, *Early Annals of the English in Bengal* (1895); and *Old Fort William in Bengal* (1906); *Imperial Gazetteer of India* (Oxford, 1908), *s.v.* "Calcutta."

CALDANI, LEOPOLDO MARCO ANTONIO (1725-1813), Italian anatomist and physician, was born at Bologna in 1725. After studying under G.B. Morgagni at Padua, he began to teach practical medicine at Bologna, but in consequence of the intrigues of which he was the object he returned to Padua, where in 1771 he succeeded Morgagni in the chair of anatomy. He continued to lecture until 1805 and died at Padua in 1813. His works include *Institutiones pathologicae* (1772), *Institutiones physiologicae* (1773) and *Icones anatomicae* (1801-1813).

His brother, PETRONIO MARIA CALDANI (1735-1808), was professor of mathematics at Bologna, and was described by J. le R. D'Alembert as the "first geometer and algebraist of Italy."

CALDECOTT, RANDOLPH (1846-1886), English artist and illustrator, was born at Chester on the 22nd of March 1846. From 1861 to 1872 he was a bank clerk, first at Whitchurch in Shropshire, afterwards at Manchester; but devoted all his spare time to the cultivation of a remarkable artistic faculty. In 1872 he migrated to London, became a student at the Slade School and finally adopted the artist's profession. He gained immediately a wide reputation as a prolific and original illustrator, gifted with a genial, humorous faculty, and he succeeded also, though in less degree, as a painter and sculptor. His health gave way in 1876, and after prolonged suffering he died in Florida on the 12th of February 1886. His chief book illustrations are as follows:—*Old Christmas* (1876) and *Bracebridge Hall* (1877), both by Washington Irving; *North Italian Folk* (1877), by Mrs Comyns Carr; *The Harz Mountains* (1883); *Breton Folk* (1879), by Henry Blackburn; picture-books (*John Gilpin, The House that Jack Built*, and other children's favourites) from 1878 onwards; *Some Aesop's Fables with Modern Instances, &c.* (1883). He held a roving commission for the *Graphic*, and was an occasional contributor to *Punch*. He was a member of the Royal Institute of Painters in Water-colours.

See Henry Blackburn, *Randolph Caldecott, Personal Memoir of his Early Life* (London, 1886).

CALDER, SIR ROBERT, Bart. (1745-1818), British admiral, was born at Elgin, in Scotland, on the 2nd of July 1745 (o.s.). He belonged to a very ancient family of Morayshire, and was the second son of Sir Thomas Calder of Muirton. He was educated at the grammar school of Elgin, and at the age of fourteen entered the British navy as midshipman. In 1766 he was serving as lieutenant of the "Essex," under Captain the Hon. George Faulkner, in the West Indies. Promotion came slowly, and it was not till 1782 that he attained the rank of post-captain. He acquitted himself honourably in the various services to which he was called, but for a long time had no opportunity [v.04 p.0984] of distinguishing himself. In 1796 he was named captain of the fleet by Sir John Jervis, and took part in the great battle off Cape St Vincent (February 14, 1797). He was selected as bearer of the despatches announcing the victory, and on that occasion was knighted by George III. He also received the thanks of parliament, and in the following year was created a baronet. In 1799 he became rear-admiral; and in 1801 he was despatched with a small squadron in pursuit of a French force, under Admiral Gantheaume, conveying supplies to the French in Egypt. In this pursuit he was not successful, and returning home at the peace he struck his flag. When the war again broke out he was recalled to service, was promoted vice-admiral in 1804, and was employed in the following year in the blockade of the ports of Ferrol and Corunna, in which (amongst other ports) ships were preparing for the invasion of England by Napoleon I. He held his position with a force greatly inferior to that of the enemy, and refused to be enticed out to sea. On its becoming known that the first movement directed by Napoleon was the raising of the blockade of Ferrol, Rear-Admiral Stirling was ordered to join Sir R. Calder and cruise with him to intercept the fleets of France and Spain on their passage to Brest. The approach of the enemy was concealed by a fog; but on the 22nd of July 1805 their fleet came in sight. It still outnumbered the British force; but Sir Robert entered into action. After a combat of four hours, during which he captured two Spanish ships, he gave orders to discontinue the action. He offered battle again on the two following days, but the challenge was not accepted. The French admiral Villeneuve, however, did not pursue his voyage, but took refuge in Ferrol. In the judgment of Napoleon, his scheme of invasion was baffled by this day's action; but much indignation was felt in England at the failure of Calder to win a complete victory. In consequence of the strong feeling against him at home he demanded a court-martial. This was held on the 23rd of December, and resulted in a severe reprimand of the vice-admiral for not having done his utmost to renew the engagement, at the same time acquitting him of both cowardice and disaffection. False expectations had been raised in England by the mutilation of his despatches, and of this he indignantly complained in his defence. The tide of feeling, however, turned again; and in 1815, by way of public testimony to his services, and of acquittal of the charge made against him, he was appointed commander of Portsmouth. He died at Holt, near Bishop's Waltham, in Hampshire, on the 31st of August 1818.

See *Naval Chronicle*, xvii.; James, *Naval History*, iii. 356-379 (1860).

CALDER, an ancient district of Midlothian, Scotland. It has been divided into the parishes of Mid-Calder (pop. in 1901 3132) and West-Calder (pop. 8092), East-Calder belonging to the parish of Kirknewton (pop. 3221). The whole locality owes much of its commercial importance and prosperity to the enormous development of the mineral oil industry. Coal-mining is also extensively pursued, sandstone and limestone are worked, and paper-mills flourish. Mid-Calder, a town on the Almond (pop. 703), has an ancient church, and John Spottiswood (1510-1585), the Scottish reformer, was for many years minister. His sons—John, archbishop of St Andrews, and James (1567-1645), bishop of Clogher—were both born at Mid-Calder. West-Calder is situated on Breich Water, an affluent of the Almond, 151/2 m. S.W. of Edinburgh by the Caledonian railway, and is the chief centre of the district. Pop. (1901) 2652. At Addiewell, about 11/2 m. S.W., the manufacture of ammonia, naphtha, paraffin oil and candles is carried on, the village practically dating from 1866, and having in 1901 a population of 1591. The Highland and Agricultural Society have an experimental farm at Pumpherston (pop. 1462). The district contains several tumuli, old ruined castles and a Roman camp in fair preservation.

CALDERON, RODRIGO (d. 1621), COUNT OF OLIVA AND MARQUES DE LAS SIETE IGLESIAS, Spanish favourite and adventurer, was born at Antwerp. His father, Francisco Calderon, a member of a family ennobled by Charles V., was a captain in the army who became afterwards *comendador mayor* of Aragon, presumably by the help of his son. The mother was a Fleming, said by Calderon to have been a lady by birth and called by him Maria Sandelin. She is said by others to have been first the mistress and then the wife of Francisco Calderon. Rodrigo is said to have been born out of wedlock. In 1598 he entered the service of the duke of Lerma as secretary. The accession of Philip III. in that year made Lerma, who had unbounded influence over the king, master of Spain. Calderon, who was active and unscrupulous, made himself the trusted agent of Lerma. In the general scramble for wealth among the worthless intriguers who governed in the name of Philip III., Calderon was conspicuous for greed, audacity and insolence. He was created count of Oliva, a knight of Santiago, commendador of Ocana in the order, secretary to the king (*secretario de camara*), was loaded with plunder, and made an advantageous marriage with Ines de Vargas. As an insolent upstart he was peculiarly odious to the enemies of Lerma. Two religious persons, Juan de Santa Maria, a Franciscan, and Mariana de San Jose, prioress of La Encarnacion, worked on the queen Margarita, by whose influence Calderon was removed from the secretaryship in 1611. He, however, retained the favour of Lerma, an indolent man to whom Calderon's activity was indispensable. In 1612 he was sent on a special mission to Flanders, and on his return was made marques de las Siete Iglesias in 1614. When the queen Margarita died in that year in childbirth, Calderon was accused of having used witchcraft against her. Soon after it became generally known that he had ordered the murder of one Francisco de Juaras. When Lerma was driven from court in 1618 by the intrigues of his own son, the duke of Uceda, and the king's confessor, the Dominican Aliaga, Calderon was seized upon as an expiatory victim to satisfy public clamour. He was arrested, despoiled, and on the 7th of January 1620 was savagely tortured to make him confess to the several charges of murder and witchcraft brought against him. Calderon confessed to the murder of Juaras, saying that the man was a pander, and adding that he gave the particular reason by word of mouth since it was more fit to be spoken than written. He steadfastly denied all the other charges of murder and the witchcraft. Some hope of pardon seems to have remained in his mind till he heard the bells tolling for Philip III. in March 1621. "He is dead, and I too am dead" was his resigned comment. One of the first measures of the new reign was to order his execution. Calderon met his fate firmly and with a show of piety on the 21st of October 1621, and this bearing, together with his broken and prematurely aged appearance, turned public sentiment in his favour. The magnificent devotion of his wife helped materially to placate the hatred he had aroused. Lord Lytton made Rodrigo Calderon the hero of his story *Calderon the Courtier*.

See Modests de la Fuente, *Historia General Espana* (Madrid, 1850-1867), vol. xv. pp. 452 et seq.; Quevedo, *Obras* (Madrid, 1794), vol. x.—*Grandes Anales de Quince Dias*. A curious contemporary French pamphlet on him, *Histoire admirable et declin pitoyable advenue en la personne d'unfawory de la Cour d'Espagne,* is reprinted by M.E. Fournier in *Varietes historiques* (Paris, 1855), vol. i.

(D. H.)

CALDERON DE LA BARCA, PEDRO (1600-1681), Spanish dramatist and poet, was born at Madrid on the 17th of January 1600. His mother, who was of Flemish descent, died in 1610; his father, who was secretary to the treasury, died in 1615. Calderon was educated at the Jesuit College in Madrid with a view to taking orders and accepting a family living; abandoning this project, he studied law at Salamanca, and competed with success at the literary fetes held in honour of St Isaidore at Madrid (1620-1632). According to his biographer, Vera Tassis, Calderon served with the Spanish army in Italy and Flanders between 1625 and 1635; but this statement is contradicted by numerous legal documents which prove that Calderon resided at Madrid during these years. Early in 1629 his brother Diego was stabbed by an actor who took sanctuary in the convent of the Trinitarian nuns; Calderon and his friends broke into the cloister and attempted to seize the offender. This violation was denounced by the fashionable preacher, Hortensio Felix Paravicino (*q.v.*), in a sermon preached before Philip IV.; [v.04 p.0985] Calderon retorted by introducing into *El Principe constante* a mocking reference (afterwards cancelled) to Paravicino's gongoristic verbiage, and was committed to prison. He was soon released, grew rapidly in reputation as a playwright, and, on the death of Lope de Vega in 1635, was recognized as the foremost Spanish dramatist of the age. A volume of his plays, edited by his brother Jose in 1636, contains such celebrated and diverse productions as *La Vida es sueno, El Purgatorio de San Patricia, La Devocion de la cruz, La Dama duende* and *Peor esta que estaba*. In 1636-1637 he was made a knight of the order of Santiago by Philip IV., who had already commissioned from him a series of spectacular plays for the royal theatre in the Buen Retiro. Calderon was almost as popular with the general public as Lope de Vega had been in his zenith; he was, moreover, in high favour at court, but this royal patronage did not help to develop the finer elements of his genius. On the 28th of May 1640 he joined a company of mounted cuirassiers recently raised by Olivares, took part in the Catalonian campaign, and distinguished himself by his gallantry at Tarragona; his health failing, he retired from the army in November 1642, and three years later was awarded a special military pension in recognition of his services in the field. The history of his life during the next few years is obscure. He appears to have been profoundly affected by the death of his mistress—the mother of his son Pedro Jose—about the year 1648-1649; his long connexion with the theatre had led him into temptations, but it had not diminished his instinctive spirit of devotion, and he now sought consolation in religion. He became a tertiary of the order of St Francis in 1650, and finally reverted to his original intention of joining the priesthood. He was ordained in 1651, was presented to a living in the parish of San Salvador at Madrid, and, according to his statement made a year or two later, determined to give up writing for the stage. He did not adhere to this resolution after his preferment to a prebend at Toledo in 1653, though he confined himself as much as possible to the composition of *autos sacramentales*—allegorical pieces in which the mystery of the Eucharist was illustrated dramatically, and which were performed with great pomp on the feast of Corpus Christi and during the weeks immediately ensuing. In 1662 two of Calderon's *autos*—*Las ordenes militares* and *Misticay real Babilonia*—were the subjects of an inquiry by the Inquisition; the former was censured, the manuscript copies were confiscated, and the condemnation was not rescinded till 1671. Calderon was appointed honorary chaplain to Philip IV, in 1663, and the royal favour was continued to him in the next reign. In his eighty-first year he wrote his last secular play, *Hado y Divisa de Leonido y Marfisa*, in honour of Charles II.'s marriage to Marie-Louise de Bourbon. Notwithstanding his position at court and his universal popularity throughout Spain, his closing years seem to have been passed in poverty. He died on the 25th of May 1681.

Like most Spanish dramatists, Calderon wrote too much and too speedily, and he was too often content to recast the productions of his predecessors. His *Saber del mal y del bien* is an adaptation of Lope de Vega's play, *Las Mudanzas de la fortuna y sucesos de Don Beltran de Aragon*; his *Selva confusa* is also adapted from a play of Lope's which bears the same title; his *Encanto sin encanto* derives from Tirso de Molina's *Amar par senas*, and, to take an extreme instance, the second act of his *Cabellos de Absalon* is transferred almost bodily from the third act of Tirso's *Venganza de Tamar*. It would be easy to add other examples of Calderon's lax methods, but it is simple justice to point out that he committed no offence against the prevailing code of literary morality. Many of his contemporaries plagiarized with equal audacity, but with far less success. Sometimes, as in *El Alcalde de Zalamea*, the bold procedure is completely justified by the result; in this case by his individual treatment he transforms one of Lope de Vega's rapid improvisations into a finished masterpiece. It was not given to him to initiate a great dramatic movement; he came at the end of a literary revolution, was compelled to accept the conventions which Lope de Vega had imposed on the Spanish stage, and he accepted them all the more readily since they were peculiarly suitable to the display of his splendid and varied gifts. Not a master of observation nor an expert in invention, he showed an unexampled skill in contriving ingenious variants on existing themes; he had a keen dramatic sense, an unrivalled dexterity in manipulating the mechanical resources of the stage, and in addition to these minor indispensable talents he was endowed with a lofty philosophic imagination and a wealth of poetic diction. Naturally, he had the defects of his great qualities; his ingenuity is apt to degenerate into futile embellishment; his employment of theatrical devices is the subject of his own good-humoured satire in *No hay burlas con el amor*; his philosophic intellect is more interested in theological mysteries than in human passions; and the delicate beauty of his style is tinged with a wilful preciosity. Excelling Lope de Vega at many points, Calderon falls below his great predecessor in the delineation of character. Yet in almost every department of dramatic art Calderon has obtained a series of triumphs. In the symbolic drama he is best represented by *El Principe constante*, by *El Magico prodigioso* (familiar to English readers in Shelley's free translation), and by *La Vida es sueno*, perhaps the most profound and original of his works. His tragedies are more remarkable for their acting qualities than for their convincing truth, and the fact that in *La Nina de Gomez Arias* he interpolates an entire act borrowed from Velez de Guevara's play of the same title seems to indicate that this kind of composition awakened no great interest in him; but in *El Medico de sa honra* and *El Mayor monstruo los celos* the theme of jealousy is handled with sombre power, while *El Alcalde de Zalamea* is one of the greatest tragedies in Spanish literature. Calderon is seen to much less advantage in the spectacular plays—*dramas de tramoya*—which he wrote at the command of Philip IV.; the dramatist is subordinated to the stage-carpenter, but the graceful fancy of the poet preserves even such a mediocre piece as *Los Tres Mayores prodigies* (which won him his knighthood) from complete oblivion. A greater opportunity is afforded in the more animated *comedias palaciegas*, or melodramatic pieces destined to be played before courtly audiences in the royal palace: *La Banda y la flor* and *El Galan fantasma* are charming illustrations of Calderon's genial conception and refined artistry. His historical plays (*La Gran Cenobia, Las armas de la hermosura*, &c.) are the weakest of all his formal dramatic productions; *El Golfo de la sirenas* and *La Purpura de la rosa* are typical *zarzuelas*, to be judged by the standard of operatic libretti, and the *entremeses* are lacking in the lively humour which should characterize these dramatic interludes. On the other hand, Calderon's faculty of ingenious stagecraft is seen at its best in his "cloak-and-sword" plays (*comedias de capa y espada*) which are invaluable pictures of contemporary society. They are conventional, no doubt, in the sense that all representations of a specially artificial society must be conventional; but they are true to life, and are still as interesting as when they first appeared. In this kind *No siempre lo peor es cierto, La Dama duende, Una casa con dos puertas mala es de guardar* and *Guardate del agua mansa* are almost unsurpassed. But it is as a writer of *autos sacramentales* that Calderon defies rivalry: his intense devotion, his subtle intelligence, his sublime lyrism all combine to produce such marvels of allegorical poetry as *La Cena del rey Baltasar, La Vina del Senor* and *La Serpiente de metal*. The *autos* lingered on in Spain till 1765, but they may be said to have died with Calderon, for his successors merely imitated him with a tedious fidelity. Almost alone among Spanish poets, Calderon had the good fortune to be printed in a fairly correct and readable edition (1682-1691), thanks to the enlightened zeal of his admirer, Juan de Vera Tassis y Villaroel, and owing to this happy accident he came to be regarded generally as the first of Spanish dramatists. The publication of the plays of Lope de Vega and of Tirso de Molina has affected the critical estimate of Calderon's work; he is seen to be inferior to Lope de Vega in creative power, and inferior to Tirso de Molina in variety of conception. But, setting aside the extravagances of his admirers, he is admittedly an exquisite poet, an expert in the dramatic form, and a typical representative of the [v.04 p.0986] devout, chivalrous, patriotic and artificial society in which he moved.

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(J. F.-K.)

CALDERWOOD, DAVID (1575-1650), Scottish divine and historian, was born in 1575. He was educated at Edinburgh, where he took the degree of M.A. in 1593. About 1604 he became minister of Crailing, near Jedburgh, where he became conspicuous for his resolute opposition to the introduction of Episcopacy. In 1617, while James was in Scotland, a Remonstrance, which had been drawn up by the Presbyterian clergy, was placed in Calderwood's hands. He was summoned to St Andrews and examined before the king, but neither threats nor promises could make him deliver up the roll of signatures to the Remonstrance. He was deprived of his charge, committed to prison at St Andrews and afterwards removed to Edinburgh. The privy council ordered him to be banished from the kingdom for refusing to acknowledge the sentence of the High Commission. He lingered in Scotland, publishing a few tracts, till the 27th of August 1619, when he sailed for Holland. During his residence in Holland he published his *Altare Damascenum*. Calderwood appears to have returned to Scotland in 1624, and he was soon afterwards appointed minister of Pencaitland, in the county of Haddington. He continued to take an active part in the affairs of the church, and introduced in 1649 the practice, now confirmed by long usage, of dissenting from the decision of the Assembly, and requiring the protest to be entered in the record. His last years were devoted to the preparation of a *History of the Church of Scotland*. In 1648 the General Assembly urged him to complete the work he had designed, and voted him a yearly pension of L800. He left behind him a historical work of great extent and of great value as a storehouse of authentic materials for history. An abridgment, which appears to have been prepared by himself, was published after his death. An excellent edition of the complete work was published by the Wodrow Society, 8 vols., 1842-1849. The manuscript, which belonged to General Calderwood Durham, was presented to the British Museum. Calderwood died at Jedburgh on the 29th of October 1650.

CALDERWOOD, HENRY (1830-1897), Scottish philosopher and divine, was born at

Peebles on the 10th of May 1830. He was educated at the Royal High school, and later at the university of Edinburgh. He studied for the ministry of the United Presbyterian Church, and in 1856 was ordained pastor of the Greyfriars church, Glasgow. He also examined in mental philosophy for the university of Glasgow from 1861 to 1864, and from 1866 conducted the moral philosophy classes at that university, until in 1868 he became professor of moral philosophy at Edinburgh. He was made LL.D. of Glasgow in 1865. He died on the 19th of November 1897. His first and most famous work was *The Philosophy of the Infinite* (1854), in which he attacked the statement of Sir William Hamilton that we can have no knowledge of the Infinite. Calderwood maintained that such knowledge, though imperfect, is real and ever-increasing; that Faith implies Knowledge. His moral philosophy is in direct antagonism to Hegelian doctrine, and endeavours to substantiate the doctrine of divine sanction. Beside the data of experience, the mind has pure activity of its own whereby it apprehends the fundamental realities of life and combat. He wrote in addition *A Handbook of Moral Philosophy, On the Relations of Mind and Brain, Science and Religion, The Evolution of Man's Place in Nature*. Among his religious works the best-known is his *Parables of Our Lord*, and just before his death he finished a *Life of David Hume* in the "Famous Scots" series. His interests were not confined to religious and intellectual matters; as the first chairman of the Edinburgh school board, he worked hard to bring the Education Act into working order. He published a well-known treatise on education. In the cause of philanthropy and temperance he was indefatigable. In politics he was at first a Liberal, but became a Liberal Unionist at the time of the Home Rule Bill.

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