Robust Scale Estimate for the Generalized Gaussian Probability Density Function

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Problem

- Robust Regression $v = f(\mathbf{u}, \theta) + \epsilon$ with:

$$\mathcal{P}(\epsilon|\theta, \sigma) = \mathcal{P}(\epsilon|\mathcal{C}, \theta, \sigma) \cdot \mathcal{P}(\mathcal{C}) + \mathcal{P}(\epsilon|\overline{\mathcal{C}}) \cdot \mathcal{P}(\overline{\mathcal{C}})$$

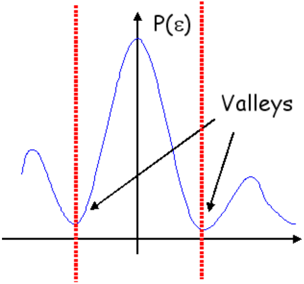
- Robust Scale estimate σ: important for the location estimate θ
- Example of object recognition:

High proportion of outliers: occlusion and cluttered background ($\mathcal{P}(\overline{\mathcal{C}})$>50%).

Recent approaches for scale estimation

- Learning of inliers and outliers statistics (Hasler et al. (2003))
  - Inliers: parametric modellings (Gaussian, Laplacian)
  - Outliers: nonparametric modelling from learning.
- Iterative estimation (with θ)

Detection of valleys and robust estimate of the scale (MAD) on the segmented residuals (Wang et al. (2004)).

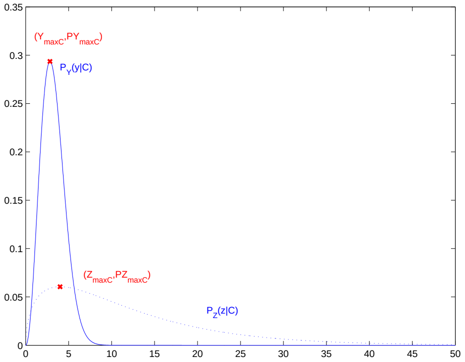


1. Hasler et al. Outlier modeling in image matching, IEEE Trans. on Pattern Analysis Machine Intelligence, March 2003.
2. Wang et al. Robust adaptive-scale parametric model estimation for computer vision. Trans. on Pattern Analysis and Machine Intelligence, Nov. 2004.

New Approach

- Hypothesis on inliers: R. V. $X \sim \mathcal{P}_X(x|\mathcal{C}) = \mathcal{GG}(0, \sigma, \alpha)$,
  - α known ($\alpha = 0.5$ Gaussian, $\alpha = 1$ Laplacian),
  - σ to estimate.

- Compute new variables from indpt. samples of X:



  - $Z = \sum_{n=1}^{\mathbf{n}} X_n^2$,
  - $Y = \sqrt{\sum_{n=1}^{\mathbf{n}} X_n^2}$
  - $Z_{\max \mathcal{C}}$ and $Y_{\max \mathcal{C}}$ prop. to σ

New Approach

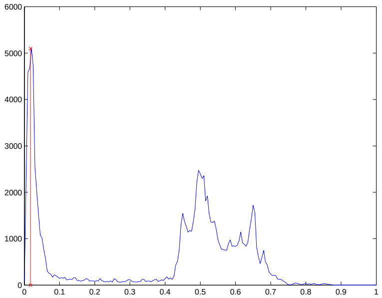
- Estimates:

$$\begin{cases} \sigma_Z = \left(\frac{Z_{\max\mathcal{C}}}{\mathbf{n}\alpha-1}\right)^{\alpha} \left[\frac{\Gamma(3\alpha)}{\Gamma(\alpha)}\right]^{1/2}, & \mathbf{n}\alpha > 1 \\ \sigma_Y = \frac{Y_{\max\mathcal{C}}}{(\mathbf{n}-1)^{\alpha}\cdot\alpha^{\alpha}} \left[\frac{\Gamma(3\alpha)}{\Gamma(\alpha)}\right]^{1/2}, & \mathbf{n} > 1 \end{cases}$$

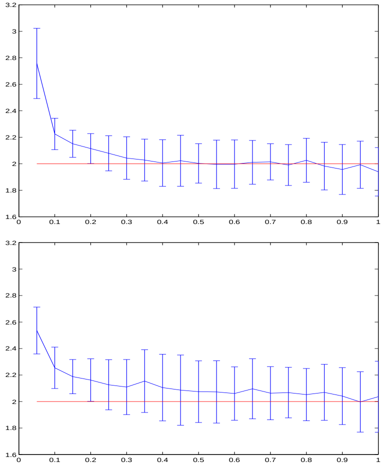
- Finding maxima $Z_{\max\mathcal{C}}$ and $Y_{\max\mathcal{C}}$ using Meanshift
  - Variable Bandwidth estimation with Sheather-Jones plug-in

Dealing with Outliers

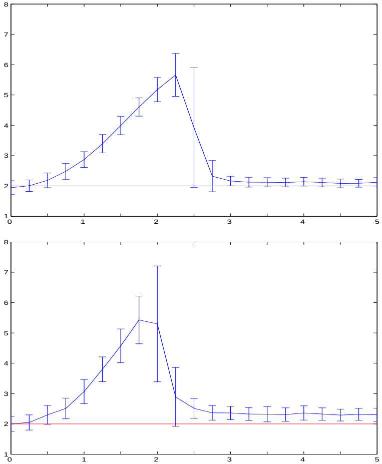
- From samples of X, compute samples of Y and Z and their bandwidth
- Outliers in samples of X, consequently in samples of Y and Z
- Robust scale estimate: search (Meanshift) for the first maximum from 0



Robustness to outliers



Robustness to outliers: Estimates σ_Y (top) and σ_Z (bottom) with standard deviation, w.r.t. the proportion of inliers $\mathcal{P}_Y(\mathcal{C}) = \mathcal{P}_Z(\mathcal{C})$.



Robustness to pseudo-outliers: Estimates σ_Y (top) and σ_Z (bottom) with standard deviation, w.r.t. the mean of the gaussian of pseudo-outliers expressed as a multiplicative of σ (i.e. abscissa equal to 2 means $\mu = 2\sigma$).

Robust Estimation

- Iterative estimation of θ and σ (RANSAC).

  Repeat B times

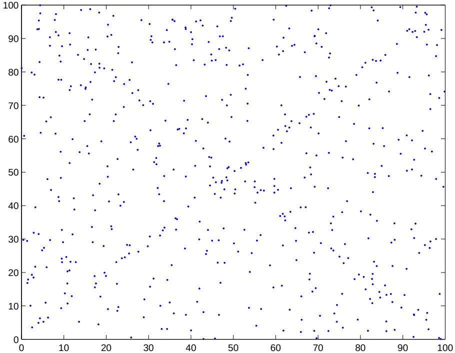
  1. From a bootstrap subset of data, standard (LS) estimation of $\theta^{(b)}$
  2. Estimate $\sigma^{(b)}$ from the residuals computed with $\theta^{(b)}$
  3. $\forall b$, compute the objective function:

$$\mathcal{J}(\theta^{(b)}, \sigma^{(b)}) = \frac{\sum_{i=1}^{\mathbf{I}} \mathbf{1}\{|\epsilon_i| < 2.5\sigma\}}{\mathbf{I} \cdot \sigma} \tag{1}$$

- Select $(\hat{\theta}, \hat{\sigma})$ by optimization:

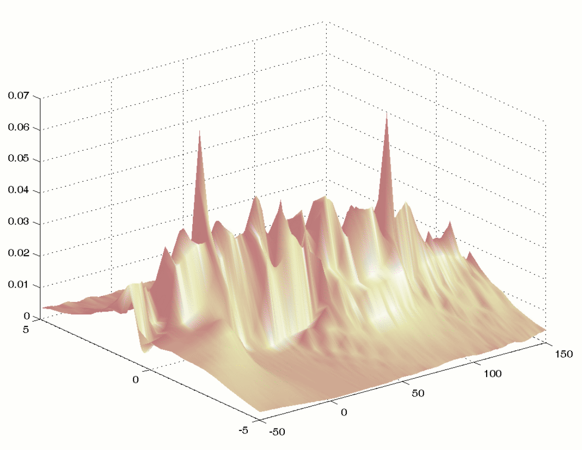
$$(\hat{\theta}, \hat{\sigma}) = \arg\max \mathcal{J}(\theta, \sigma)$$

Application to line fitting



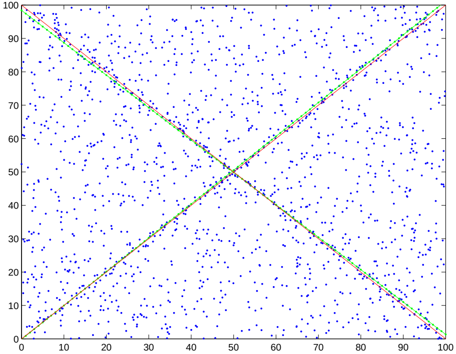
Observations

Application to line fitting



Objective function $\mathcal{J}(\theta, \sigma)$

Application to line fitting

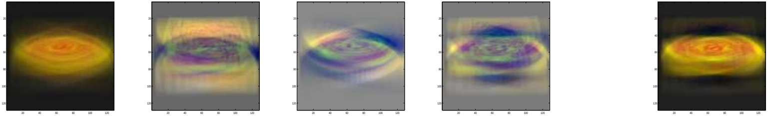


Estimates (green) and ground truth (red)

Application to Robust Object Recognition

t_0 t_{90} t_{180} t_{270}

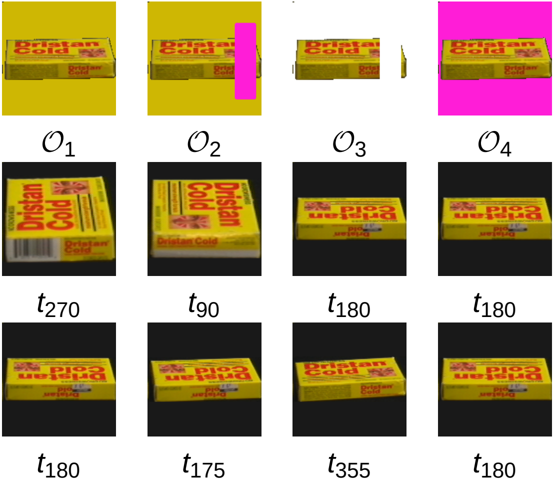
Figure: Example of training colour images for one object varying under different viewpoints.



μ $\mathbf{u}_1$ $\mathbf{u}_2$ $\mathbf{u}_3$ $\hat{t}_0 = \mu + [\mathbf{u}_1 \mathbf{u}_2 \mathbf{u}_3]^T \hat{\theta}$

Figure: Mean and eigenvectors selected to represent the object class, and the reconstruction of one of the training template in this eigenspace.

Application to Robust Object Recognition



Observations (top), recognition performed with M-estimators (middle), recognition with simultaneous robust estimation of the scale and location parameters (bottom).

Final comments

- Novel Robust Scale parameter estimation
- Combine with robust location parameter estimate (RANSAC)
- Application to Robust recognition in images
- Future work: choice of objective function, and number of models

Technical Report:
www.mee.tcd.ie/∼dahyot/pdf/AS2005.pdf