GR^{2}T Vs L$_2$E with nuisance scale

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GR^{2}T and L$_2$E

Assuming a Normal distribution for the error $\epsilon \sim p_{\epsilon|\nu} \equiv \mathcal{N}(\epsilon; 0, \nu^2)$, and assuming that its occurrences computed with N observations, noted $\{\epsilon^{(i)}(\theta)\}_{i=1,\cdots,N}$, are depending on a latent variable θ of interest, θ can be inferred using the following objectives functions:

- Generalised Relaxed Radon Transform (GR^{2}T) [3] augmented with a prior on the nuisance scale ν chosen here as the log-normal distribution:

$$\hat{\theta}_{GR^2T} \longleftarrow \arg\max_{\theta,\nu} \left\{ GR^2T(\theta,\nu) = \left(\frac{1}{N} \sum_{i=1}^{N} p_{\epsilon|\nu}(\epsilon^{(i)}(\theta)) \right) \times p_\nu(\nu) \right\}$$

- Integrated Square Error L$_2$E [4]:

$$\hat{\theta}_{L_2E} \longleftarrow \arg\min_{\theta,\nu} \left\{ L_2E(\theta,\nu) = \underbrace{\|p_{\epsilon|\nu}\|^2}_{=\frac{1}{2\nu\sqrt{\pi}}} - 2 \left(\frac{1}{N} \sum_{i=1}^{N} p_{\epsilon|\nu}(\epsilon^{(i)}(\theta)) \right) \right\}$$

- the non robust Maximum Likelihood used for comparison:

$$\hat{\theta}_{ML} \longleftarrow \arg\max_{\theta,\nu} \left\{ ML(\theta,\nu) = \left(\prod_{i=1}^{N} p_{\epsilon|\nu}(\epsilon^{(i)}(\theta)) \right) \right\}$$

Regression

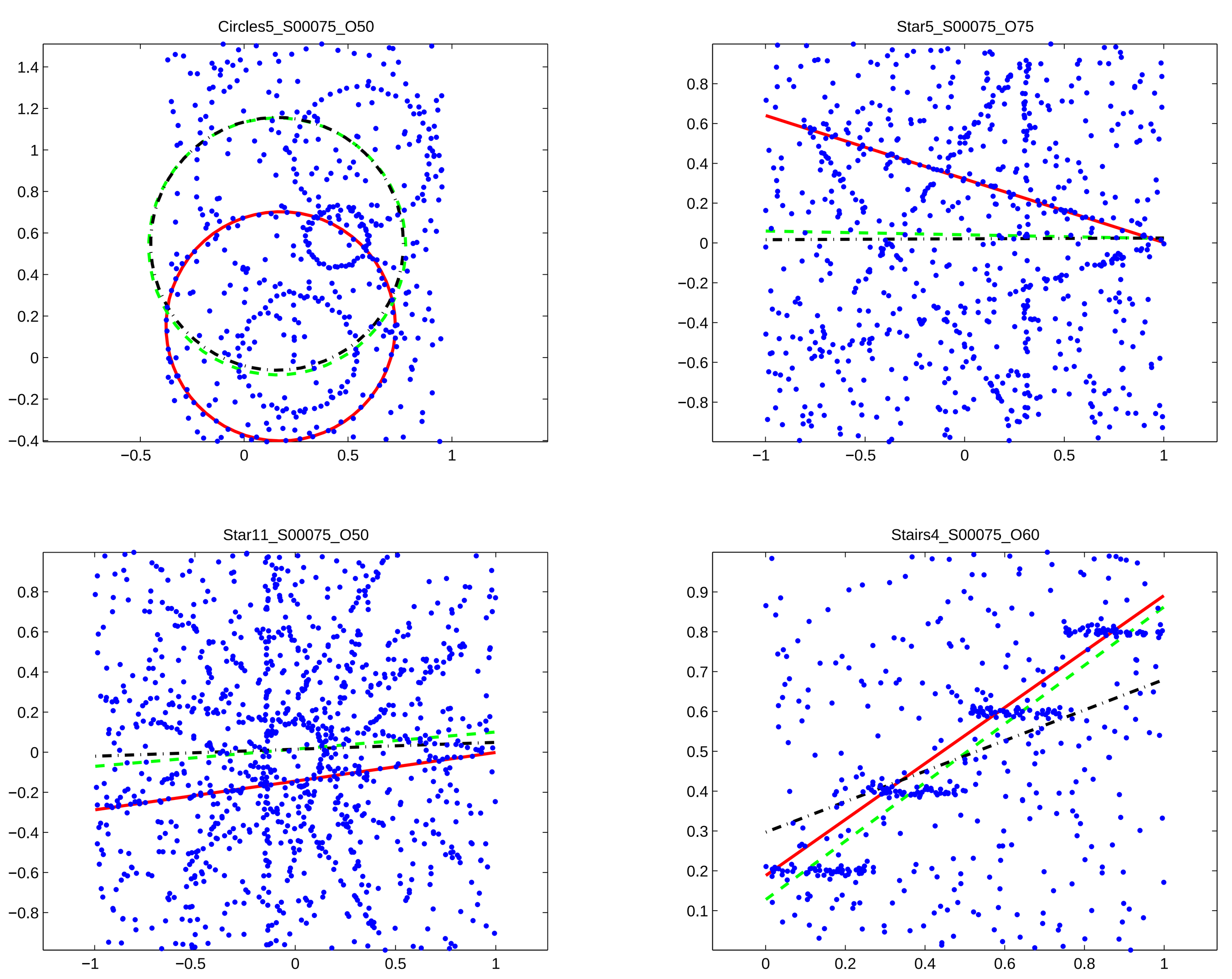

FIGURE 1: **Robust Regression with Outliers and Pseudo-Outliers**: GR^{2}T with Scale Prior (red), L$_2$E (green, dashed), ML (black, dot-dashed).

For a 2D point cloud $\{x^{(i)} = (x_1^{(i)}, x_2^{(i)})\}_{i=1,\cdots,N}$, the residuals are:

- for a line $\epsilon^{(i)}(\theta) = x_1^{(i)} - (\theta_1 + \theta_2\, x_2^{(i)})$ with $\theta = (\theta_1, \theta_2)$,
- for a circle $\epsilon^{(i)}(\theta) = \sqrt{(x_1^{(i)} - \theta_1)^2 + (x_2^{(i)} - \theta_2)^2} - \theta_3$ with $\theta = (\theta_1, \theta_2, \theta_3)$

Registration

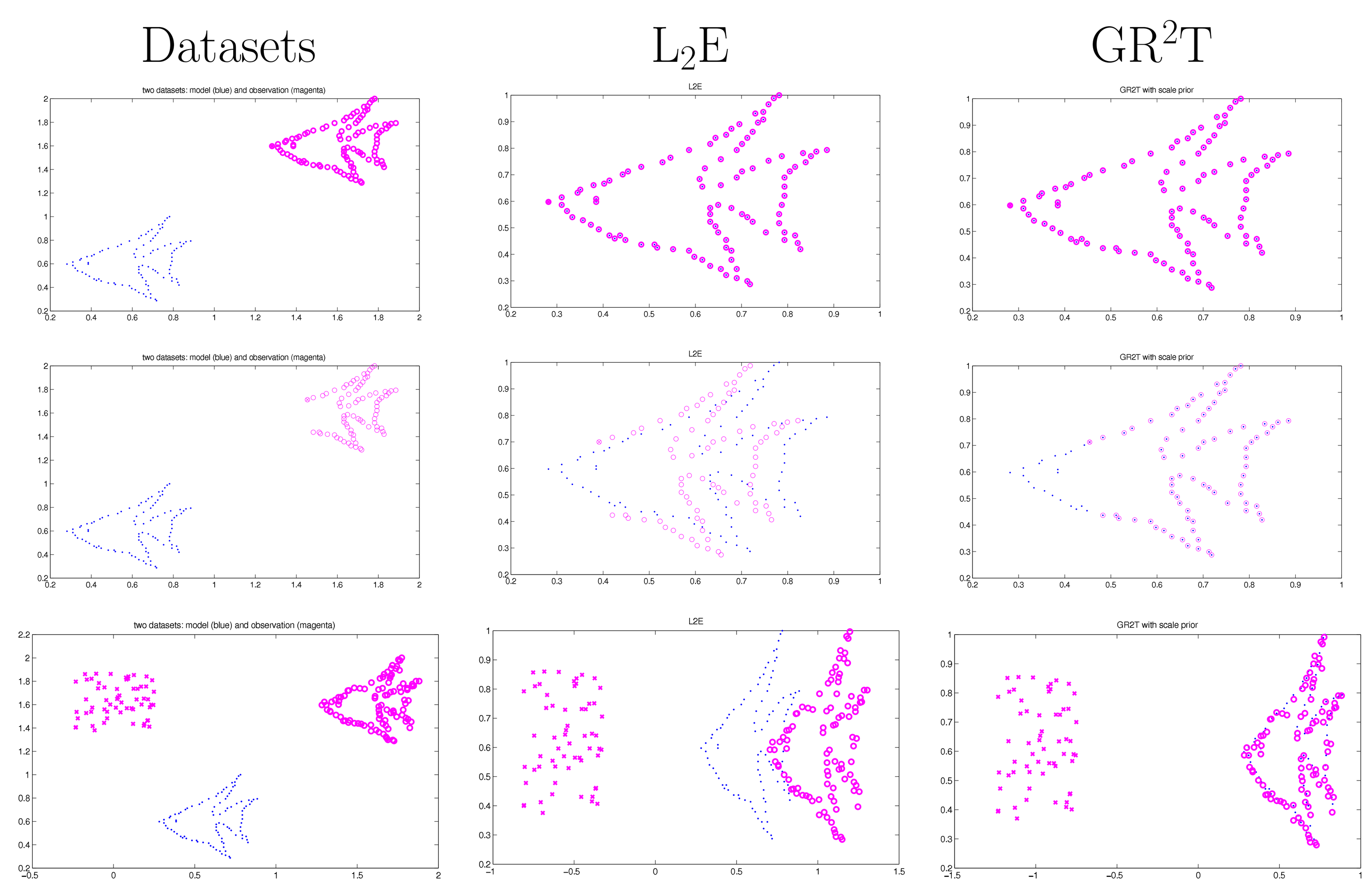

FIGURE 2: **Rigid Registration (translation) of 2D point clouds.** Model point cloud (blue), target point cloud (pink) with outliers highlighted by cross markers.

For registering a 2D point cloud $\{x^{(j)}\}_{j=1,\cdots,N_x}$ onto another $\{y^{(i)}\}_{i=1,\cdots,N_y}$, the $(N_x \times N_y)$ residuals are $\epsilon^{(i,j)} = \|y^{(i)} - (x^{(j)} + \theta)\|$ with $\theta = (\theta_1, \theta_2)^T$ the latent translation vector in $\mathbb{R}^2$.

Non-rigid transformations can also be tackled for instance using morphable models [1, 2].

References

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[2] C. Arellano and R. Dahyot. Robust bayesian fitting of 3d morphable model. In *Proceedings of the 10th European Conference on Visual Media Production*, CVMP '13, pages 9:1–9:10, New York, NY, USA, 2013. ACM.

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[4] D. W. Scott. Parametric statistical modeling by minimum integrated square error. *Technometrics*, 43(3):pp. 274–285, 2001.