Fitting a parametric family of models to sample semi variograms

Prof. Rozenn Dahyot
Trinity College Dublin, Ireland

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Table 1 presents the values of average semi variogram for the meuse dataset.

	np	dist	gamma	dir.hor	dir.ver	id
1	57	79.29244	0.1234479	0	0	var1
2	299	163.97367	0.2162185	0	0	var1
3	419	267.36483	0.3027859	0	0	var1
4	457	372.73542	0.4121448	0	0	var1
5	547	478.47670	0.4634128	0	0	var1
6	533	585.34058	0.5646933	0	0	var1
7	574	693.14526	0.5689683	0	0	var1
8	564	796.18365	0.6186769	0	0	var1
9	589	903.14650	0.6471479	0	0	var1
10	543	1011.29177	0.6915705	0	0	var1
11	500	1117.86235	0.7033984	0	0	var1
12	477	1221.32810	0.6038770	0	0	var1
13	452	1329.16407	0.6517158	0	0	var1
14	457	1437.25620	0.5665318	0	0	var1
15	415	1543.20248	0.5748227	0	0	var1

Table 1: Average semiVariogram of log(zinc) in the Meuse data set.

1. Why are we not satisfied by the sample semivariogram ? why do we need to fit a parametric model ?
2. A function used for modelling the variogram is the negative exponential ('Exp'):

$$\gamma(h,\theta) = c\left\{1-\exp\left(\frac{-h}{r}\right)\right\} \quad \text{with} \quad \theta=\{c,r\} \tag{1}$$

   with sill c and distance parameter that defines the spatial extent of the model. Plot model (Eq. 1).
3. Show that model (Eq. 1) cannot have a nugget[1].
4. What is the value of the sill[2]?

[1]Nugget: The value at which the semi-variogram (almost) intercepts the y-value ($h \to 0$).
[2]Sill: The value at which the model first flattens out ($h \to \infty$).

5. The Exponential Model (Eq. 1) can be extended as follow:

$$\gamma(h,\theta) = c_0 + c\,\left\{1 - \exp\left(\frac{-h}{r}\right)\right\} \quad \text{with} \quad \theta = \{c_0, c, r\} \tag{2}$$

What is the nugget ? What is the sill ?

6. How to estimate the parameters c_0, c and r using values in Table 1 (cf. Fig. 1)?

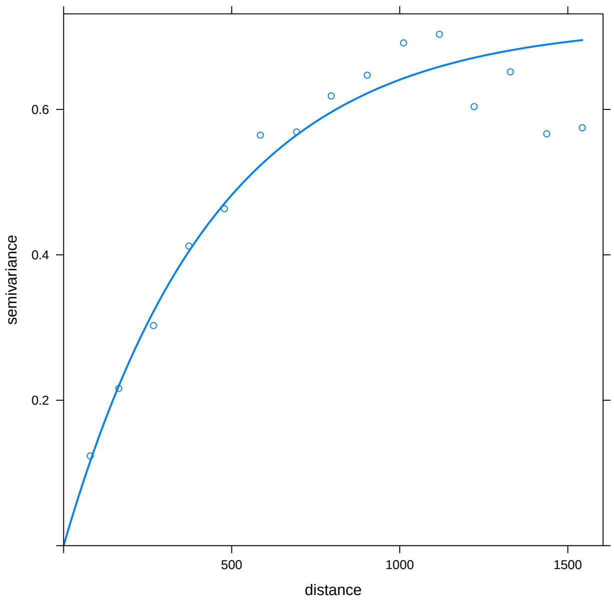

Figure 1: Fitted exponential variogram to data in table 1.

7. Example of other parametric models for variograms:

$$\gamma(h) = c\left\{1 - \exp\left(\frac{-h^2}{r^2}\right)\right\} \quad \text{(Gaussian model)} \tag{3}$$

and

$$\gamma(h) = \begin{cases} c\left\{\frac{3h}{2a} - \frac{1}{2}\left(\frac{h}{a}\right)^3\right\} & \text{for } h \leq a \\ c \quad \text{for } h > a \end{cases} \quad \text{(Spherical model)} \tag{4}$$

Show these variogram models are bounded.

8. Unbounded models for variograms also exist e.g.:

$$\gamma(h) = w\,h^{\alpha} \quad \text{for } 0 < \alpha < 2 \tag{5}$$

Give an example of process that has an unbounded variogram.

