The Dirac function I

Definition

The Dirac function is defined as:

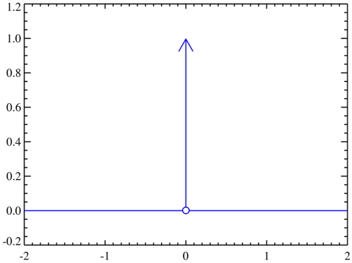
$$\delta(t) = \begin{cases} 0, & t \neq 0 \\ \infty, & t = 0 \end{cases}$$

$\delta(t)$ is a **generalised function** and only the notation $< f|\delta >$ actually makes sense:

$$\int_{-\infty}^{\infty} f(t)\ \delta(t)\ dt = f(0)$$

where f is continuous.

The Dirac function II



Dirac function

The Dirac function III

Properties of the Dirac function:

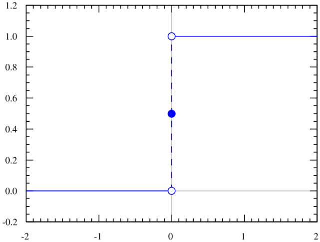
- $$\int_{-\infty}^{\infty} \delta(t)\ dt = 1$$
- $\delta(t)$ can be understood as the limit of a sequence of Normal probability density functions where the standard deviation goes to 0.
- Sampling a function f can be written as:

$$\int_{-\infty}^{+\infty} f(t)\ \delta(t - t_0)\ dt = f(t_0)$$

The Dirac function IV

- The Heaviside function is the primitive of the Dirac function:

$$H(x) = \int_{-\infty}^{x} \delta(t)\, dt$$



Heaviside function

- $\delta(x)$ (also $H(x)$) is referred as a **generalised function** .

Delta Functions - Introduction to Generalised Functions , by R. F. Hoskins, 1999