Linear Smoothing: Conclusion and Extensions II

Extension: Regularised basis approach

- When fitting the model $\mu(t) = \sum_{k=1}^{K} \theta_k \ \phi_k(t)$, one may add a smoothness constraint to ensure that the estimated function is smooth.
- If the basis of functions is differentiable then $D^2\mu(t)$ can be computed and a constraint (prior) can be added to the cost function $\mathcal{C}$ to estimate the coefficients:

$$\mathcal{C}(\theta_1, \cdots, \theta_K) = \sum_{i=1}^{n} (s^{(i)} - \mu(t^{(i)}))^2 + \lambda \int [D^2\mu(t)]^2 \ dt$$

Linear Smoothing: Conclusion and Extensions III

- Another example, consider growth, exponential growth or decay (e.g. population, radioactive decay, economic indicators), we know that such system follows an ODE:

$$Lx = -\gamma x + Dx = 0$$

then fitting the model $\mu(t) = \sum_{k=1}^{K} \theta_k \ \phi_k(t)$ can be constrained with

$$\mathcal{C}(\theta_1, \cdots, \theta_K) = \sum_{j=1}^{n} (s^{(i)} - \mu(t^{(i)})^2 + \lambda \int [L\mu(t)]^2 \ dt$$

where γ is unknown it can be estimated.

Linear Smoothing: Conclusion and Extensions IV

- Depending of the problem, derivatives can be used for regularisation, adding prior for the type of solutions we are looking for (e.g. smoothness, or subject to a known ODE).
- Derivatives can also be the subject of interest in the study i.e. For instance in mechanics, if you collect spatial positions over time (trajectories), one may be interested in studying the velocity (first derivative) or acceleration (second derivatives).