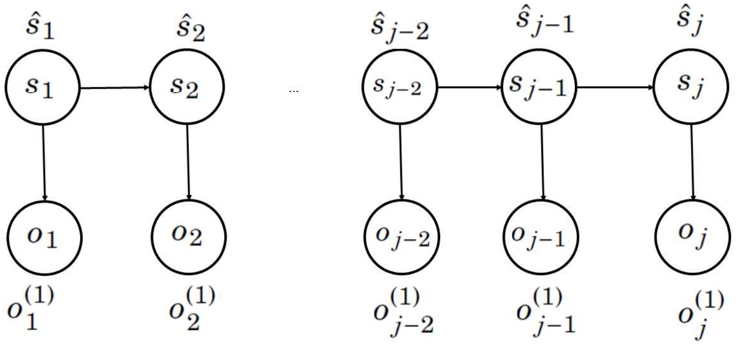
Filters I



Filters II

Given a hidden temporal stochastic process $(s_1, \cdots, s_J)$ and observed one $(o_1, \cdots, o_J)$ linked by

$$\begin{cases} p_{s_1}(s_1) & \text{Initial state} \\ p_{s_j|s_{j-1}}(s_j|s_{j-1}) & \text{transition (order 1)} \\ p_{o_j|s_j}(o_j|s_j) & \text{link between observed variable and hidden state} \end{cases}$$

Estimate the posterior $p_{s_j|o_{1:j}}$. Hint: Express $p_{s_j|o_{1:j}}(s_j|o_{1:j})$ w.r.t. $p_{s_{j-1}|o_{1:j-1}}(s_{j-1}|o_{1:j-1})$ (notation $o_{1:j-1} = (o_1, \cdots, o_{j-1})$).

Filters III

$p_{s_j|o_{1:j}}(s_j|o_{1:j})$

$= \int p(s_j, s_{j-1}|o_{1:j})\, ds_{j-1}$ (Total probability theorem)

$= \frac{1}{p(o_{1:j})} \int p(s_t, s_{t-1}, o_{1:j})\, ds_{j-1}$ (Bayes)

$= \frac{1}{p(o_{1:j})} \int p(s_j, s_{j-1}, o_j, o_{1:j-1})\, ds_{j-1}$

$= \frac{p(o_{1:j-1})}{p(o_{1:j})} \int p(o_j|s_j, s_{j-1}, o_{1:j-1})\, p(s_j|s_{j-1}, o_{1:j-1})\, p(s_{j-1}|o_{1:j-1})\, ds_{j-1}$ (Bay

$= \frac{p(o_{1:j-1})}{p(o_{1:j})} \int p(o_j|s_j)\, p(s_j|s_{j-1})\, p(s_{j-1}|o_{1:j-1})\, ds_{j-1}$

Filters IV

Hence

$$p(s_j|o_{1:j}) = \frac{p(o_{1:j-1})}{p(o_{1:j})}\, p(o_j|s_j) \int p(s_j|s_{j-1})\, p(s_{j-1}|o_{1:j-1})\, ds_{j-1}$$

given a hidden temporal stochastic process $(s_1, \cdots, s_J)$ and observed one $(o_1, \cdots, o_J)$ linked by

$$\begin{cases} p_{s_1}(s_1) & \text{Initial state} \\ p_{s_j|s_{j-1}}(s_j|s_{j-1}) & \text{transition (order 1)} \\ p_{o_j|s_j}(o_j|s_j) & \text{link between observed variable and hidden state} \end{cases}$$

Filters V

In the general case, we have the following integral to solve:

$$p(s_j|o_{1:j}) = \frac{p(o_{1:j-1})}{p(o_{1:j})}\, p(o_j|s_j) \int p(s_j|s_{j-1})\, p(s_{j-1}|o_{1:j-1})\, ds_{j-1}$$

1. Prediction:

$$p(s_j|o_{1:j-1}) = \int p(s_j|s_{j-1})\, p(s_{j-1}|o_{1:j-1})\, ds_{j-1}$$

2. Update

$$p(s_j|o_{1:j}) \propto p(o_j|s_j)\, p(s_j|o_{1:j-1})$$

Filters VI

- Having collected observations $\{o_t^{(1)}\}_{t=1,\cdots,j-1}$ (also noted $o_{1:j-1}^{(1)}$)), the probability density function to predict is s_j is

$$p(s_j|o_{1:j-1}^{(1)})$$

- Once the new observation has been collected at time j, $o_j^{(1)}$, the probability density function taking this update into account is

$$p(s_j|o_{1:j}^{(1)})$$

Filters VII