

Fourier transform I

The Fourier transform is an important tool to analyse signals, discrete or continuous.

 A Wavelet tour of signal processing , by S. Mallat, second edition

Definition (Fourier transform)

The **Fourier integral** measures *how much* oscillations at the frequency ω there is the function f :

$$F(\omega) = \int_{-\infty}^{\infty} f(t) \exp(-i\omega t) dt$$

Fourier transform II

Definition (Inverse Fourier transform)

The **Inverse Fourier** transform can be computed:

$$f(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(\omega) \exp(i\omega t) d\omega$$

Exercises: Fourier transform

Compute the Fourier transforms of:

- 1 $f(t) = \begin{cases} 1 & -T < t < T \\ 0 & \text{otherwise} \end{cases}$
- 2 $f(t) = \delta(t)$
- 3 $f(t) = \delta(t - t_0)$

Fourier transform III

4 $f(t) = \exp(-at)$ $h(t)$ (h is the heaviside step function)

5 $f(t) = \cos(\omega_0 t)$

6 Show

1 Linearity:

$$a x(t) + b y(t) \leftrightarrow a X(\omega) + b Y(\omega)$$

2 Time shifting:

$$x(t - t_0) \leftrightarrow \exp(-i\omega t_0) X(\omega)$$

Fourier transform IV

Definition (Convolution)

The convolution of two functions $x(t)$ and $h(t)$ gives a function $y(t)$ such that:

$$y(t) = x(t) * h(t) = \int_{-\infty}^{+\infty} x(\tau) h(t - \tau) d\tau$$

Theorem (Convolution)

*If $y(t) = x(t) * h(t)$ then:*

$$Y(\omega) = X(\omega) \cdot H(\omega)$$

Prove this theorem.

Two-dimensional Fourier transform I

Definition (2D Fourier Transform)

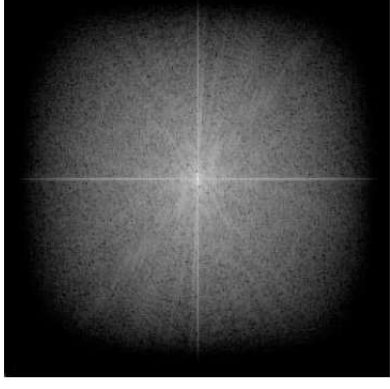
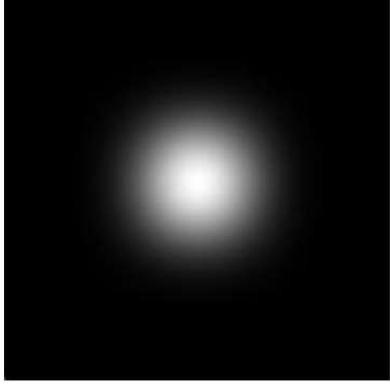
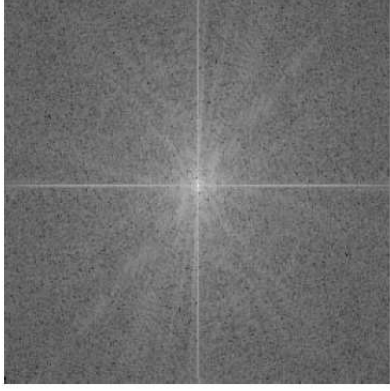
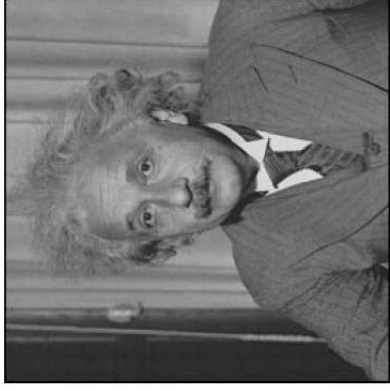
$$F(\omega_x, \omega_y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) \exp(-i\omega_x x) \exp(-i\omega_y y) dx dy$$

Remember that Spectral analysis is well suited for signal that is observed at regular interval. In particular, 2D Fourier Transform is suitable to analyse images:

Definition (image)

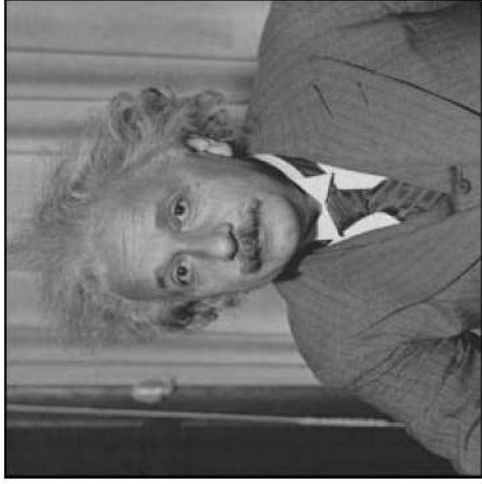
An image can be understood as a stochastic process $s(x)$ that has been observed at J locations $\{x_j\}_{j=1, \dots, J}$ lying on a regular 2D grid. For grey level images, the state space of s is all integers between 0 (black) and 255 (white).

Two-dimensional Fourier transform II



Low pass filtering

Two-dimensional Fourier transform III



High pass filtering

