Solution: Diffusion equation I

1. Using Forward difference and the second derivative difference:

$$rc \cdot \frac{v(x_n, t_{m+1}) - v(x_n, t_m)}{h_t} = \frac{v(x_{n+1}, t_m) - 2v(x_n, t_m) + v(x_{n-1}, t_m)}{h_x^2}$$

with $t_{m+1} = t_m + h_t$ and $x_{n+1} = x_n + h_x$. It is equivalent as:

$$v(x_n, t_{m+1}) = v(x_n, t_m) + \frac{h_t}{rch_x^2} \cdot (v(x_{n+1}, t_m) - 2v(x_n, t_m) + v(x_{n-1}, t_m))$$

the initial conditions are $v(x_n, t_0 = 0) = 0V \quad \forall n > 0$, and $v(x_0 = 0, t_m) = 5V \ \forall m$ to start to compute the algorithm.

Solution: Diffusion equation II

2. So $x_0 = 0$ and $x_{20} = l$ so $h_x = \frac{l}{20} = \frac{2000}{20} = 100\mu m$ or $10^{-4}m$. We generate the sequence of locations $x_{n+1} = x_n + h_x$.

Solution: Diffusion equation III

3. `xls/DiffEq.xls`
   From the simulation, you see that:
   - there is propagation for $k \leq 0.5$
   - For $k > 0.5$, the signal value appears to oscillate along the wire. This solution does not make sense: so you should choose the time step h_t and space step h_x such that $k \leq 0.5$ (optimal value is $k = 0.5$).

Solution: Diffusion equation IV

4. We want to verify that $t_m \propto x_n$. For each time slot, we determine the first location where the voltage is less that the threshold (here I chose $v < 0.5v$).