Assignment II - CS7ET01

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1. **Statistical Moments.** Consider the function $f:\ (x,y) \in \mathbb{R}^2 \rightarrow f(x,y)$ defined as:

$$f(x,y) = \begin{cases} \frac{1}{\pi R^2} & \forall (x,y):\ (x-1)^2+(y-2)^2 \leq R^2 \\ 0 & \text{otherwise} \end{cases}$$

where $R > 0$ is a constant.

   (a) Show that f is a probability density function.

   (b) The central moments (in 2D) are defined as:

$$\theta_{n,m} = \int\int (x-\mu_x)^n (y-\mu_y)^m\ f(x,y)\ dx\ dy$$

   with $\mu_x = \int\int x^1 y^0\ f(x,y)\ dx\ dy$ and $\mu_y = \int\int x^0 y^1\ f(x,y)\ dx\ dy$.
   Compute the first, and second central moments of f.

   (c) Plot f and give a geometric meaning to its moments.

2. **PCA.** Read *Moghaddam B, Pentland A (1997) Probabilistic visual learning for object representation. IEEE Trans Pattern Anal Machine Intell 19(7):696–710.* Explain how the PCA is used for modelling a probability density function (max 30 lines).

3. **Inversion method.** Consider the Laplace density function:

$$f(x|\mu,b) = \frac{1}{2b}\exp\left(-\frac{|x-\mu|}{b}\right)$$

   (a) Compute the cumulative p. d. f. $F(x)$

   (b) Compute the inverse cumulative p. d. f. $F^{-1}(x)$

   (c) Use matlab to generate samples of the laplace distribution using the inversion method.

4. **Numerical Integration.** Consider the following probability density function

$$p(x) = \frac{1}{\sqrt{2\pi}\sigma}\exp\frac{-x^2}{2\sigma^2}$$

   (a) Compute numerically the probability that $x \in [-\sigma;\sigma]$ using the trapezoidal and the Simpson methods for different values of h.

   (b) Compute numerically the probability that $x \in [-2\sigma;2\sigma]$ using the trapezoidal and the Simpson methods for different values of h.

   (c) Compute numerically the probability that $x \in [-3\sigma;3\sigma]$ using the trapezoidal and the Simpson methods for different values of h.

5. **Random numbers.**

   (a) Read `http://www.mathworks.com/moler/random.pdf`, and summarize it (max. 30 lines).

   (b) Consider the surface domain defined by $D:\ x^2+y^2 \leq 3^2$ and $f(x,y) = \exp(-x^3+2y^2)$ for $(x,y) \in D$, 0 otherwise.

   i. Propose a way to randomly find n points in D.
   ii. Solve with Matlab the Monte Carlo integration of f.

6. **Fourier.**
   (a) Is it possible to discretely sample the function $y(t) = \cos(a\ t)$ without losing any information? (i.e., can we sample $y(t)$ discretely, and be able to reconstruct the continuous $y(t)$ back from its discrete samples?)
   (b) If the answer is Yes - then what is the maximal allowed distance between the samples? If the answer is No - explain why.