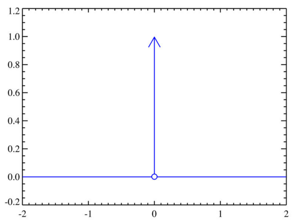
The Dirac function I

Definition

The Dirac function is defined as:

$$\delta(t) = \begin{cases} 0, & t \neq 0 \\ \infty, & t = 0 \end{cases}$$

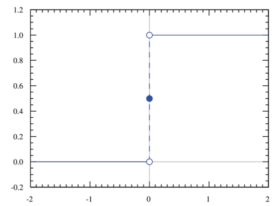


The Dirac function II

Properties of the Dirac function:

- $$\int_{-\infty}^{\infty} \delta(t)\ dt = 1$$
- The Heaviside function is the primitive of the Dirac function:

$$H(x) = \int_{-\infty}^{x} \delta(t)\ dt$$



The Dirac function III

- Sampling a function f can be written as:

$$\int_{-\infty}^{+\infty} f(t)\ \delta(t-t_0)\ dt = f(t_0)$$

- The Laplace Transform of the Dirac function is:

$$\int_{0}^{\infty} \delta(t)\ \exp(-st)\ dt = 1$$

Vector Space I

Let V be a set on which addition and scalar multiplication are defined. Assume the following conditions to be true for any $\mathbf{u}$, $\mathbf{v}$, $\mathbf{w} \in V$ and scalars r, s (belong $\mathbb{R}$ or $\mathbb{C}$):

1. $\mathbf{u}+\mathbf{v} = \mathbf{v}+\mathbf{u}$ (commutativity)
2. $(\mathbf{u}+\mathbf{v})+\mathbf{w} = \mathbf{u}+(\mathbf{v}+\mathbf{w})$ (associativity)
3. $\exists \mathbf{0}_V \in V$ such that $\mathbf{u}+\mathbf{0}_V = \mathbf{u}$, $\forall \mathbf{u} \in V$
4. $\forall \mathbf{u} \in V$, $\exists -\mathbf{u}$ such that $\mathbf{u}+(-\mathbf{u}) = \mathbf{0}_V$.
5. $r(\mathbf{u}+\mathbf{v}) = r\mathbf{u}+r\mathbf{v}$
6. $(r+s)\mathbf{u} = r\mathbf{u}+s\mathbf{u}$
7. $(rs)\mathbf{u} = r(s\mathbf{u})$
8. $1 \cdot \mathbf{u} = \mathbf{u}$

Then V is a vector space.

Vector Space II

Example (Vector spaces V)

- Set $\{0\}$
- Set $\mathbb{R}^n$
- Set of matrices $\mathbf{M}_{mn}$
- Set of polynomial functions P_n
- Set of continuous functions $C[a,b]$.

Vector Space III

Lets take the set of polynomial functions P_n of degree n:

$$p(x) = a_0 + a_1\ x + a_2\ x^2 + \cdots + a_n\ x^n$$

$p(x)$ is an element of the vector space P_n.
The monomonial basis is the basis of function $e_i(x) = x^i$. So any vector $p(x) \in P_n$ can be written as a linear combination over the basis $\{e_i(x)\}_{i=1,\cdots n}$.

Vector Space IV

Definition

A basis $\{e_i\}$ in a vector space is a set of vectors that can described any vector in that vector space. Moreover each basis vector is independent from all the other basis vectors (i.e. you cannot express a basis vector as a linear combination of the others). An orthogonal basis has the property:

$$<e_i, e_j> = \delta_{ij}$$

Vector Space V

Vector space can be of finite dimension (e.g. $\mathbb{R}^2$ is of dimension 2) or on infinite dimension (e.g. the set of all polynoms).

Definition

An integral transform $\mathcal{T}$ is defined as:

$$\mathcal{T}[f(t)] \equiv F(u) = \int_{t_1}^{t_2} K(t,u)\ f(t)\ dt$$

where K is the kernel function. This can also be understood as a linear combination ('continuous sum') over a basis of functions K.

The Laplace transform is an example of integral transform with $K(t,u) = \exp(-ut)$. The Fourier transform is another example.