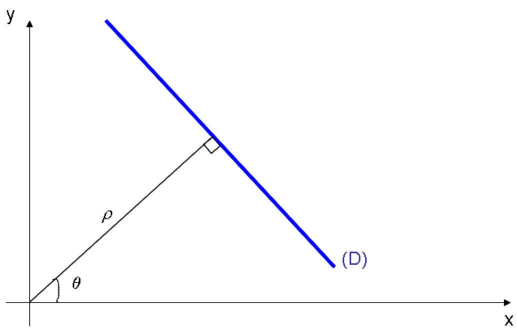
Standard Hough Transform I



Equation of a line (D): $\rho = x \cos\theta + y \sin\theta$

Standard Hough Transform II

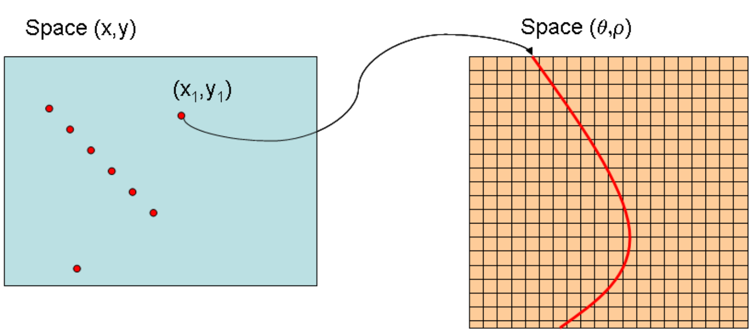
Proposed in 1962, the Hough transform aims to estimate lines from a set of points $\mathscr{S} = \{(x_i, y_i)\}_{i \cdots n}$.

1. Create a 2 dimensionnal array for the variable (θ, ρ) with zeros entries.
2. for each observation (x_i, y_i), increment all the cells in the array such that

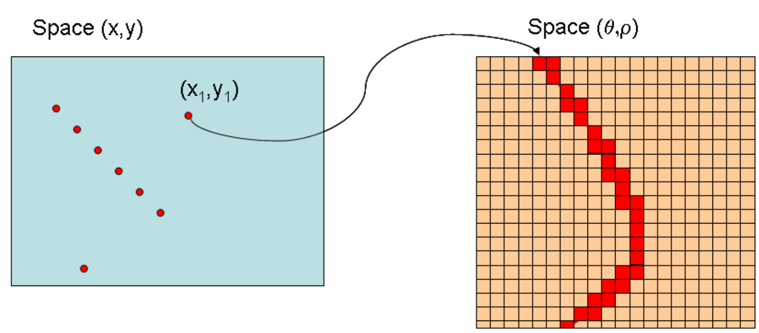
$$\rho = x_i \cos\theta + y_i \sin\theta$$

   This array is then a two dimensional histogram estimate of $p(\theta, \rho)$.
3. Detect the maxima in the $p(\theta, \rho)$

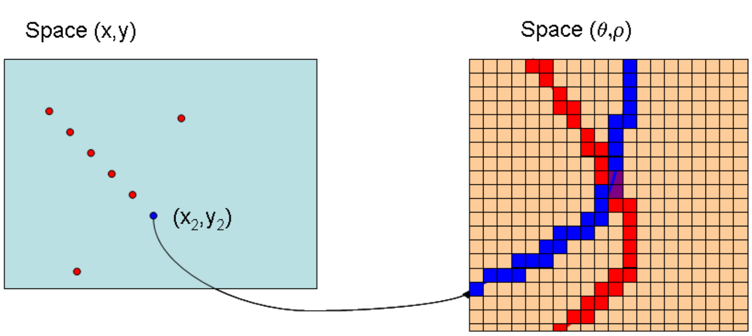
Standard Hough Transform III



Standard Hough Transform IV



Standard Hough Transform V



Standard Hough Transform VI

Hough transform interpretation using relation between variables x, y, θ, ρ

if you consider the r.v. (θ, ρ, x, y) then:

$$\begin{aligned} f(\theta,\rho,x,y) &= f(\rho|\theta,x,y)\ f(\theta,x,y) \\ &= \delta(\rho - x\cos\theta - y\sin\theta)\ f(\theta,x,y) \\ &\simeq \delta(\rho - x\cos\theta - y\sin\theta)\ f(\theta)\ f(x,y) \quad \text{assuming } \theta,x,y \text{ independent} \end{aligned}$$

- Given θ, x, y we can infer ρ, consequently $f(\rho|\theta,x,y) = \delta(\rho - x\cos\theta - y\sin\theta)$.
- $f(\theta) = \frac{1}{\pi}$: uniform distribution in $[-\pi/2; \pi/2]$ (we dont know anything about θ).
- $f(x,y)$ p.d.f. of (x,y): it can be estimated from the observations $\{(x_i, y_i)\}$.

Then

$$f(\theta,\rho) = \int\int \frac{1}{\pi} \delta(\rho - x\cos\theta - y\sin\theta)\ f(x,y)\ dx\ dy$$

Radon transform

Definition (Radon tranform)

The radon transform of a function $g(x,y)$ is defined as:

$$R[g](\theta,\rho) = \int\int \delta(\rho - x\cos\theta - y\sin\theta)\ g(x,y)\ dx\ dy$$

So the Hough transform can be understood as being proportional to the Radon transform of the probability density function of the spatial coordinates (x,y).

Conclusion

To useful tricks that we used are:

- When you look for a marginal $f(z)$ of a random variable Z, express the marginal as an integral of a joint probability function of Z and other random variables you know about. For instance when $Z = g(X,Y)$ (g a function) and you know about X and Y then:

$$f(z) = \int\int f(x,y,z)\ dx\ dy$$

- Then, using the equality $f(x,y,z) = f(z|x,y)\ f(x,y)$, we use a second trick using the Dirac function:

$$f(z|x,y) = \delta(z - g(x,y))$$

(Given X and Y, thanks to the relation g, you can infer Z.)