Introduction

Today[1]:

- Classification
- Cumulative density function
- Transformation between variables.
- Hough and Radon tranform.

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Classification I

Consider the sample space set of all the students of IET

$$\Omega_{IET} = \{\omega_i\}_{i=1,\cdots,13}$$

where ω_i is the student i. A camera is filming one student of IET, and from the audio signal $\mathbf{x}_a$ and visual signal $\mathbf{x}_v$ (we note $\mathbf{x} = (\mathbf{x}_a, \mathbf{x}_v)$) we observe, we want to infer the identity of this student.

Definition (Maximum a Posteriori Classifier)

Considering a observed random variable $\mathbf{x}$

$$\hat{\omega}_{MAP} = \arg\max_{\omega\in\Omega} P(\omega|\mathbf{x}) = \arg\max_{\omega\in\Omega} \frac{P(\mathbf{x}|\omega)P(\omega)}{P(\mathbf{x})}$$

Classification II

Applied to our problem of IET students recognition, we need to estimate $P(\omega_i|\mathbf{x})$ for all students i and select the one that has the maximum posterior probability.
Questions:

1. What would you choose to model the marginals $\{P(\omega_i)\}$?
2. What would you do to estimate $P(\mathbf{x}|\omega_i)$?

Classification III

One easy method proposed for classification is the K nearest neighbour.

1. Draw in the feature space the training samples corresponding to each student.
2. When you have a new observation $\mathbf{x}$ find its K nearest neighbours.
3. Among those K nearest neighbours, find the class that is the most represented.

Transformation in between two random variables $Y = g(X)$

Definition (Cumulative density function)

The cumulative density function of a random variable X with p.d.f. $f_X(x)$ is defined as

$$F_X(x) = P(X \leq x) = \int_{-\infty}^{x} f_X(t)\, dt$$

Theorem (Transformations in between two random variables)

Consider two random variables X and Y such that $Y = g(X)$ with g a monotonic an differentiable function ($g'(x) > 0,\ \forall x$). If $F_X(x)$ and $F_Y(y)$ are the respective c.d.fs. then

$$F_X(g^{-1}(y)) = F_Y(y)$$

Exercises

1. Prove the previous theorem.
2. Consider $X \sim \mathcal{N}(0,1)$ and $Y = aX + b$ with $a > 0$.
   1. What is the probability density function $f_Y(y)$?
   2. Compute mean and variance of Y.

   Consider the random variable $Z = X + Y$. Compute $f_Z(z)$ w.r.t. $f_X(x)$ and $f_Y(y)$.