## Introduction

The Fourier transform is an important tool to analyse signals, discrete or continuous.

A Wavelet tour of signal processing , by S. Mallat, second edition

## Fourier transform

**Definition (Fourier transform)**

The Fourier integral measures *how much* oscillations at the frequency $\omega$ there is the function $f$:

$$F(\omega) = \int_{-\infty}^{\infty} f(t)\ \exp^{(-\imath\omega t)}\ dt$$

## Inverse Fourier transform

**Definition (Fourier transform)**

The Inverse Fourier transform can be computed:

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)\ \exp^{(\imath\omega t)}\ d\omega$$

## Exercises: Fourier transform

Compute the Fourier transforms of:

1. $$f(t) = \begin{cases} 1 & -T < t < T \\ 0 & \text{otherwise} \end{cases}$$
2. $f(t) = \delta(t)$
3. $f(t) = \delta(t - t_0)$
4. $f(t) = \exp(-at)\ h(t)$ ($h$ is the heaviside step function)
5. $f(t) = \cos(\omega_0 t)$

## Properties of the Fourier Transform

1. Linearity:
$$a\ x(t) + b\ y(t) \leftrightarrow a\ X(\omega) + b\ Y(\omega)$$
2. Time shifting:
$$x(t - t_0) \leftrightarrow \exp(-\imath\omega t_0)X(\omega)$$

Questions: Proove the time shifting property.

## Convolution

**Definition (Convolution)**

The convolution of two functions $x(t)$ and $h(t)$ gives a function $y(t)$ such that:

$$y(t) = x(t) * h(t) = \int_{-\infty}^{+\infty} x(\tau)\ h(t-\tau)\ d\tau$$

**Theorem (Convolution)**

*If $y(t) = x(t) * h(t)$ then:*

$$Y(\omega) = X(\omega)\cdot H(\omega)$$

Proove this theorem.