Trinity College Dublin
Coláiste na Tríonóide, Baile Átha Cliath
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ASSIGNMENT 5: BASIC COLLISION RESPONSE

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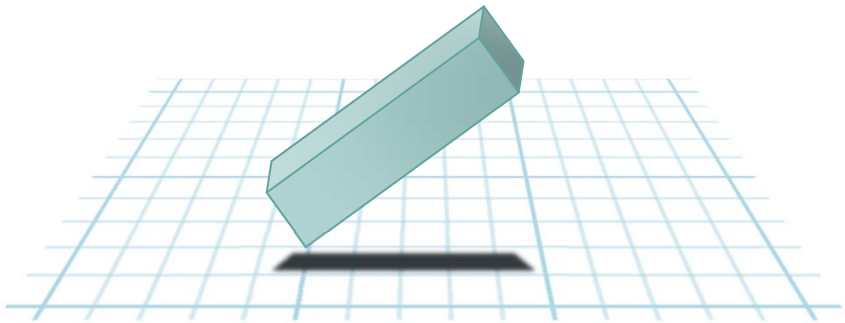
ASSIGNMENT 5: COLLISION RESPONSE

Implement collision response system for a rigid body.

- Basic requirement requires correct simulation of a convex object falling onto an infinite plane



Required Reading: [Baraff] or [Hecker]

Minimum requirements:

- Minimum scene: convex object 3D object e.g. cube colliding against infinite plane
- Rigid Body state (from previous)
- Unconstrained State update (from previous)
- Detect a point of collision and collision normal (from previous)
- **Impulse magnitude j calculation**
- Application of impulsive force and torque

Additional Features + Options:

- Object-object collisions
- General objects i.e. beyond a cube
- c.o.m. and Inertia tensor computation
- Backtracking, adaptive timesteps, springs or some corrective measure
- Any additional features/customizations
- Cool demos

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NOTES ON COMPLETING THIS ASSIGNMENT

BASIC STEPS (OVERVIEW)

1. **Make sure you read the following or equivalent*:**
   - [Baraff] Colliding Contact OR
   - [Hecker] Rigid Body Dynamics
2. **Get a collision point, $\boldsymbol{p}$ and normal, $\hat{\boldsymbol{n}}$**
   - This is input from narrow-phase
3. **Calculate the Impulse Magnitude, j**
   - You will need to know the inertial tensor (see Lecture 2)
4. **Apply an Impulse, $\boldsymbol{J} = j\hat{\boldsymbol{n}}$, to update the angular and linear momenta of each colliding rigid body**

*: Texts by Eberley, Millington, Ericson or Erleben et al will also cover this material but may take a slightly different approach

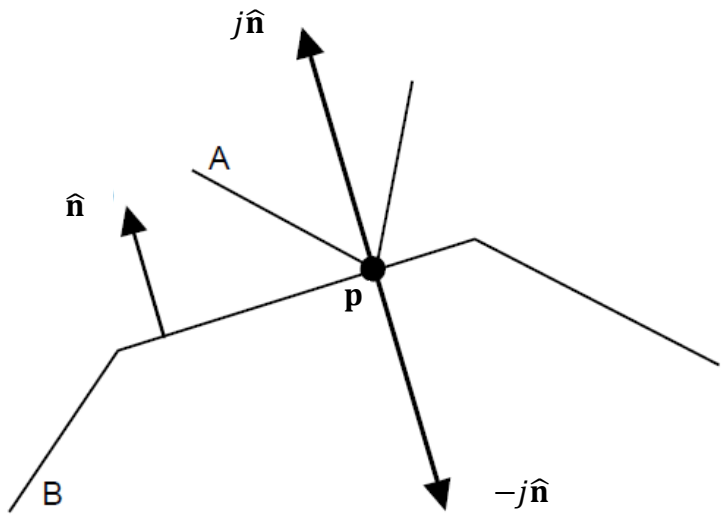
COLLISION RESPONSE

Calculate instantaneous change in rigid body state (linear and angular momenta)

Based on impulse J

We require:

- Point of contact, $\boldsymbol{p}$
- Direction (contact normal), $\hat{\boldsymbol{n}}$
- Magnitude of impulse, j



$\boldsymbol{p}$ and $\hat{\boldsymbol{n}}$ are obtained as part of contact modelling. j is calculated based on a number of variables …

IMPULSE MAGNITUDE EQUATION

N.B. YOU WILL NEED TO IMPLEMENT THIS

$$j = \frac{-(1+\epsilon)\ v_{rel}^{-}}{m_A^{-1} + m_B^{-1} + \hat{\boldsymbol{n}}\left(\boldsymbol{I}_A^{-1}(\boldsymbol{r}_A \times \hat{\boldsymbol{n}})\right) \times \boldsymbol{r}_A + \hat{\boldsymbol{n}}\left(\boldsymbol{I}_B^{-1}(\boldsymbol{r}_B \times \hat{\boldsymbol{n}})\right) \times \boldsymbol{r}_B}$$

$$\boldsymbol{I}_A^{-1} = \boldsymbol{R}_A \boldsymbol{I}_{bodyA}^{-1} \boldsymbol{R}_A^T \quad \text{and} \quad \boldsymbol{I}_B^{-1} = \boldsymbol{R}_B \boldsymbol{I}_{bodyB}^{-1} \boldsymbol{R}_B^T$$

$$\boldsymbol{r}_A = \boldsymbol{p}_A - \boldsymbol{x}_A \quad \text{and} \quad \boldsymbol{r}_B = \boldsymbol{p}_B - \boldsymbol{x}_B$$

$$v_{rel}^{-} = \hat{\boldsymbol{n}}(\dot{\boldsymbol{p}}_A^{-} - \dot{\boldsymbol{p}}_B^{-})$$

$$\dot{\boldsymbol{p}}_A = \boldsymbol{v}_A + \boldsymbol{\omega}_A \times (\boldsymbol{p}_A - \boldsymbol{x}_A) \quad \text{and} \quad \dot{\boldsymbol{p}}_B = \boldsymbol{v}_B + \boldsymbol{\omega}_B \times (\boldsymbol{p}_B - \boldsymbol{x}_B)$$

j: impulse magnitude

ϵ: coefficient of restitution

m_A, m_B: mass of object A and B

$\mathbf{R}_A, \mathbf{R}_B$: orientation of A and B

$\boldsymbol{\omega}_A, \boldsymbol{\omega}_B$: angular velocity of A and B

$\mathbf{x}_A, \mathbf{x}_B$: centre of mass position of A and B

$\mathbf{v}_A, \mathbf{v}_B$: velocity of A and B

v_{rel}^{-}: pre-collision relative velocity of contact points

$\mathbf{I}_A, \mathbf{I}_B$: world space inertial tensor

$\mathbf{I}_{bodyA}, \mathbf{I}_{bodyB}$: object space inertial tensor

$\hat{\mathbf{n}}$: contact plane normal

$\mathbf{p}_A, \mathbf{p}_B$: contact point on A and B

IMPULSE MAGNITUDE EQUATION

WHERE THE VARIABLES COME FROM

$$j = \frac{-(1+\epsilon)\, v_{rel}^{-}}{m_A^{-1} + m_B^{-1} + \hat{n}\left(I_A^{-1}(r_A \times \hat{n})\right) \times r_A + \hat{n}\left(I_B^{-1}(r_B \times \hat{n})\right) \times r_B}$$

$$I_A^{-1} = R_A I_{bodyA}^{-1} R_A^T \quad \text{and} \quad I_B^{-1} = R_B I_{bodyB}^{-1} R_B^T$$

$$r_A = p_A - x_A \quad \text{and} \quad r_B = p_B - x_B$$

$$v_{rel}^{-} = \hat{n}(\dot{p}_A^{-} - \dot{p}_B^{-})$$

$$\dot{p}_A = v_A + \omega_A \times (p_A - x_A) \quad \text{and} \quad \dot{p}_B = v_B + \omega_B \times (p_B - x_B)$$

Constants

Intermediate Variables

Rigid Body State

Contact Model

j: impulse magnitude

ϵ: coefficient of restitution

m_A, m_B: mass of object A and B

$\mathbf{R}_A, \mathbf{R}_B$: orientation of A and B

$\boldsymbol{\omega}_A, \boldsymbol{\omega}_B$: angular velocity of A and B

$\mathbf{x}_A, \mathbf{x}_B$: centre of mass position of A and B

$\mathbf{v}_A, \mathbf{v}_B$: velocity of A and B

v_{rel}^{-}: pre-collision relative velocity of contact points

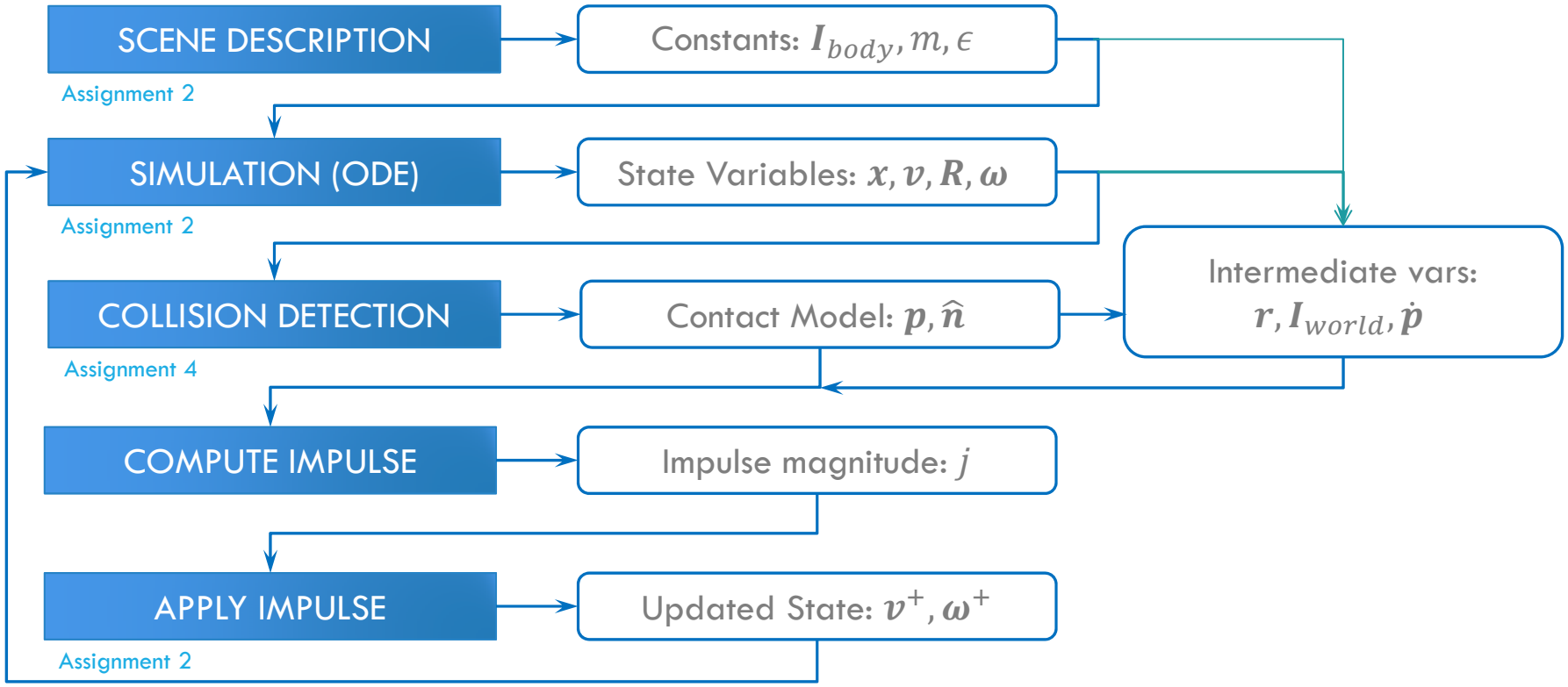
$\mathbf{I}_A, \mathbf{I}_B$: world space inertial tensor

$\mathbf{I}_{bodyA}, \mathbf{I}_{bodyB}$: object space inertial tensor

$\hat{\mathbf{n}}$: contact plane normal

$\mathbf{p}_A, \mathbf{p}_B$: contact point on A and B

COLLISION RESPONSE: INPUTS AND OUTPUTS



IMPLEMENTATION

For ease of coding, Baraff et al break down the impulse equation as follows:

$$j = \frac{N}{t_1 + t_2 + t_3 + t_4}$$

where

- $N = -(1+\epsilon)v_{rel}$
- $t_1 = \frac{1}{m_A}$
- $t_2 = \frac{1}{m_B}$
- $t_3 = \hat{\mathbf{n}}\left(\left(\mathbf{I}_A^{-1}(\mathbf{r}_A \times \hat{\mathbf{n}})\right) \times \mathbf{r}_A\right)$
- $t_4 = \hat{\mathbf{n}}\left(\left(\mathbf{I}_B^{-1}(\mathbf{r}_B \times \hat{\mathbf{n}})\right) \times \mathbf{r}_B\right)$

```
void collision(Contact *c, double epsilon)
{
    triple  padot = pt_velocity(c->a, c->p), /* ṗ_a^-(t_0) */
            pbdot = pt_velocity(c->b, c->p), /* ṗ_b^-(t_0) */
            n = c->n,                        /* n̂(t_0) */
            ra = p - c->a->x,                /* r_a */
            rb = p - c->b->x;                /* r_b */
    double  vrel = n * (padot - pbdot),      /* v_rel^- */
            numerator = -(1 + epsilon) * vrel;

    /* We'll calculate the denominator in four parts */
    double  term1 = 1 / c->a->mass,
            term2 = 1 / c->b->mass,
            term3 = n * ((c->a->Iinv * (ra ^ n)) ^ ra),
            term4 = n * ((c->b->Iinv * (rb ^ n)) ^ rb);

    /* Compute the impulse magnitude */

    double  j = numerator / (term1 + term2 + term3 + term4);
    triple  force = j * n;

    /* Apply the impulse to the bodies */
    c->a->P += force;
    c->b->P -= force;
    c->a->L += ra ^ force;
    c->b->L -= rb ^ force;

    /* recompute auxiliary variables */
    c->a->v = c->a->P / c->a->mass;
    c->b->v = c->b->P / c->b->mass;

    c->a->omega = c->a->Iinv * c->a->L;
    c->b->omega = c->b->Iinv * c->b->L;
}
```

NOTES

For efficiency, we typically store..

- $\frac{1}{m}$ (inverse mass) instead of m
- and $\mathbf{I}_{body}^{-1}$(inverse of inertia tensor in body space) instead of $\mathbf{I}_{body}$

For immovable objects (e.g. ground plane) use

- Mass: $\frac{1}{m} = 0$ i.e. infinite mass
- Inertial Tensor: $\mathbf{I}_{body}^{-1} = \mathbf{0}$ i.e. infinite moment of inertia

zero matrix (all entries 0)

- t_2 and t_4 are 0 in this case

APPLYING IMPULSE

Impulse magnitude is given by:

$$j = \frac{-(1+\epsilon)\ v_{rel}^{-}}{m_A^{-1} + m_B^{-1} + \hat{\boldsymbol{n}}\left(\boldsymbol{I}_A^{-1}(\boldsymbol{r}_A \times \hat{\boldsymbol{n}})\right) \times \boldsymbol{r}_A + \hat{\boldsymbol{n}}\left(\boldsymbol{I}_B^{-1}(\boldsymbol{r}_B \times \hat{\boldsymbol{n}})\right) \times \boldsymbol{r}_B}$$

The actual impulse vector is simply $\boldsymbol{J} = j\hat{\boldsymbol{n}}$

This is applied to the objects as follows:

- Change in Linear momentum is directly equal to the impulse:

$$\Delta \boldsymbol{P} = \boldsymbol{J} \quad \Leftrightarrow \quad \Delta \boldsymbol{v} = \boldsymbol{J} m^{-1}$$

- Change in Angular momentum is equal to the imulsive torque ($\boldsymbol{\tau}_{\mathrm{IMPULSE}}$):

$$\Delta \boldsymbol{L} = (\boldsymbol{r} \times \boldsymbol{J}) \quad \Leftrightarrow \quad \Delta \boldsymbol{\omega} = \boldsymbol{I}^{-1}(\boldsymbol{r} \times \boldsymbol{J})$$

REFERENCES

REQUIRED READING (either one of):

[Hecker] Chris Hecker has a few Gamasutra Articles on the topic that are relatively easy reading:

- In particular, "Physics Part 4 – The 3rd Dimension"
- Available http://chrishecker.com/Rigid_Body_Dynamics

[Baraff] Baraff Witkin Kass Lecture 2001

- Rigid Body Dynamics Notes - Collision and Contact
- http://www.pixar.com/companyinfo/research/pbm2001/

ALSO SEE:

Erik Neumann's MyPhysics Lab: http://www.myphysicslab.com/collision.html

Glen Fiedler "Physics in 3D" : http://gafferongames.com/game-physics/physics-in-3d/