Communicating Transactions

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Outline

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- Isolation:
  - good: provides coherent semantics
  - bad: limits concurrency
  - bad: limits co-operation between transactions and their environments

Communicating Transactions

- We *drop isolation* to *increase concurrency*
  - There is no limit on the communication between a transaction and its environment
- These new transactional systems guarantee:
  - Atomicity: Each transaction will either run in its entirety or not at all
  - Consistency: When faults are detected the transaction is automatically rolled-back, *together with all effects of the transaction on its environment*
  - Durability: After *all transactions that have interacted* commit, their effects are permanent (coordinated checkpointing)

Outline

Introduction

TransCCS

Liveness and safety properties

Compositional semantics

TransCCS

An extension of CCS with communicating transactions.

1. Simple language: 2 additional language constructs and 3 additional reduction rules.
2. Intricate concurrent and transactional behaviour:
   - encodes nested, restarting, and non-restarting transactions
   - does not limit communication between transactions
3. Simple behavioural theory: based on properties of systems:
   - *Safety* property: nothing bad happens
   - *Liveness* property: something good happens

TransCCS

Syntax:	P, Q	$::=$	$\sum \mu_i.P_i$	guarded choice
		$\mid$	$P \mid Q$	parallel
		$\mid$	$\nu a.P$	hiding
		$\mid$	$\mu X.P$	recursion
		$\mid$	$[\![P \rhd_k Q]\!]$	transaction (k bound in P)
		$\mid$	co k	commit

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- execute P to completion ($\text{co } k$)
- subject to random aborts
- if aborted roll back all effects of P and initiate Q

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Transaction $[\![P \triangleright_k Q]\!]$

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- roll back includes . . . environmental impact of P

Rollbacks and Commits

Co-operating actions: $\boxed{a \longleftarrow \text{needs co-operation of} \longrightarrow \overline{a}}$

Rollbacks and Commits

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$$T_a \mid T_b \mid T_c \mid P_d \mid P_e$$

where

$$\begin{aligned}
T_a &= [\![\overline{d}.\overline{b}.(\text{co } k_1 \mid a) \triangleright_{k_1} \mathbf{0}]\!] \\
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- if T_c aborts, what roll-backs are necessary?

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- if T_c aborts, what roll-backs are necessary?
- When can action a be considered permanent?
- When can code R_d be considered permanent?

Reduction semantics main rules

R-Comm

$$\frac{a_i = \overline{b}_j}{\sum_{i \in I} a_i.P_i \mid \sum_{j \in J} b_j.Q_j \rightarrow P_i \mid Q_j}$$

Communication

R-Co

$$\frac{}{\llbracket P \mid \text{co } k \triangleright_k Q \rrbracket \rightarrow P}$$

Commit

R-Ab

$$\frac{}{\llbracket P \triangleright_k Q \rrbracket \rightarrow Q}$$

Random abort

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Commit

R-AB

$$\frac{}{[\![P \triangleright_k Q]\!] \to Q}$$

Random abort

R-EMB

$$\frac{k \notin R}{[\![P \triangleright_k Q]\!] \mid R \to [\![P \mid R \triangleright_k Q \mid R]\!]}$$

Embed

Simple Example

Convention:

- ω: I am happy
- $\mho$: I am sad

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$$\xrightarrow{\text{R-Emb}} \llbracket a.c.\omega + e.\mho \mid \overline{a}.\overline{c}.co\ k + \overline{e} \rhd_k a.c.\omega + e.\mho \mid r \rrbracket$$

$$\xrightarrow{\text{R-Comm}} \llbracket c.\omega \mid \overline{c}.co\ k \rhd_k a.c.\omega + e.\mho \mid r \rrbracket$$

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$$\xrightarrow{\text{R-CO}} \omega$$

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Simple Example (a second trace)

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$$\xrightarrow{\text{R-COMM}} [\![\quad \mho \quad \rhd_k a.c.\omega + e.\mho \mid r]\!] \quad \text{(Deadlocked)}$$

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$$\xrightarrow{\text{R-COMM}} [\![\quad \mho \quad \rhd_k a.c.\omega + e.\mho \mid r]\!]$$

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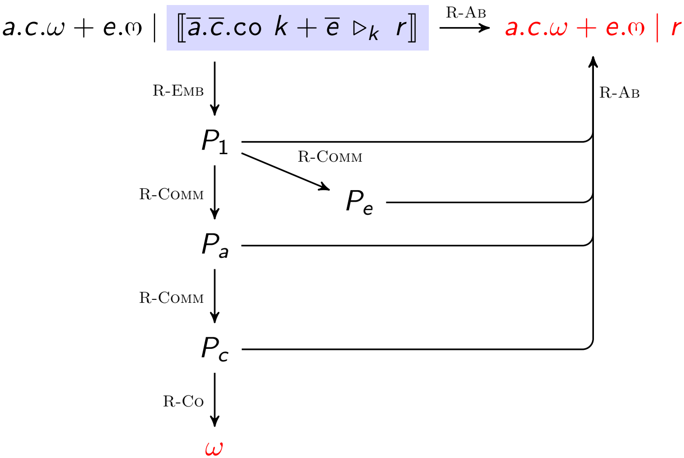
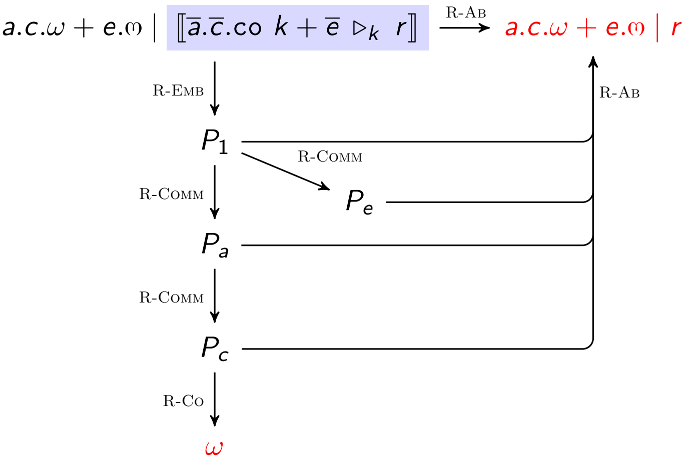
$$\xrightarrow{\text{R-COMM}} [\![\mho \rhd_k a.c.\omega + e.\mho \mid r]\!]$$

$$\xrightarrow{\text{R-AB}} a.c.\omega + e.\mho \mid r$$ (The environment is restored)

Simple Example (all traces)

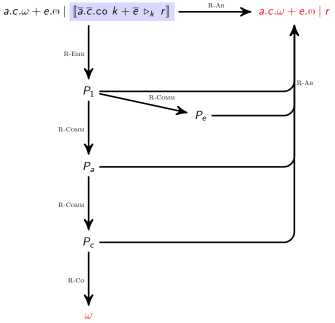


Simple Example (all traces)



Will never be sad: $\mho$ assuming r does not contain $\overline{e}$

Aborting transactions



A commit step makes the effects of the transaction permanent (**Durability**)

An abort step:

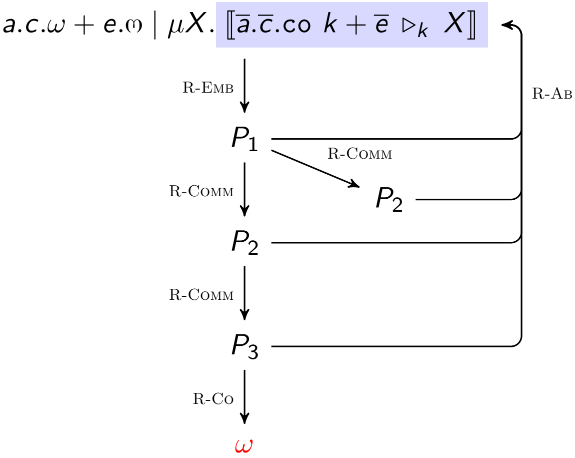
- restarts the transaction
- rolls-back embedded processes to their state before embedding (**Consistency**)
- does not roll-back actions that happened before embedding
- does not affect non-embedded processes

The behavioural theory will show the **Atomicity** property.

Restarting transactions

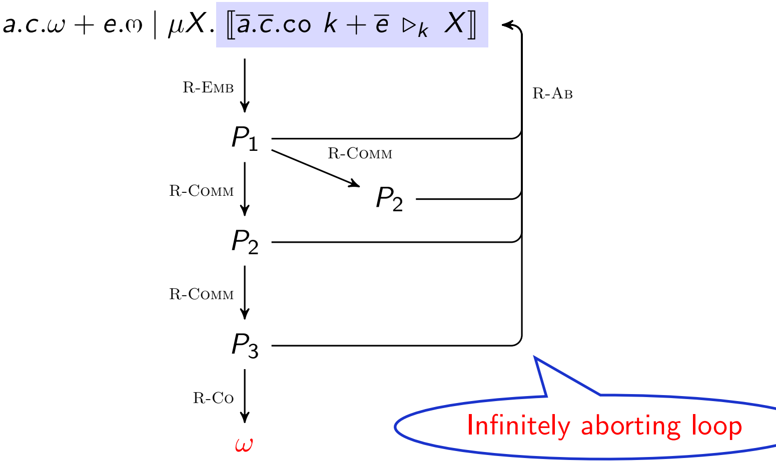
$$a.c.\omega + e.\mho \mid \mu X. \llbracket \overline{a}.\overline{c}.co\ k + \overline{e} \triangleright_k X \rrbracket$$

Restarting transactions



Will never be sad: $\mho$

Restarting transactions



Will never be sad: ℧

Double Embedding

$$[\![a.\mathrm{co}\ k \mid b \rhd_k \mathbf{0}]\!] \mid [\![\overline{a}.\mathrm{co}\ l \mid c \rhd_l \mathbf{0}]\!]$$

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$$\xrightarrow{\text{R-EMB}} \left[\!\!\left[a.\mathrm{co}\ k \mid b \mid [\![\bar{a}.\mathrm{co}\ l \mid c \rhd_l \mathbf{0}]\!] \rhd_k [\![\bar{a}.\mathrm{co}\ l \mid c \rhd_l \mathbf{0}]\!]\right]\!\!\right]$$

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$$\xrightarrow{\text{R-EMB}} \Big[\!\!\Big[a.co\ k \mid b \mid [\![\bar{a}.co\ l \mid c \rhd_l \mathbf{0}]\!] \rhd_k [\![\bar{a}.co\ l \mid c \rhd_l \mathbf{0}]\!] \Big]\!\!\Big]$$

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$$\xrightarrow{\text{R-COMM}} \Big[\!\!\Big[b \mid [\![co\ k \mid co\ l \mid c \rhd_l a.co\ k]\!] \rhd_k [\![\bar{a}.co\ l \mid c \rhd_l \mathbf{0}]\!] \Big]\!\!\Big]$$

Double Embedding

$$\llbracket a.\mathsf{co}\ k \mid b \rhd_k \mathbb{0} \rrbracket \mid \llbracket \bar{a}.\mathsf{co}\ l \mid c \rhd_l \mathbb{0} \rrbracket$$

$$\xrightarrow{\text{R-EMB}} \left\llbracket a.\mathsf{co}\ k \mid b \mid \llbracket \bar{a}.\mathsf{co}\ l \mid c \rhd_l \mathbb{0} \rrbracket \rhd_k \llbracket \bar{a}.\mathsf{co}\ l \mid c \rhd_l \mathbb{0} \rrbracket \right\rrbracket$$

$$\xrightarrow{\text{R-EMB}} \left\llbracket b \mid \llbracket a.\mathsf{co}\ k \mid \bar{a}.\mathsf{co}\ l \mid c \rhd_l a.\mathsf{co}\ k \rrbracket \rhd_k \llbracket \bar{a}.\mathsf{co}\ l \mid c \rhd_l \mathbb{0} \rrbracket \right\rrbracket$$

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$$\xrightarrow{\text{R-CO}} \left\llbracket b \mid \mathsf{co}\ k \mid c \rhd_k \llbracket \bar{a}.\mathsf{co}\ l \mid c \rhd_l \mathbb{0} \rrbracket \right\rrbracket$$

Double Embedding

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$$\xrightarrow{\text{R-COMM}} \left[\!\!\left[b \mid [\![\mathrm{co}\ k \mid \mathrm{co}\ l \mid c \rhd_l a.\mathrm{co}\ k]\!] \rhd_k [\![\bar{a}.\mathrm{co}\ l \mid c \rhd_l \mathbb{0}]\!] \right]\!\!\right]$$

$$\xrightarrow{\text{R-CO}} \left[\!\!\left[b \mid \mathrm{co}\ k \mid c \rhd_k [\![\bar{a}.\mathrm{co}\ l \mid c \rhd_l \mathbb{0}]\!] \right]\!\!\right]$$

$$\xrightarrow{\text{R-CO}} b \mid c$$

Double Embedding

$$[\![a.co\ k \mid b \rhd_k \mathbb{0}]\!] \mid [\![\bar{a}.co\ l \mid c \rhd_l \mathbb{0}]\!]$$

$$\xrightarrow{\text{R-EMB}} \left[\!\!\left[a.co\ k \mid b \mid [\![\bar{a}.co\ l \mid c \rhd_l \mathbb{0}]\!] \rhd_k [\![\bar{a}.co\ l \mid c \rhd_l \mathbb{0}]\!] \right]\!\!\right]$$

$$\xrightarrow{\text{R-EMB}} \left[\!\!\left[b \mid [\![a.co\ k \mid \bar{a}.co\ l \mid c \rhd_l a.co\ k]\!] \rhd_k [\![\bar{a}.co\ l \mid c \rhd_l \mathbb{0}]\!] \right]\!\!\right]$$

$$\xrightarrow{\text{R-COMM}} \left[\!\!\left[b \mid [\![co\ k \mid co\ l \mid c \rhd_l a.co\ k]\!] \rhd_k [\![\bar{a}.co\ l \mid c \rhd_l \mathbb{0}]\!] \right]\!\!\right]$$

$$\xrightarrow{\text{R-CO}} \left[\!\!\left[b \mid co\ k \mid c \rhd_k [\![\bar{a}.co\ l \mid c \rhd_l \mathbb{0}]\!] \right]\!\!\right]$$

$$\xrightarrow{\text{R-CO}} b \mid c$$

Outline

Safety properties

Safety: "Nothing bad will happen" [Lamport'77]

- A safety property can be formulated as a *safety test* T^{ω} which signals on channel ω when it detects the bad behaviour

Examples:

- $\mu X.(a.X + e.\omega)$ can not perform e while performing any sequence of as
- $T^{\omega} = e.\omega \mid \overline{a}.\overline{b}$ can not perform e when a followed by b is offered.

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  - This is the negation of passing a "may test" [DeNicola-Hennessy'84]

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  - This is the negation of passing a "may test" [DeNicola-Hennessy'84]

Examples:

- $I_3 = \mu X. \llbracket a.b.co\ k + \overline{e} \triangleright_k\ X \rrbracket$ passes safety test T^{ω}
- $I_4 = \mu X. \llbracket a.b.co\ k \mid \overline{e} \triangleright_k\ X \rrbracket$ does not pass safety test T^{ω}

Safety

Definition (Basic Observable)

$P{\Downarrow}_{\omega}$ iff there exists P' such that $P \rightarrow^* P' \mid \omega$

- Basic observable actions are *permanent*

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Definition (P Passes safety test T^{ω})

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Definition (P Passes safety test $T^{\mho}$)

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Definition (Safety Preservation)

$$S \sqsubseteq_{\text{safe}} I \quad \text{when} \quad \forall T^{\mho}.\quad S \text{ cannot } T^{\mho} \quad \text{implies} \quad I \text{ cannot } T^{\mho}$$

Safety preservation: Examples

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\begin{aligned}
S_{ab} &= \mu X. [\![a.b.\text{co } k \rhd_k X]\!] \\
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Liveness

Liveness: "Something good will eventually happen" [Lamport'77]

- A liveness property can be formulated as a *liveness test* T^ω which detects and reports good behaviour on ω.

Examples:

- $T^\omega = \overline{a}.\overline{b}.\omega$ can do an a then a b
- $\mu X.\ [\![\overline{a}.\overline{b}.(\omega \mid \text{co } l) \triangleright_l X]\!]$ can eventually do an a, b uninterrupted?
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Dilemma: What does this mean?

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Does $\mu X.\ [\![a.b.\text{co}\ k \triangleright_k X]\!]$ pass liveness test $T^{\omega}_{ab} = \overline{a}.\overline{b}.\omega$?

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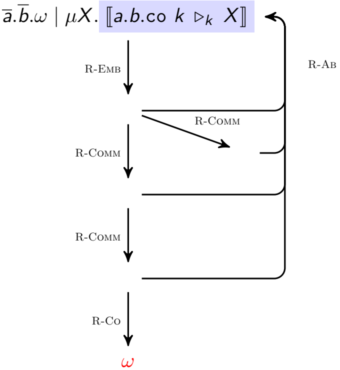

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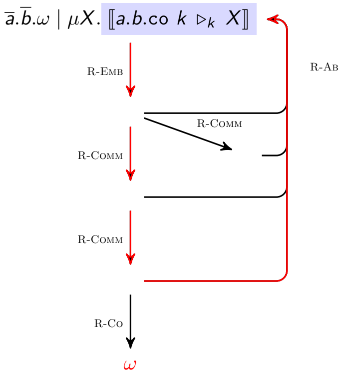

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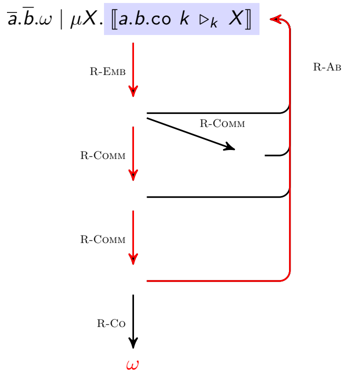

- **must-testing:** NO because of infinite loop
- **should-testing:** YES

Liveness testing

Definition (P Passes liveness test T^ω [Rensink-Vogler'07])

$$P \,\mathrm{shd}\, T^\omega \quad \text{when} \quad \forall R.\quad P \mid T^\omega \rightarrow^* R \quad \text{implies} \quad R{\Downarrow_\omega}$$

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Proof techniques:

Require characterisations using “traces” and “refusals”

Outline

Compositional Semantics

- The embedding rule is simple but entangles the processes
- We need to reason about the behaviour of $P|Q$ in terms of P and Q

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$\xrightarrow{\texttt{emb}\ k}$	$[\![a.c.\mathbb{0} \rhd_k a.c.\mathbb{0}]\!]^\circ$	$\vert$	$\xrightarrow{\texttt{emb}\ k}$	$[\![\overline{a}.\overline{c}.\mathrm{co}\ k \mid \overline{a}.\mathbb{0} \rhd_k e]\!]$
$\xrightarrow{k(a)}$	$[\![c.\mathbb{0} \rhd_k a.c.\mathbb{0}]\!]^\circ$	$\vert$	$\xrightarrow{k(\overline{a})}$	$[\![\overline{c}.\mathrm{co}\ k \mid \overline{a}.\mathbb{0} \rhd_k e]\!]$

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\begin{array}{rl|rl}
 & a.c.\mathbb{0} & & [\![\overline{a}.\overline{c}.\mathrm{co}\ k \mid \overline{a}.\mathbb{0} \rhd_k e]\!] \\
\xrightarrow{\texttt{emb}\ k} & [\![a.c.\mathbb{0} \rhd_k a.c.\mathbb{0}]\!]^\circ & \xrightarrow{\texttt{emb}\ k} & [\![\overline{a}.\overline{c}.\mathrm{co}\ k \mid \overline{a}.\mathbb{0} \rhd_k e]\!] \\
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\xrightarrow{\mathrm{co}\ k} & \mathbb{0} & \xrightarrow{\mathrm{co}\ k} & \overline{a}.\mathbb{0}
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\xrightarrow{k(a)} & [\![c.\mathbb{0} \rhd_k a.c.\mathbb{0}]\!]^\circ & & \xrightarrow{k(\overline{a})} & [\![\overline{a}.\overline{c}.\text{co } k \mid \mathbb{0} \rhd_k e]\!] \\
\xrightarrow{\texttt{ab } k} & a.c.\mathbb{0} & & \xrightarrow{\texttt{ab } k} & e
\end{array}
$$

Compositional Semantics: may-testing

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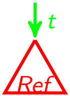
$$\exists P'. \quad P \stackrel{t}{\Rightarrow}_{CL} P' \quad \text{and} \quad \mathcal{L}(P') \cap \mathit{Ref} = \emptyset$$



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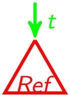

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Characterisation of should-testing:

$$S \sqsubseteq_{\text{live}} I \quad \text{iff} \quad \mathcal{F}(S) \supseteq \mathcal{F}(I)$$

Simple Examples

Let $S_{ab} = \mu X. [\![a.b.\mathsf{co}\ k \rhd_k X]\!]$ $\mathcal{L}(S_{ab}) = \{\epsilon, ab\}$

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- $S_{ab} \eqsim_{\text{safe}} I_2 = \mu X. \llbracket a.b.\mathsf{co}\ k + e \rhd_k X \rrbracket$ $\mathcal{L}(I_2) = \mathcal{L}(S_{ab})$
  $S_{ab} \eqsim_{\text{live}} I_2$ $\mathcal{F}(I_2) = \mathcal{F}(S_{ab})$

Summary

- TransCCS: a language for communicating/co-operative transactions
- simple reduction semantics using an *embedding* rule
- behavioural theories for preservation of
  - safety properties
  - liveness properties
- characterisations which allow
  - proofs of equivalences
  - equational laws

References:

- *Communicating Transactions*, Concur 2010
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Future work:

- Reference implementation
- Extension to Haskell
- PhD Scholarship position funded by Microsoft Research, UK

THANK YOU!