Real Reward Testing for Probabilistic Processes

Matthew Hennessy

(joint work with Y. Deng, R. van Glabbeek, C. Morgan)

QAPL, Saarbrucken, April 2011

TRINITY COLLEGE DUBLIN
Coláiste na Tríonóide, Baile Átha Cliath

Outline

Testing

Outcomes: resolutions v derivations

Failure simulations

Results

Outline

Testing

Outcomes: resolutions v derivations

Failure simulations

Results

Probabilistic labelled transition systems

Intensional semantics:

A process is a distribution in an pLTS

pLTSs:

$\langle S, \mathsf{Act}_\tau, \longrightarrow \rangle$

- S - states
- $\longrightarrow \subseteq S \times \mathsf{Act}_\tau \times \mathcal{D}(S)$

$\mathcal{D}(S)$: Mappings $\Delta : S \rightarrow [0,1]$ with $\sum_{s \in S} \Delta(s) = 1$

$s_1 \xrightarrow{\mu} \Delta$: process s_1

- can perform action μ
- with probability $\Delta(s_2)$ it continues as process s_2

Probabilistic labelled transition systems

Intensional semantics:

A process is a distribution in an pLTS

pLTSs:

$\langle S, \text{Act}_\tau, \longrightarrow \rangle$

- S - states
- $\longrightarrow \subseteq S \times \text{Act}_\tau \times \mathcal{D}(S)$

$\mathcal{D}(S)$: Mappings $\Delta : S \to [0,1]$ with $\sum_{s \in S} \Delta(s) = 1$

$s_1 \xrightarrow{\mu} \Delta$: process s_1

- can perform action μ
- with probability $\Delta(s_2)$ it continues as process s_2

Probabilistic labelled transition systems

Intensional semantics:

A process is a distribution in an pLTS

pLTSs:

$\langle S, \mathsf{Act}_\tau, \longrightarrow \rangle$

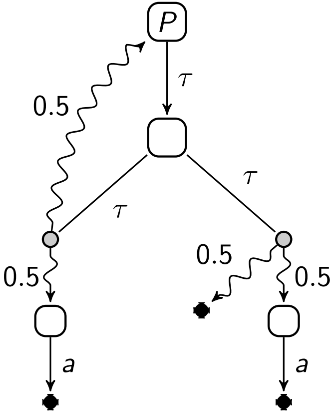
- S - states
- $\longrightarrow \subseteq S \times \mathsf{Act}_\tau \times \mathcal{D}(S)$

$\mathcal{D}(S)$: Mappings $\Delta : S \rightarrow [0, 1]$ with $\sum_{s \in S} \Delta(s) = 1$

$s_1 \xrightarrow{\mu} \Delta$: process s_1

- can perform action μ
- with probability $\Delta(s_2)$ it continues as process s_2

Example



Testing scenario

- a set of processes $\mathcal{P}roc$
- a set of tests $\mathcal{T}$
- a set of ordered outcomes $\mathcal{O}$
- $\mathcal{A}pply : \mathcal{T} \times \mathcal{P}roc \to \mathcal{P}^{+}(\mathcal{O})$ – the non-empty set of possible results of applying a test to a process

Testing preorders:

- Optimistic: $P \sqsubseteq_{\mathrm{pmay}} Q$ if $\bigsqcup \mathcal{A}pply(T, P) \leq \bigsqcup \mathcal{A}pply(T, Q)$ for every test T
- Pessimistic: $P \sqsubseteq_{\mathrm{pmust}} Q$ if $\bigsqcap \mathcal{A}pply(T, P) \leq \bigsqcap \mathcal{A}pply(T, Q)$ for every test T

Testing scenario

- a set of processes $\mathcal{P}roc$
- a set of tests $\mathcal{T}$
- a set of ordered outcomes $\mathcal{O}$
- $\mathcal{A}pply : \mathcal{T} \times \mathcal{P}roc \to \mathcal{P}^{+}(\mathcal{O})$ – the non-empty set of possible results of applying a test to a process

Testing preorders:

- Optimistic: $P \sqsubseteq_{\mathrm{pmay}} Q$ if $\bigsqcup \mathcal{A}pply(T, P) \leq \bigsqcup \mathcal{A}pply(T, Q)$ for every test T
- Pessimistic: $P \sqsubseteq_{\mathrm{pmust}} Q$ if $\bigsqcap \mathcal{A}pply(T, P) \leq \bigsqcap \mathcal{A}pply(T, Q)$ for every test T

Variations on Testing preorders

Standard testing:

Use $\mathcal{O} = \{\top, \bot\}$ with $\bot \leq \top$

Probabilistic testing:

Use as $\mathcal{O}$ the unit interval $[0, 1]$ with worse $\rightarrow p \leq q \leftarrow$ better

Modal testing:

Use as $\mathcal{O}$ the interval $[0, 1]^{\Omega}$

- Ω a countable set of *qualities* $\{\omega_1, \omega_2, \omega_3 \ldots \ldots\}$
- each assigned a probability $p_i \in [0, 1]$

Variations on Testing preorders

Standard testing:

Use $\mathcal{O} = \{\top, \bot\}$ with $\bot \leq \top$

Probabilistic testing:

Use as $\mathcal{O}$ the unit interval $[0, 1]$ with worse $\rightarrow p \leq q \leftarrow$ better

Modal testing:

Use as $\mathcal{O}$ the interval $[0, 1]^{\Omega}$

- Ω a countable set of *qualities* $\{\omega_1, \omega_2, \omega_3 \ldots\ldots\}$
- each assigned a probability $p_i \in [0, 1]$

Variations on Testing preorders

Standard testing:

Use $\mathcal{O} = \{\top, \ \bot\}$ with $\bot \leq \top$

Probabilistic testing:

Use as $\mathcal{O}$ the unit interval $[0, 1]$ with worse $\rightarrow p \leq q \leftarrow$ better

Modal testing:

Use as $\mathcal{O}$ the interval $[0, 1]^{\Omega}$

- Ω a countable set of *qualities* $\{\omega_1, \omega_2, \omega_3 \ldots\ldots\}$
- each assigned a probability $p_i \in [0, 1]$

Reward testing

Intuition:

Associate with each success modality $\omega_i \in \Omega$ a reward weighting h_i

Reward preorders:

- $\Delta \sqsubseteq^{\Omega}_{\mathbf{nr}\text{must}} \Theta$ if for every Ω-test $\mathcal{T}$ and non-negative reward tuple $h \in [0,1]^{\Omega}$, $\prod h \cdot \mathcal{A}(\mathcal{T},\Delta) \leq \prod h \cdot \mathcal{A}(\mathcal{T},\Gamma)$.
- $\Delta \sqsubseteq^{\Omega}_{\mathbf{rr}\text{must}} \Theta$ if for every Ω-test $\mathcal{T}$ and real reward tuple $h \in [-1,1]^{\Omega}$, $\prod h \cdot \mathcal{A}(\mathcal{T},\Delta) \leq \prod h \cdot \mathcal{A}(\mathcal{T},\Gamma)$.

Main result:

In finitary convergent pLTSs

$$\Delta \sqsubseteq^{\Omega}_{\mathbf{nr}\text{must}} \Theta \iff \Delta \sqsubseteq^{\Omega}_{\mathbf{rr}\text{must}} \Theta$$

Reward testing

Intuition:

Associate with each success modality $\omega_i \in \Omega$ a reward weighting h_i

Reward preorders:

- $\Delta \sqsubseteq^{\Omega}_{\mathbf{nr}\text{must}} \Theta$ if for every Ω-test $\mathcal{T}$ and non-negative reward tuple $h \in [0,1]^{\Omega}$,
  $\prod h \cdot \mathcal{A}(\mathcal{T}, \Delta) \leq \prod h \cdot \mathcal{A}(\mathcal{T}, \Gamma)$.
- $\Delta \sqsubseteq^{\Omega}_{\mathbf{rr}\text{must}} \Theta$ if for every Ω-test $\mathcal{T}$ and real reward tuple $h \in [-1,1]^{\Omega}$,
  $\prod h \cdot \mathcal{A}(\mathcal{T}, \Delta) \leq \prod h \cdot \mathcal{A}(\mathcal{T}, \Gamma)$.

Main result:

In finitary convergent pLTSs

$$\Delta \sqsubseteq^{\Omega}_{\mathbf{nr}\text{must}} \Theta \iff \Delta \sqsubseteq^{\Omega}_{\mathbf{rr}\text{must}} \Theta$$

Reward testing

Intuition:

Associate with each success modality $\omega_i \in \Omega$ a reward weighting h_i

Reward preorders:

- $\Delta \sqsubseteq^{\Omega}_{\mathbf{nr}\text{must}} \Theta$ if for every Ω-test $\mathcal{T}$ and non-negative reward tuple $h \in [0,1]^{\Omega}$,
  $\bigsqcap h \cdot \mathcal{A}(\mathcal{T}, \Delta) \leq \bigsqcap h \cdot \mathcal{A}(\mathcal{T}, \Gamma)$.
- $\Delta \sqsubseteq^{\Omega}_{\mathbf{rr}\text{must}} \Theta$ if for every Ω-test $\mathcal{T}$ and real reward tuple $h \in [-1,1]^{\Omega}$,
  $\bigsqcap h \cdot \mathcal{A}(\mathcal{T}, \Delta) \leq \bigsqcap h \cdot \mathcal{A}(\mathcal{T}, \Gamma)$.

Main result:

In finitary convergent pLTSs

$$\Delta \sqsubseteq^{\Omega}_{\mathbf{nr}\text{must}} \Theta \iff \Delta \sqsubseteq^{\Omega}_{\mathbf{rr}\text{must}} \Theta$$

Reward testing

Intuition:

Associate with each success modality $\omega_i \in \Omega$ a reward weighting h_i

Reward preorders:

- $\Delta \sqsubseteq^{\Omega}_{\mathbf{nr}\text{must}} \Theta$ if for every Ω-test $\mathcal{T}$ and non-negative reward tuple $h \in [0,1]^{\Omega}$,
  $\bigsqcap h \cdot \mathcal{A}(\mathcal{T}, \Delta) \leq \bigsqcap h \cdot \mathcal{A}(\mathcal{T}, \Gamma)$.
- $\Delta \sqsubseteq^{\Omega}_{\mathbf{rr}\text{must}} \Theta$ if for every Ω-test $\mathcal{T}$ and real reward tuple $h \in [-1,1]^{\Omega}$,
  $\bigsqcap h \cdot \mathcal{A}(\mathcal{T}, \Delta) \leq \bigsqcap h \cdot \mathcal{A}(\mathcal{T}, \Gamma)$.

Main result:

In finitary convergent pLTSs

$$\Delta \sqsubseteq^{\Omega}_{\mathbf{nr}\text{must}} \Theta \quad \Longleftrightarrow \quad \Delta \sqsubseteq^{\Omega}_{\mathbf{rr}\text{must}} \Theta$$

Outline

Testing

Outcomes: resolutions v derivations

Failure simulations

Results

Applying tests to processes $\mathcal{A}pply(T, P)$

Run the combined process $(T \| P)$

Nondeterministic case:

- Nondeterministic $(T \| P)$ resolved to a set of deterministic executions
- Each execution succeeds or fails
- Each execution contributes $\top$ or $\bot$ to $\mathcal{A}pply(T, P)$

Reward case:

- Probabilistic $(T \| P)$ resolved to a set of deterministic but probabilistic executions
- Each execution contributes a modal vector of probabilities to $\mathcal{A}pply(T, P)$
- Formalisation: resolutions, policies, strategies, derivations, ...

Applying tests to processes $\mathcal{A}pply(T, P)$

Run the combined process $(T \| P)$

Nondeterministic case:

- Nondeterministic $(T \| P)$ resolved to a set of deterministic executions
- Each execution succeeds or fails
- Each execution contributes $\top$ or $\bot$ to $\mathcal{A}pply(T, P)$

Reward case:

- Probabilistic $(T \| P)$ resolved to a set of deterministic but probabilistic executions
- Each execution contributes a modal vector of probabilities to $\mathcal{A}pply(T, P)$
- Formalisation: resolutions, policies, strategies, derivations, ...

Applying tests to processes $\mathcal{A}pply(T, P)$

Run the combined process $(T \| P)$

Nondeterministic case:

- Nondeterministic $(T \| P)$ resolved to a set of deterministic executions
- Each execution succeeds or fails
- Each execution contributes $\top$ or $\bot$ to $\mathcal{A}pply(T, P)$

Reward case:

- Probabilistic $(T \| P)$ resolved to a set of deterministic but probabilistic executions
- Each execution contributes a modal vector of probabilities to $\mathcal{A}pply(T, P)$
- Formalisation: resolutions, policies, strategies, derivations, ...

Applying tests to processes $\mathcal{A}pply(T, P)$

Run the combined process $(T \| P)$

Nondeterministic case:

- Nondeterministic $(T \| P)$ resolved to a set of deterministic executions
- Each execution succeeds or fails
- Each execution contributes $\top$ or $\bot$ to $\mathcal{A}pply(T, P)$

Reward case:

- Probabilistic $(T \| P)$ resolved to a set of deterministic but probabilistic executions
- Each execution contributes a modal vector of probabilities to $\mathcal{A}pply(T, P)$
- Formalisation: resolutions, policies, strategies, derivations, ...

Resolutions

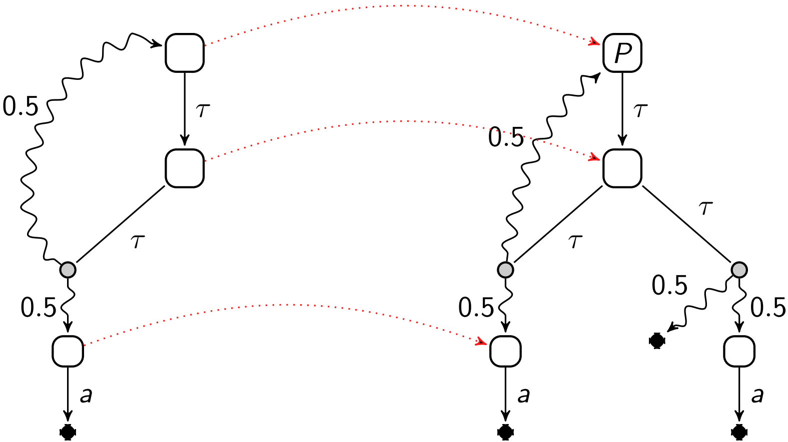
Resolution of process Δ from pLTS $\mathbf{M}$:

- a deterministic pLTS $\mathbf{R} = \langle R, \Omega_\tau, \rightarrow_R \rangle$ but probabilistic
- a process Θ_Δ in $\mathbf{R}$
- a resolving function $f : \mathbf{R} \rightarrow \mathbf{M}$ explaining how process Θ_Δ in $\mathbf{R}$ is an execution of Δ in $\mathbf{M}$

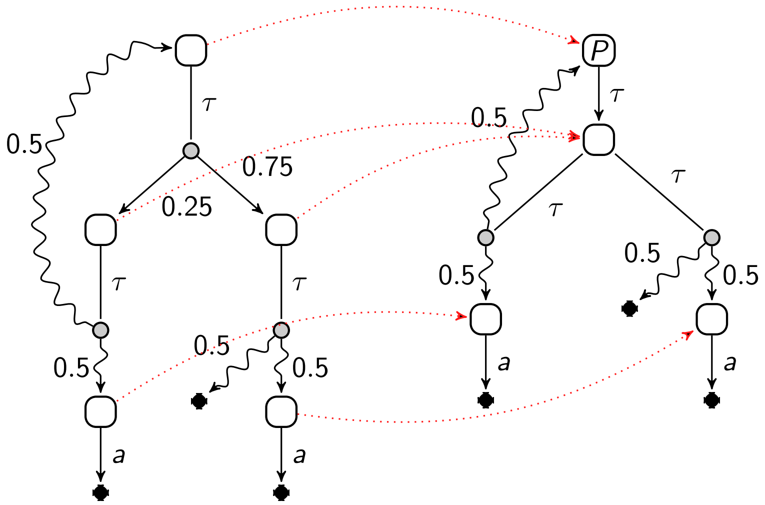
Calculating outcomes:

- Each resolution $\mathbf{R}$ of process Δ determines an outcome $o_\mathbf{R}$
- $\mathcal{A}pply^{\text{res}}(T|P) = \{\, o_\mathbf{R} \mid \mathbf{R} \text{ is a resolution of } T \| P \,\}$

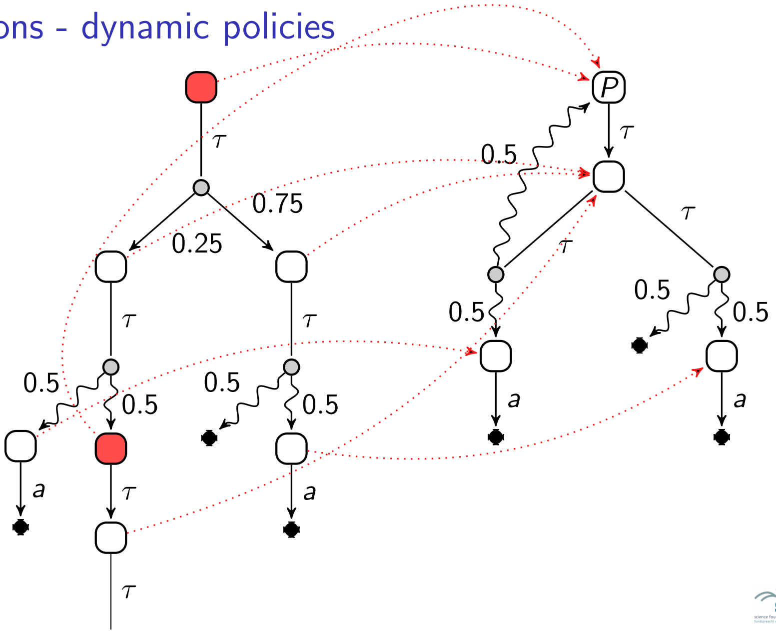
Resolutions



Resolutions - interpolations



Resolutions - dynamic policies



Derivations in a pLTS

$$s \xrightarrow{\alpha} \Theta$$

Generalisation:

- Distributions to distributions: $\Delta \xrightarrow{\alpha} \Theta$
- Internal moves: $\Delta \xrightarrow{\tau} \Delta_1 \xrightarrow{\tau} \ldots \xrightarrow{\tau} \Delta_k$
- In the limit: $\Delta \xrightarrow{\tau} \Delta_1 \xrightarrow{\tau} \ldots \xrightarrow{\tau} \Delta_k \xrightarrow{\tau} \ldots \Delta_\infty$

Derivations in a pLTS

$$\boxed{s \xrightarrow{\alpha} \Theta}$$

Generalisation:

- Distributions to distributions: $\Delta \xrightarrow{\alpha} \Theta$
- Internal moves: $\Delta \xrightarrow{\tau} \Delta_1 \xrightarrow{\tau} \ldots \xrightarrow{\tau} \Delta_k$
- In the limit: $\Delta \xrightarrow{\tau} \Delta_1 \xrightarrow{\tau} \ldots \xrightarrow{\tau} \Delta_k \xrightarrow{\tau} \ldots \Delta_\infty$

Derivations in a pLTS

$$\boxed{s \xrightarrow{\alpha} \Theta}$$

Generalisation:

- Distributions to distributions: $\Delta \xrightarrow{\alpha} \Theta$
- Internal moves: $\Delta \xrightarrow{\tau} \Delta_1 \xrightarrow{\tau} \ldots \xrightarrow{\tau} \Delta_k$
- In the limit: $\Delta \xrightarrow{\tau} \Delta_1 \xrightarrow{\tau} \ldots \xrightarrow{\tau} \Delta_k \xrightarrow{\tau} \ldots \Delta_\infty$

Derivations in a pLTS

$$\boxed{s \xrightarrow{\alpha} \Theta}$$

Generalisation:

- Distributions to distributions: $\Delta \xrightarrow{\alpha} \Theta$
- Internal moves: $\Delta \xrightarrow{\tau} \Delta_1 \xrightarrow{\tau} \ldots \xrightarrow{\tau} \Delta_k$
- In the limit: $\Delta \xrightarrow{\tau} \Delta_1 \xrightarrow{\tau} \ldots \xrightarrow{\tau} \Delta_k \xrightarrow{\tau} \ldots \Delta_\infty$

Lifting relations

From $\mathcal{R} \subseteq S \times \mathcal{D}(S)$, to $\text{lift}(\mathcal{R}) \subseteq \mathcal{D}(S) \times \mathcal{D}(S)$

$\Delta \;\; \text{lift}(\mathcal{R})\, \Theta$ whenever

- $\Delta = \sum_{i \in I} p_i \cdot s_i$, I a finite index set
- For each $i \in I$ there is a distribution Θ_i s.t. $s_i \;\; \mathcal{R} \;\; \Theta_i$
- $\Theta = \sum_{i \in I} p_i \cdot \Theta_i$
- $\sum_{i \in I} p_i = 1$

Many different formulations

Note: in decomposition $\sum_{i \in I} p_i \cdot s_i$ states s_i are not necessarily unique

Lifting relations

From $\mathcal{R} \subseteq S \times \mathcal{D}(S)$, to $\text{lift}(\mathcal{R}) \subseteq \mathcal{D}(S) \times \mathcal{D}(S)$

$\boxed{\Delta \;\; \text{lift}(\mathcal{R})\, \Theta}$ whenever

- $\Delta = \sum_{i \in I} p_i \cdot s_i$, I a finite index set
- For each $i \in I$ there is a distribution Θ_i s.t. $s_i \;\; \mathcal{R} \;\; \Theta_i$
- $\Theta = \sum_{i \in I} p_i \cdot \Theta_i$
- $\sum_{i \in I} p_i = 1$

Many different formulations

Note: in decomposition $\sum_{i \in I} p_i \cdot s_i$ states s_i are not necessarily unique

Lifting relations

From $\mathcal{R} \subseteq S \times \mathcal{D}(S)$, to $\text{lift}(\mathcal{R}) \subseteq \mathcal{D}(S) \times \mathcal{D}(S)$

$$\boxed{\Delta \quad \text{lift}(\mathcal{R})\, \Theta} \qquad \text{whenever}$$

- $\Delta = \sum_{i \in I} p_i \cdot s_i$, I a finite index set
- For each $i \in I$ there is a distribution Θ_i s.t. $s_i \ \mathcal{R} \ \Theta_i$
- $\Theta = \sum_{i \in I} p_i \cdot \Theta_i$
- $\sum_{i \in I} p_i = 1$

Many different formulations

Note: in decomposition $\sum_{i \in I} p_i \cdot s_i$ states s_i are not necessarily unique

Lifting relations

From $\mathcal{R} \subseteq S \times \mathcal{D}(S)$, to $\text{lift}(\mathcal{R}) \subseteq \mathcal{D}(S) \times \mathcal{D}(S)$

$$\boxed{\Delta \ \ \text{lift}(\mathcal{R})\, \Theta} \qquad \text{whenever}$$

- $\Delta = \sum_{i \in I} p_i \cdot s_i$, I a finite index set
- For each $i \in I$ there is a distribution Θ_i s.t. $s_i \ \ \mathcal{R} \ \ \Theta_i$
- $\Theta = \sum_{i \in I} p_i \cdot \Theta_i$
- $\sum_{i \in I} p_i = 1$

Many different formulations
Note: in decomposition $\sum_{i \in I} p_i \cdot s_i$ states s_i are not necessarily unique

Lifting actions: from $s \xrightarrow{\mu} \Theta$ to $\Delta \xrightarrow{\mu} \Theta$

$$\Delta \xrightarrow{\mu} \Theta$$

- Δ represents a cloud of possible process states
- each possible state must be able to perform μ
- all possible residuals combine to Θ

Examples:

- $(a.b + a.c)\,{}_{\frac{1}{2}}\!\oplus\, a.d \quad \xrightarrow{a} \quad b\,{}_{\frac{1}{2}}\!\oplus\, d$
- $(a.b + a.c)\,{}_{\frac{1}{2}}\!\oplus\, a.d \quad \xrightarrow{a} \quad (b\,{}_{\frac{1}{2}}\!\oplus\, c)\,{}_{\frac{1}{2}}\!\oplus\, d$
- $(a.b + a.c)\,{}_{\frac{1}{2}}\!\oplus\, a.d \quad \xrightarrow{a} \quad (b\,{}_{p}\!\oplus\, c)\,{}_{\frac{1}{2}}\!\oplus\, d$
- $(\tau.a + \tau.b)\,{}_{\frac{1}{2}}\!\oplus\, (\tau.a + \tau.c) \quad \xrightarrow{\tau} \quad a\;{}_{\frac{1}{2}}\!\oplus\; (b\;{}_{\frac{1}{2}}\!\oplus\; c)$

Lifting actions: from $s \xrightarrow{\mu} \Theta$ to $\Delta \xrightarrow{\mu} \Theta$

$$\Delta \xrightarrow{\mu} \Theta$$

- Δ represents a cloud of possible process states
- each possible state must be able to perform μ
- all possible residuals combine to Θ

Examples:

- $(a.b + a.c)\ {}_{\frac{1}{2}}\!\oplus\ a.d \quad \xrightarrow{a} \quad b\ {}_{\frac{1}{2}}\!\oplus\ d$
- $(a.b + a.c)\ {}_{\frac{1}{2}}\!\oplus\ a.d \quad \xrightarrow{a} \quad (b\ {}_{\frac{1}{2}}\!\oplus\ c)\ {}_{\frac{1}{2}}\!\oplus\ d$
- $(a.b + a.c)\ {}_{\frac{1}{2}}\!\oplus\ a.d \quad \xrightarrow{a} \quad (b\ {}_{p}\!\oplus\ c)\ {}_{\frac{1}{2}}\!\oplus\ d$
- $(\tau.a + \tau.b)\ {}_{\frac{1}{2}}\!\oplus\ (\tau.a + \tau.c) \quad \xrightarrow{\tau} \quad a\ {}_{\frac{1}{2}}\!\oplus\ (b\ {}_{\frac{1}{2}}\!\oplus\ c)$

Lifting actions: from $s \xrightarrow{\mu} \Theta$ to $\Delta \xrightarrow{\mu} \Theta$

$$\Delta \xrightarrow{\mu} \Theta$$

- Δ represents a cloud of possible process states
- each possible state must be able to perform μ
- all possible residuals combine to Θ

Examples:

- $(a.b + a.c) \,_{\frac{1}{2}}\!\oplus\, a.d \quad \xrightarrow{a} \quad b \,_{\frac{1}{2}}\!\oplus\, d$
- $(a.b + a.c) \,_{\frac{1}{2}}\!\oplus\, a.d \quad \xrightarrow{a} \quad (b \,_{\frac{1}{2}}\!\oplus\, c) \,_{\frac{1}{2}}\!\oplus\, d$
- $(a.b + a.c) \,_{\frac{1}{2}}\!\oplus\, a.d \quad \xrightarrow{a} \quad (b \,_{p}\!\oplus\, c) \,_{\frac{1}{2}}\!\oplus\, d$
- $(\tau.a + \tau.b) \,_{\frac{1}{2}}\!\oplus\, (\tau.a + \tau.c) \quad \xrightarrow{\tau} \quad a \,_{\frac{1}{2}}\!\oplus\, (b \,_{\frac{1}{2}}\!\oplus\, c)$

Lifting actions: from $s \xrightarrow{\mu} \Theta$ to $\Delta \xrightarrow{\mu} \Theta$

$$\Delta \xrightarrow{\mu} \Theta$$

- Δ represents a cloud of possible process states
- each possible state must be able to perform μ
- all possible residuals combine to Θ

Examples:

- $(a.b + a.c) \,_{\frac{1}{2}}\!\oplus\, a.d \xrightarrow{a} b \,_{\frac{1}{2}}\!\oplus\, d$
- $(a.b + a.c) \,_{\frac{1}{2}}\!\oplus\, a.d \xrightarrow{a} (b \,_{\frac{1}{2}}\!\oplus\, c) \,_{\frac{1}{2}}\!\oplus\, d$
- $(a.b + a.c) \,_{\frac{1}{2}}\!\oplus\, a.d \xrightarrow{a} (b \,_{p}\!\oplus\, c) \,_{\frac{1}{2}}\!\oplus\, d$
- $(\tau.a + \tau.b) \,_{\frac{1}{2}}\!\oplus\, (\tau.a + \tau.c) \xrightarrow{\tau} a \,_{\frac{1}{2}}\!\oplus\, (b \,_{\frac{1}{2}}\!\oplus\, c)$

Lifting actions: from $s \xrightarrow{\mu} \Theta$ to $\Delta \xrightarrow{\mu} \Theta$

$$\Delta \xrightarrow{\mu} \Theta$$

- Δ represents a cloud of possible process states
- each possible state must be able to perform μ
- all possible residuals combine to Θ

Examples:

- $(a.b + a.c) \,_{\frac{1}{2}}\!\oplus\, a.d \quad \xrightarrow{a} \quad b \,_{\frac{1}{2}}\!\oplus\, d$
- $(a.b + a.c) \,_{\frac{1}{2}}\!\oplus\, a.d \quad \xrightarrow{a} \quad (b \,_{\frac{1}{2}}\!\oplus\, c) \,_{\frac{1}{2}}\!\oplus\, d$
- $(a.b + a.c) \,_{\frac{1}{2}}\!\oplus\, a.d \quad \xrightarrow{a} \quad (b \,_{p}\!\oplus\, c) \,_{\frac{1}{2}}\!\oplus\, d$
- $(\tau.a + \tau.b) \,_{\frac{1}{2}}\!\oplus\, (\tau.a + \tau.c) \quad \xrightarrow{\tau} \quad a \,_{\frac{1}{2}}\!\oplus\, (b \,_{\frac{1}{2}}\!\oplus\, c)$

Lifting actions: from $s \xrightarrow{\mu} \Theta$ to $\Delta \xrightarrow{\mu} \Theta$

$$\Delta \xrightarrow{\mu} \Theta$$

- Δ represents a cloud of possible process states
- each possible state must be able to perform μ
- all possible residuals combine to Θ

Examples:

- $(a.b + a.c)\ {}_{\frac{1}{2}}\oplus\ a.d \quad \xrightarrow{a} \quad b\ {}_{\frac{1}{2}}\oplus\ d$
- $(a.b + a.c)\ {}_{\frac{1}{2}}\oplus\ a.d \quad \xrightarrow{a} \quad (b\ {}_{\frac{1}{2}}\oplus\ c)\ {}_{\frac{1}{2}}\oplus\ d$
- $(a.b + a.c)\ {}_{\frac{1}{2}}\oplus\ a.d \quad \xrightarrow{a} \quad (b\ {}_{p}\oplus\ c)\ {}_{\frac{1}{2}}\oplus\ d$
- $(\tau.a + \tau.b)\ {}_{\frac{1}{2}}\oplus\ (\tau.a + \tau.c) \quad \xrightarrow{\tau} \quad a\ {}_{\frac{1}{2}}\oplus\ (b\ {}_{\frac{1}{2}}\oplus\ c)$

Derivations in a pLTS

Derivations from $(T\|P)$ to Θ: $(T\|P) \Longrightarrow\!\!\!\!\succ \Theta$

$$
\begin{array}{llll}
(T\|P) & = & \Delta_0^{\rightarrow} + & \Delta_0^{\text{stop}} \\
\Delta_0^{\rightarrow} & \xrightarrow{\tau} & \Delta_1^{\rightarrow} + & \Delta_1^{\text{stop}} \\
\cdots & & \cdots & \\
\Delta_k^{\rightarrow} & \xrightarrow{\tau} & \Delta_{(k+1)}^{\rightarrow} + & \Delta_{(k+1)}^{\text{stop}} \\
\cdots & & & \cdots \\
\cdots & & & \cdots
\end{array}
$$

Total: $\Theta = \sum_{k=0}^{\infty} \Delta_k^{\text{stop}}$

- Δ^{stop}: all states in Δ which
  - are successful $s \xrightarrow{\omega}$
  - or are stuck $s \not\xrightarrow{\tau}$
- $\Delta^{\rightarrow}$: all other states, which can proceed $s \xrightarrow{\tau}$

note: subdistributions

Derivations in a pLTS

Derivations from $(T \| P)$ to Θ: $(T \| P) \Longrightarrow\!\!\!\!\!> \Theta$

$$
\begin{array}{llll}
(T \| P) & = & \Delta_0^{\rightarrow} + & \Delta_0^{\text{stop}} \\
\Delta_0^{\rightarrow} & \xrightarrow{\tau} & \Delta_1^{\rightarrow} + & \Delta_1^{\text{stop}} \\
\dots & & \dots & \\
\Delta_k^{\rightarrow} & \xrightarrow{\tau} & \Delta_{(k+1)}^{\rightarrow} + & \Delta_{(k+1)}^{\text{stop}} \\
\dots & & & \dots \\
\dots & & & \dots \\
\hline
\end{array}
$$

Total: $\Theta = \sum_{k=0}^{\infty} \Delta_k^{\text{stop}}$

- Δ^{stop}: all states in Δ which
  - are successful $s \xrightarrow{\omega}$
  - or are stuck $s \not\xrightarrow{\tau}$
- $\Delta^{\rightarrow}$: all other states, which can proceed $s \xrightarrow{\tau}$

note: subdistributions

Derivations in a pLTS

Derivations from $(T \| P)$ to Θ: $(T \| P) \Longrightarrow\!\!\!\!\!\!\!\!\!\!\!\! \rightarrowtail \Theta$

$$
\begin{array}{llll}
(T \| P) & = & \Delta_0^{\rightarrow} + & \Delta_0^{\mathsf{stop}} \\
\Delta_0^{\rightarrow} & \xrightarrow{\tau} & \Delta_1^{\rightarrow} + & \Delta_1^{\mathsf{stop}} \\
\ldots & & \ldots & \\
\Delta_k^{\rightarrow} & \xrightarrow{\tau} & \Delta_{(k+1)}^{\rightarrow} + & \Delta_{(k+1)}^{\mathsf{stop}} \\
\ldots & & & \ldots \\
\ldots & & & \ldots
\end{array}
$$

Total: $\Theta = \sum_{k=0}^{\infty} \Delta_k^{\mathsf{stop}}$

- Δ^{stop}: all states in Δ which
  - are successful $s \xrightarrow{\omega}$
  - or are stuck $s \not\xrightarrow{\tau}$
- $\Delta^{\rightarrow}$: all other states, which can proceed $s \xrightarrow{\tau}$

note: subdistributions

Derivations in a pLTS

Derivations from $(T\|P)$ to Θ: $\boxed{(T\|P) \Longrightarrow \Theta}$

$$
\begin{array}{llll}
(T\|P) & = & \Delta_0^{\rightarrow} + & \Delta_0^{\text{stop}} \\
\Delta_0^{\rightarrow} & \xrightarrow{\tau} & \Delta_1^{\rightarrow} + & \Delta_1^{\text{stop}} \\
\dots & & \dots & \\
\Delta_k^{\rightarrow} & \xrightarrow{\tau} & \Delta_{(k+1)}^{\rightarrow} + & \Delta_{(k+1)}^{\text{stop}} \\
\dots & & & \dots \\
\dots & & & \dots \\
\hline
 & & \text{Total:} & \Theta = \sum_{k=0}^{\infty} \Delta_k^{\text{stop}}
\end{array}
$$

- Δ^{stop}: all states in Δ which
  - are successful $s \xrightarrow{\omega}$
  - or are stuck $s \not\xrightarrow{\tau}$
- $\Delta^{\rightarrow}$: all other states, which can proceed $s \xrightarrow{\tau}$

note: subdistributions

Applying tests to processes: $\mathcal{A}pply^{der}(T, P)$

- find all executions from $(T \| P)$: $\boxed{(T \| P) \Longrightarrow\!\!\!\succ \Theta}$
- calculate contribution of each Θ

$$\mathcal{A}pply^{der}(T, P) = \{ \text{contribute}(\Theta) \mid (T \| P) \Longrightarrow\!\!\!\succ \Theta \}$$

Resolutions v. Derivations:

In an arbitrary pLTS

$$\mathcal{A}pply^{der}(T, P) = \mathcal{A}pply^{res}(T, P)$$

because

- bijection between resolutions $\mathbf{R}$ of process Δ and executions $\Delta \Longrightarrow\!\!\!\succ \Theta$
- outcome $o_{\mathbf{R}}$ the same as contribute(Θ)

Applying tests to processes: $\mathcal{A}pply^{der}(T, P)$

- find all executions from $(T \| P)$: $(T \| P) \Longrightarrow\!\!\!\succ \Theta$
- calculate contribution of each Θ

$$\mathcal{A}pply^{der}(T, P) = \{\, \text{contribute}(\Theta) \mid (T \| P) \Longrightarrow\!\!\!\succ \Theta \,\}$$

Resolutions v. Derivations:

In an arbitrary pLTS

$$\mathcal{A}pply^{der}(T, P) = \mathcal{A}pply^{res}(T, P)$$

because

- bijection between resolutions $\mathbf{R}$ of process Δ and executions $\Delta \Longrightarrow\!\!\!\succ \Theta$
- outcome $o_{\mathbf{R}}$ the same as contribute(Θ)

sfi science foundation ireland

Applying tests to processes: $\mathcal{A}pply^{der}(T, P)$

- find all executions from $(T \| P)$: $(T \| P) \Longrightarrow\!\!\!\succ \Theta$
- calculate contribution of each Θ

$$\mathcal{A}pply^{der}(T, P) = \{ \text{contribute}(\Theta) \mid (T \| P) \Longrightarrow\!\!\!\succ \Theta \}$$

Resolutions v. Derivations:

In an arbitrary pLTS

$$\mathcal{A}pply^{der}(T, P) = \mathcal{A}pply^{res}(T, P)$$

because

- bijection between resolutions **R** of process Δ and executions $\Delta \Longrightarrow\!\!\!\succ \Theta$
- outcome $o_{\mathbf{R}}$ the same as contribute(Θ)

Applying tests to processes: $\mathcal{A}pply^{der}(T, P)$

- find all executions from $(T \| P)$: $\boxed{(T \| P) \Longrightarrow\!\!\succ \Theta}$
- calculate contribution of each Θ

$$\mathcal{A}pply^{der}(T, P) = \{\, \text{contribute}(\Theta) \;\mid\; (T \| P) \Longrightarrow\!\!\succ \Theta \,\}$$

Resolutions v. Derivations:

In an arbitrary pLTS

$$\mathcal{A}pply^{der}(T, P) \;=\; \mathcal{A}pply^{res}(T, P)$$

because

- bijection between resolutions $\mathbf{R}$ of process Δ and executions $\Delta \Longrightarrow\!\!\succ \Theta$
- outcome $o_{\mathbf{R}}$ the same as contribute(Θ)

Outline

Failure simulation preorder

$\lhd_{FS}$ is the largest relation in $S \times \mathcal{D}_{sub}(S)$ such that if $s \lhd_{FS} \Theta$ then

(i) whenever $\overline{s} \stackrel{\alpha}{\Longrightarrow} \Delta'$, for $\alpha \in \mathsf{Act}_\tau$, then there is a $\Theta' \in \mathcal{D}_{sub}(S)$ with $\Theta \stackrel{\alpha}{\Longrightarrow} \Theta'$ and $\Delta' \; \mathsf{lift}(\lhd_{FS}) \; \Theta'$,

(ii) and whenever $\overline{s} \Longrightarrow \stackrel{A}{\not\longrightarrow}$ then $\Delta \Longrightarrow \stackrel{A}{\not\longrightarrow}$.

Note: Definition based on derivations

$$\Delta \quad \sqsubseteq_{FS} \quad \Theta$$

whenever there is a $\Delta^\natural$ with $\Delta \Longrightarrow \Delta^\natural$ and $\Theta \; \overline{\lhd_{FS}} \; \Delta^\natural$.

Soundness and completeness:

In a finitary pLTS

$$\Delta \sqsubseteq_{FS} \Theta \quad \Longleftrightarrow \quad \Delta \sqsubseteq^{\Omega}_{\mathbf{nr}must} \Theta$$

from concur 2009

Failure simulation preorder

$\lhd_{FS}$ is the largest relation in $S \times \mathcal{D}_{sub}(S)$ such that if $s \lhd_{FS} \Theta$ then

(i) whenever $\overline{s} \stackrel{\alpha}{\Longrightarrow} \Delta'$, for $\alpha \in \mathsf{Act}_\tau$, then there is a $\Theta' \in \mathcal{D}_{sub}(S)$ with $\Theta \stackrel{\alpha}{\Longrightarrow} \Theta'$ and $\Delta' \; \mathsf{lift}(\lhd_{FS}) \; \Theta'$,

(ii) and whenever $\overline{s} \Longrightarrow \stackrel{A}{\not\longrightarrow}$ then $\Delta \Longrightarrow \stackrel{A}{\not\longrightarrow}$.

Note: Definition based on derivations

$$\Delta \quad \sqsubseteq_{FS} \quad \Theta$$

whenever there is a $\Delta^\natural$ with $\Delta \Longrightarrow \Delta^\natural$ and $\Theta \; \overline{\lhd_{FS}} \; \Delta^\natural$.

Soundness and completeness:

In a finitary pLTS

$$\Delta \sqsubseteq_{FS} \Theta \quad \Longleftrightarrow \quad \Delta \sqsubseteq^{\Omega}_{\text{nrmust}} \Theta$$

from concur 2009

Failure simulation preorder

$\lhd_{FS}$ is the largest relation in $S \times \mathcal{D}_{sub}(S)$ such that if $s \lhd_{FS} \Theta$ then

(i) whenever $\bar{s} \stackrel{\alpha}{\Longrightarrow} \Delta'$, for $\alpha \in \mathsf{Act}_\tau$, then there is a $\Theta' \in \mathcal{D}_{sub}(S)$ with $\Theta \stackrel{\alpha}{\Longrightarrow} \Theta'$ and $\Delta' \; \mathsf{lift}(\lhd_{FS}) \; \Theta'$,

(ii) and whenever $\bar{s} \Longrightarrow \stackrel{A}{\not\longrightarrow}$ then $\Delta \Longrightarrow \stackrel{A}{\not\longrightarrow}$.

Note: Definition based on derivations

$$\Delta \quad \sqsubseteq_{FS} \quad \Theta$$

whenever there is a $\Delta^\natural$ with $\Delta \Longrightarrow \Delta^\natural$ and $\Theta \; \overline{\lhd_{FS}} \; \Delta^\natural$.

Soundness and completeness:

In a finitary pLTS

$$\Delta \sqsubseteq_{FS} \Theta \quad \Longleftrightarrow \quad \Delta \sqsubseteq^{\Omega}_{\mathbf{nr}must} \Theta$$

from concur 2009

Outline

Divergence makes a difference

- $P = \text{rec}\, x.x \qquad Q = a.\,\mathbf{0}$
- $P \sqsubseteq_{FS} Q$ and so $P \sqsubseteq^{\Omega}_{\mathbf{nr}\text{must}} Q$
- But $P \not\sqsubseteq^{\Omega}_{\mathbf{rr}\text{must}} Q$

- Use test $t = \overline{a}.\omega$
- use real reward $h(\omega) = -1$

$$\bigsqcap h \cdot \mathcal{A}(t, P) = \bigsqcap h \cdot \{emptyDis\} = 0$$

$$\bigsqcap h \cdot \mathcal{A}(t, Q) = \bigsqcap h \cdot \{\overrightarrow{\omega}\} = -1$$

Negative rewards make a difference in the presence of divergence

Divergence makes a difference

- $P = \mathrm{rec}\, x.x \qquad Q = a.\,\mathbf{0}$
- $P \sqsubseteq_{FS} Q$ and so $P \sqsubseteq^{\Omega}_{\mathbf{nr}\mathrm{must}} Q$
- But $P \not\sqsubseteq^{\Omega}_{\mathbf{rr}\mathrm{must}} Q$

- Use test $t = \overline{a}.\omega$
- use real reward $h(\omega) = -1$

$$\bigsqcap h \cdot \mathcal{A}(t, P) = \bigsqcap h \cdot \{emptyDis\} = 0$$
$$\bigsqcap h \cdot \mathcal{A}(t, Q) = \bigsqcap h \cdot \{\overrightarrow{\omega}\} = -1$$

Negative rewards make a difference in the presence of divergence

Results

Convergence:

No state with $\overline{s} \xrightarrow{\tau} \Delta_1 \xrightarrow{\tau} \Delta_2 \xrightarrow{\tau} \ldots\ldots$

- For finitary processes,
  $\Delta \sqsubseteq^{\Omega}_{\mathbf{nr}\text{must}} \Theta$ implies $\Delta \sqsubseteq_{FS} \Theta$
  - See: Concur 2009 paper
- For finitary convergent processes,
  $\Delta \sqsubseteq_{FS} \Theta$ implies $\Delta \sqsubseteq^{\Omega}_{\mathbf{rr}\text{must}} \Theta$
  - simulations make proof (relatively) straightforward

Cor: For finitary convergent processes,

$$\Delta \sqsubseteq^{\Omega}_{\mathbf{nr}\text{must}} \Theta \iff \Delta \sqsubseteq^{\Omega}_{\mathbf{rr}\text{must}} \Theta$$

Results

Convergence:

No state with $\overline{s} \xrightarrow{\tau} \Delta_1 \xrightarrow{\tau} \Delta_2 \xrightarrow{\tau} \ldots\ldots$

- For finitary processes,
  $\Delta \sqsubseteq^{\Omega}_{\mathbf{nr}\mathrm{must}} \Theta$ implies $\Delta \sqsubseteq_{FS} \Theta$
  - See: Concur 2009 paper
- For finitary convergent processes,
  $\Delta \sqsubseteq_{FS} \Theta$ implies $\Delta \sqsubseteq^{\Omega}_{\mathbf{rr}\mathrm{must}} \Theta$
  - simulations make proof (relatively) straightforward

Cor: For finitary convergent processes,

$$\Delta \sqsubseteq^{\Omega}_{\mathbf{nr}\mathrm{must}} \Theta \iff \Delta \sqsubseteq^{\Omega}_{\mathbf{rr}\mathrm{must}} \Theta$$

Results

Convergence:

No state with $\overline{s} \xrightarrow{\tau} \Delta_1 \xrightarrow{\tau} \Delta_2 \xrightarrow{\tau} \ldots\ldots$

- For finitary processes,
  $\Delta \sqsubseteq^{\Omega}_{\mathbf{nr}\text{must}} \Theta$ implies $\Delta \sqsubseteq_{FS} \Theta$
  - See: Concur 2009 paper
- For finitary convergent processes,
  $\Delta \sqsubseteq_{FS} \Theta$ implies $\Delta \sqsubseteq^{\Omega}_{\mathbf{rr}\text{must}} \Theta$
  - simulations make proof (relatively) straightforward

Cor: For finitary convergent processes,

$$\Delta \sqsubseteq^{\Omega}_{\mathbf{nr}\text{must}} \Theta \iff \Delta \sqsubseteq^{\Omega}_{\mathbf{rr}\text{must}} \Theta$$

Results

Convergence:

No state with $\overline{s} \xrightarrow{\tau} \Delta_1 \xrightarrow{\tau} \Delta_2 \xrightarrow{\tau} \ldots\ldots$

- For finitary processes,
  $\Delta \sqsubseteq^{\Omega}_{\mathbf{nr}\text{must}} \Theta$ implies $\Delta \sqsubseteq_{FS} \Theta$
  - See: Concur 2009 paper
- For finitary convergent processes,
  $\Delta \sqsubseteq_{FS} \Theta$ implies $\Delta \sqsubseteq^{\Omega}_{\mathbf{rr}\text{must}} \Theta$
  - simulations make proof (relatively) straightforward

Cor: For finitary convergent processes,

$$\Delta \sqsubseteq^{\Omega}_{\mathbf{nr}\text{must}} \Theta \iff \Delta \sqsubseteq^{\Omega}_{\mathbf{rr}\text{must}} \Theta$$

THANK YOU!