Exploring probabilistic bisimulations, part I

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Outline

Outline

Concurrency theory à la Milner

- Intensional model of nondeterministic processes: LTSs
- Language for describing processes: algebra CCS
- Extensional equivalence: barbed congruence

  processes indistinguishable in all contexts
- Proof method: bisimulations to show processes equivalent
- Proof method: HML property logic to show inequivalence
- Semantic preserving transformations: equational characterisation

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- Proof method: HML property logic to show inequivalence
- Semantic preserving transformations: equational characterisation

What happens when we add probabilistic behaviour ?

Outline

Intensional models: nondeterministic processes

LTS labelled transition systems

$\langle S, \mathrm{Act}, \rightarrow \rangle$ where

(a) S states

(b) Act_τ transition labels, with distinguished τ

(c) relation $\rightarrow$ is a subset of $S \times \mathrm{Act}_\tau \times S$ $s \xrightarrow{a} t$

A process is a state in an LTS

Intensional models: probabilistic processes and nondeterministic

pLTSs: probabilistic LTSs

Segala

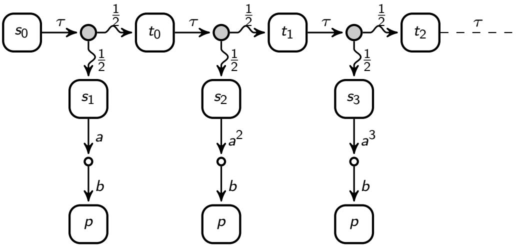
$\langle S, \mathsf{Act}_\tau, \to \rangle$, where

(a) S states

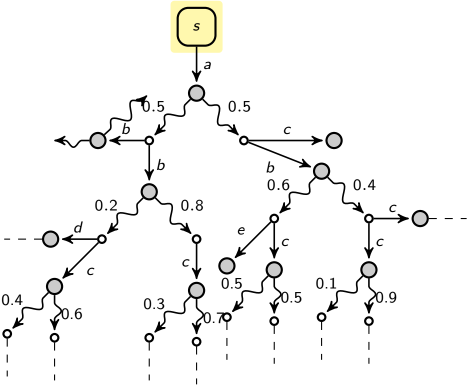
(b) Act_τ transition labels, with distinguished τ

(c) relation $\to$ is a subset of $S \times \mathsf{Act} \times \mathcal{D}(S)$ $\quad s \xrightarrow{\mu} \Delta$

A process is a distribution over states of a pLTS

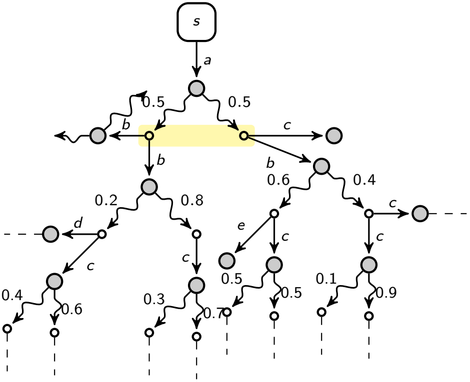


Processes are distributions in pLTSs



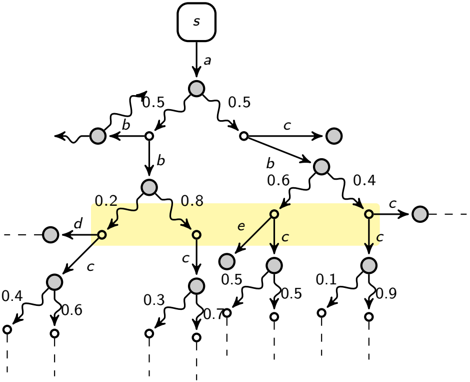
start distribution $\overline{s}$

Processes are distributions in pLTSs



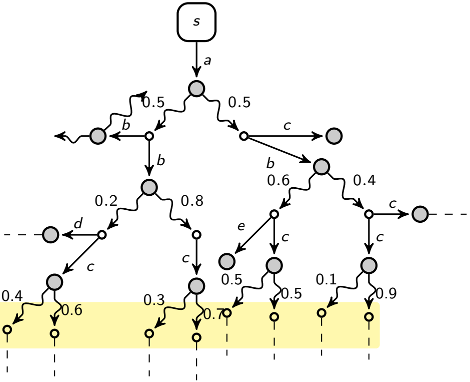
distribution after a

Processes are distributions in pLTSs



distribution after *ab*

Processes are distributions in pLTSs



distribution after *abc*

Algebras for processes

LTSs: CCS

$$P, Q \quad ::= \quad \mathbf{0} \ \mid \ \mu.P,\ \mu \in \mathsf{Act}_\tau \ \mid \ P + Q \ \mid \ \ P \mid Q \ \ \mid \ A,\ \ A \Leftarrow D(A)$$

- Every P determines an LTS
- Every P is a state in an LTS

Algebras for processes

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pLTSs: pCCS

$$P, Q \quad ::= \quad \mathbf{0} \;\mid\; \mu.P,\; \mu \in \mathsf{Act}_\tau \;\mid\; P + Q \;\mid\; P \mid Q \;\mid\; A,\; A \Leftarrow D(A)$$
$$P \,{}_p\!\oplus\, Q,\; 0 \le p \le 1$$

- Every P determines a pLTS
- Every P is a distribution in an LTS

Extensional equivalence: LTSs

$P = a.(a.(b + c) + a.b + a.c)$ $R = a.a.(b + c) + a.(a.b + a.c)$

Q: Can P and R be distinguished behaviourally?

Extensional equivalence: LTSs

$P = a.(a.(b + c) + a.b + a.c) \qquad R = a.a.(b + c) + a.(a.b + a.c)$

Q: Can P and R be distinguished behaviourally?

Contextual equivalences: $P \sim_{rbc} Q$

Very general method:

Extensional equivalence: LTSs

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Very general method:
Largest equivalence between processes which

- is preserved by language contexts
- preserves *basic observables*
- preserves *nondeterministic potential*

Extensional equivalence: LTSs

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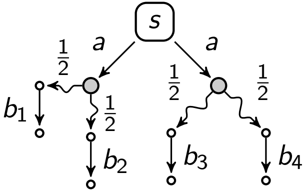
Very general method:
Largest equivalence between processes which

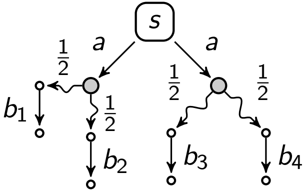
- is preserved by language contexts
- preserves *basic observables*
  - requires definition of observation predicates $P \downarrow o$
- preserves *nondeterministic potential*
  - requires reduction semantics $P \xrightarrow{\tau} P'$

Extensional equivalence: example in pLTSs



$\overline{s} \not\sim_{rbc} \overline{u}$:

Extensional equivalence: example in pLTSs



$\overline{s} \not\sim_{rbc} \overline{u}$:

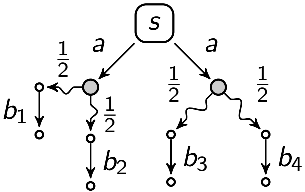
$$s \mid T \qquad\qquad T = \overline{a}.(\overline{b_1}.\omega + \overline{b_2}.\omega)$$

$$\xrightarrow{\tau} (b_1 \; {}_{\frac{1}{2}}\!\oplus \; b_2) \mid (\overline{b_1}.\omega + \overline{b_2}.\omega)$$

the same as $b_1 \mid (\overline{b_1}.\omega + \overline{b_2}.\omega) \; {}_{\frac{1}{2}}\!\oplus \; b_2 \mid ((\overline{b_1}.\omega + \overline{b_2}.\omega)$

$$\xrightarrow{\tau} \omega \; {}_{\frac{1}{2}}\!\oplus \; \omega$$

Extensional equivalence: example in pLTSs



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$$\xrightarrow{\tau} \omega \;{}_{\frac{1}{2}}\!\oplus\; \omega \qquad\qquad \downarrow^{1} \omega \;\leftarrow \text{probabilistic observable}$$

Extensional equivalence: in pLTs

Reduction barbed congruence

Largest equivalence over distributions which is

- closed wrt parallel contexts
- preserves probabilistic observations barbs
- is *reduction-closed*

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Reduction barbed congruence

Largest equivalence over distributions which is

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Observations:

$\Delta \downarrow^{p} a$ if $\Sigma\{\, \Delta(s) \mid s \xrightarrow{a} \} \geq p$

Reduction-closed: strong

if $\Delta \sim_{rbc} \Theta$ then

- $\Delta \xrightarrow{\tau} \Delta'$ implies $\Theta \xrightarrow{\tau} \Theta'$ s.t. $\Delta' \sim_{rbc} \Theta$
- conversely, $\Theta \xrightarrow{\tau} \Theta'$ implies . . . . . .

Outline

Bisimulations for LTSs a proof method

$\mathcal{R} \subseteq S \times S$ is a bisimulation in an LTS if
whenever $p \; \mathcal{R} \; q$ then

(1) $p \xrightarrow{\mu} p'$ implies $q \xrightarrow{\mu} q'$ such that $p' \; \mathcal{R} \; q'$

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Thm (Milner&Sangiorgi): In a sufficiently expressive finite-branching LTS,

$$p \sim_{rbc} q \textit{ iff } p \,\mathcal{R}\, q \textit{ for some bisimulation } \mathcal{R}$$

Bisimulations for LTSs a proof method

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Thm (Milner&Sangiorgi): In a sufficiently expressive finite-branching LTS,

$p \sim_{rbc} q$ *iff* $p \; \mathcal{R} \; q$ *for some bisimulation* $\mathcal{R}$

Proof method;

To show $p \sim_{rbc} q$:

- exhibit a bisimulation $\mathcal{R}$ containing the pair (p, q)

Bisimulations for pLTSs a proof method

Processes are distributions

Proof method:

To show $\Delta \sim_{rbc} \Theta$

- exhibit a bisimulation $\mathcal{R}$ such that $\Delta \, \mathcal{R} \, \Theta$

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Proof method:

To show $\Delta \sim_{rbc} \Theta$

- exhibit a bisimulation $\mathcal{R}$ such that $\Delta \mathcal{R} \Theta$

Problem:

- Definition of bisimulations require actions $\Delta \xrightarrow{\mu} \Theta$
- pLTSs only have actions $s \xrightarrow{\mu} \Theta$

Bisimulations for pLTSs a proof method

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- pLTSs only have actions $s \xrightarrow{\mu} \Theta$

Solution:

Lift $s \xrightarrow{\mu} \Theta$ to $\Delta \xrightarrow{\mu} \Theta$

Lifting relations: from $S \times \mathcal{D}(S)$ to $\mathcal{D}(S) \times \mathcal{D}(S)$

from $\boxed{s \xrightarrow{\mu} \Theta}$ to $\boxed{\Delta \xrightarrow{\mu} \Theta}$

Lifting relations: from $S \times \mathcal{D}(S)$ to $\mathcal{D}(S) \times \mathcal{D}(S)$

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- Δ represents a cloud of possible process states
- each possible state must be able to perform μ
- all possible residuals combine to Θ

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Examples:

- $(a.b + a.c)\ _{\frac{1}{2}}\oplus\ a.d \xrightarrow{a} b\ _{\frac{1}{2}}\oplus\ d$
- $(a.b + a.c)\ _{\frac{1}{2}}\oplus\ a.d \xrightarrow{a} (b\ _{\frac{1}{2}}\oplus\ c)\ _{\frac{1}{2}}\oplus\ d$
- $(a.b + a.c)\ _{\frac{1}{2}}\oplus\ a.d \xrightarrow{a} (b\ _p\oplus\ c)\ _{\frac{1}{2}}\oplus\ d$

Note: dynamic scheduling

Lifting relations

From $\mathcal{R} \subseteq S \times \mathcal{D}(S)$, to $\quad \mathit{lift}(\mathcal{R}) \subseteq \mathcal{D}(S) \times \mathcal{D}(S)$

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$\boxed{\Delta \quad \mathit{lift}(\mathcal{R})\ \Theta}$ whenever

- $\Delta = \sum_{i \in I} p_i \cdot \overline{s_i}$, I a finite index set
- For each $i \in I$ there is a distribution Θ_i s.t. $s_i \ \ \mathcal{R} \ \ \Theta_i$
- $\Theta = \sum_{i \in I} p_i \cdot \Theta_i$
- $\sum_{i \in I} p_i = 1$

Lifting relations

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Many different formulations
Note: in decomposition $\sum_{i \in I} p_i \cdot s_i$ states s_i are not necessarily unique

Bisimulations for pLTSs a proof method

$\mathcal{R} \subseteq \mathcal{D}(S) \times \mathcal{D}(S)$ is a bisimulation in an pLTS if whenever $\Delta \; \mathcal{R} \; \Theta$ then

(1) $\Delta \xrightarrow{\mu} \Delta'$ implies $\Theta \xrightarrow{\mu} \Theta'$ such that $\Delta' \; \mathcal{R} \; \Theta'$

(2) conversely, ...

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(2) conversely, ...

Problem:

- There is a bisimulation containing $(a \ _{0.5}\!\oplus \ b, \ \overline{\mathbf{0}})$
- $a \ _{0.5}\!\oplus \ b$ and $\overline{\mathbf{0}}$ are NOT reduction barbed congruent

Singleton relation $\mathcal{R} = (a \ _{0.5}\!\oplus \ b, \ \overline{\mathbf{0}})$ is a trivial bisimulation

Decomposable Relations

$\mathcal{R} \subseteq \mathcal{D}(S) \times \mathcal{D}(S)$ is decomposable if

- $(\Delta_1 \, {}_p\!\oplus \, \Delta_2) \; \mathcal{R} \; \Theta$ implies $\Theta = \Theta_1 \, {}_p\!\oplus \, \Theta_2$ such that $\Delta_i \mathcal{R} \Theta_i$
- $\Delta \; \mathcal{R} \; (\Theta_1 \, {}_p\!\oplus \, \Theta_2)$ implies $\Delta = \Delta_1 \, {}_p\!\oplus \, \Delta_2$ such that $\Delta_i \mathcal{R} \Theta_i$

Decomposable Relations

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- $\Delta\ \mathcal{R}\ (\Theta_1 \,{}_p\!\oplus\, \Theta_2)$ implies $\Delta = \Delta_1 \,{}_p\!\oplus\, \Delta_2$ such that $\Delta_i \mathcal{R} \Theta_i$

Examples:

- $\mathcal{R} = (a \,{}_{0.5}\!\oplus\, b,\ \mathbf{0})$ is NOT decomposable
- $\sim_{rbc}$ is decomposable

Properties:

- Every $\mathcal{R} \subseteq S \times S$ can be lifted to a decomposable $\mathit{slift}(\mathcal{R}) \subseteq \mathcal{D}(S) \times \mathcal{D}(S)$
- Every decomposable $\mathcal{R} \subseteq \mathcal{D}(S) \times \mathcal{D}(S)$ can be written as $\mathit{slift}(\mathcal{R}_s)$ for some $\mathcal{R}_s \subseteq S \times S$

Bisimulations for pLTSs at last

A decomposable $\mathcal{R} \subseteq \mathcal{D}(S) \times \mathcal{D}(S)$ is a bisimulation in an pLTS if whenever $\Delta \; \mathcal{R} \; \Theta$ then

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Result:

Thm: In a sufficiently expressive finitary pLTS,

$\Delta \sim_{rbc} \Theta$ *iff* $\Delta \; \mathcal{R} \; \Theta$ *for some bisimulation* $\mathcal{R}$

Bisimulations for pLTSs at last

A decomposable $\mathcal{R} \subseteq \mathcal{D}(S) \times \mathcal{D}(S)$ is a bisimulation in an pLTS if whenever $\Delta \mathrel{\mathcal{R}} \Theta$ then

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Result:

Thm: $\Delta \; \mathit{slift}(\sim_{segala}) \; \Theta$ iff $\Delta \mathrel{\mathcal{R}} \Theta$ for some bisimulation $\mathcal{R}$

$s \sim_{segala} t$ is state based probabilistic bisimulation à la Segala.

Bisimulations à Segala

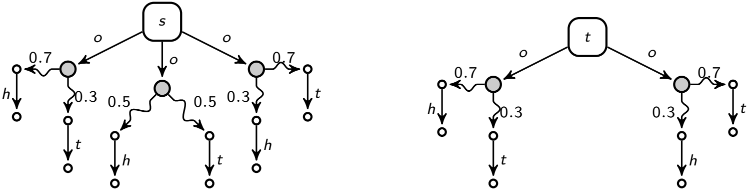
in pLTSs

An equivalence relation $\mathcal{R} \subseteq S \times S$ is an s-bisimulation if, whenever $s \; \mathcal{R} \; t$, then

- $\overline{s} \xrightarrow{\mu} \Delta$ implies $\overline{t} \xrightarrow{\mu} \Theta$ such that $\Delta(E) = \Theta(E)$ for all $\mathcal{R}$-equivalence classes E
- conversely, $\overline{t} \xrightarrow{\mu} \Theta$ implies . . . . . .

$\sim_{segala}$ is the largest s-bisimulation

Example in pLTSs: dynamic scheduling



$$\overline{s} \sim \overline{t}$$

Because

$$\overline{t} \xrightarrow{o} \overline{h} \;_{0.5}\!\oplus\; \overline{t}$$

using combined moves

Outline

Concurrency theory à la Milner

Labelled transition systems

Bisimulations

Property logics

Summary

Property logics in LTSs: proving inequivalences

HML:

$$\varphi \quad ::= \quad \texttt{tt} \;\mid\; \varphi_1 \vee \varphi_2 \;\mid\; \langle\mu\rangle\varphi, \mu \in \mathsf{Act}_\tau \;\mid\; \neg\varphi$$

$p \models \varphi$ means process p has property φ
E.G:

- $p \models \langle\mu\rangle\varphi$ if $p \xrightarrow{\mu} p'$ such that $p' \models \varphi$

Property logics in LTSs: proving inequivalences

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E.G:

- $p \models \langle \mu \rangle \varphi$ if $p \xrightarrow{\mu} p'$ such that $p' \models \varphi$

Classical result:

In a finite branching LTS, $p \sim_{rbc} q$ iff
$p \models \varphi$ implies $q \models \varphi$, for every property φ

Proof method:

To show $p \not\sim_{rbc} q$ exhibit φ such that

$$p \models \varphi \text{ and } q \not\models \varphi$$

Example

$$P = a.(a.(b + c) \; + \; a.b + a.c)$$
$$R = a.a.(b + c) \; + \; a.(a.b + a.c)$$

Q: Can P and R be distinguished behaviourally?

Example

$$P = a.(a.(b + c) \ + \ a.b + a.c)$$
$$R = a.a.(b + c) \ + \ a.(a.b + a.c)$$

Q: Can P and R be distinguished behaviourally?

$P \not\sim_{rbc} Q$ because

- $P \models \langle a \rangle(\ \langle a \rangle(\langle b \rangle \mathtt{tt} \wedge \langle c \rangle \mathtt{tt}) \ \wedge \ \langle a \rangle(\langle b \rangle \mathtt{tt} \wedge \neg \langle c \rangle \mathtt{tt}) \ )$
- $Q \not\models \langle a \rangle(\ \ldots$

Property logics in pLTSs: proving inequivalences

pHML:

$$\varphi ::= \mathtt{tt} \mid \varphi_1 \vee \varphi_2 \mid \langle\mu\rangle\varphi, \mu \in \mathsf{Act}_\tau \mid \neg\varphi \mid \varphi_1 \,{}_p\!\oplus\, \varphi_2,\ p \in [0,1]$$

$\Delta \models \varphi$ means process Δ has property φ
E.G:

- $\Delta \models \varphi_1 \,{}_p\!\oplus\, \varphi_2$ if $\Delta = \Delta_1 \,{}_p\!\oplus\, \Delta_2$ such that $\Delta_i \models \varphi_i$

Property logics in pLTSs: proving inequivalences

pHML:

$$\varphi \ ::= \ \mathtt{tt} \ \mid \ \varphi_1 \vee \varphi_2 \ \mid \ \langle\mu\rangle\varphi, \mu \in \mathrm{Act}_\tau \ \mid \ \neg\varphi$$
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$\Delta \models \varphi$ means process Δ has property φ

E.G:

- $\Delta \models \varphi_1 \, {}_p\!\oplus \, \varphi_2$ if $\Delta = \Delta_1 \, {}_p\!\oplus \, \Delta_2$ such that $\Delta_i \models \varphi_i$

Result:

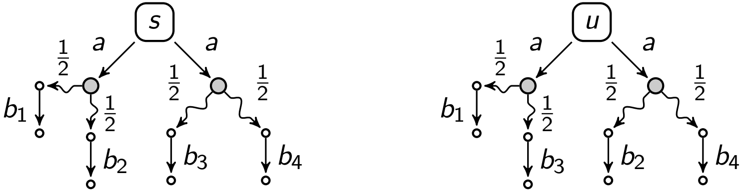
In a finitary pLTS, $\Delta \sim \Theta$ *iff*
$\Delta \models \varphi$ *implies* $\Theta \models \varphi$, *for every property* φ

Proof method:

To show $\Delta \not\sim_{rbc} \Theta$ exhibit φ such that

$$\Delta \models \varphi \text{ and } \Theta \not\models \varphi$$

Example



$\overline{s} \not\sim_{rbc} \overline{u}$ because

- $\overline{s} \models \langle a\rangle(\langle b_1\rangle\mathtt{tt}\ {}_{\frac{1}{2}}\!\oplus \langle b_2\rangle\mathtt{tt})$
- $\overline{u} \not\models \langle a\rangle(\langle b_1\rangle\mathtt{tt}\ {}_{\frac{1}{2}}\!\oplus \langle b_2\rangle\mathtt{tt})$

Outline

Summary

- Emphasis on processes as *distributions* in pLTSs
- Natural formulation of (strong) contextual behavioural equivalence
- Behavioural justification of Segalas state-based bisimulation equivalence
- Simple complete extension of HML for probabilistic processes
- Complete axiomatisation for probabilistic CCS

Summary

- Emphasis on processes as *distributions* in pLTSs
- Natural formulation of (strong) contextual behavioural equivalence
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- Simple complete extension of HML for probabilistic processes
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Future

- Algorithms ?
  - Input two processes Δ, Θ
  - Output: bisimulation containing (Δ, Θ) or a pHML distinguishing formula
- Static schedulers ?
- Weak case ?

The weak case: thoughts

Weak reduction barbed congruence $\approx_{rbc}$: easy to define

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Weak observations:

$\Delta \Downarrow^{p} a$ if $\Delta \stackrel{\tau}{\Longrightarrow} \Delta'$ such that $\Delta' \downarrow^{p} a$

Weak reduction-closed:

if $\Delta \approx_{rbc} \Theta$ then

- $\Delta \stackrel{\tau}{\Longrightarrow} \Delta'$ implies $\Theta \stackrel{\tau}{\Longrightarrow} \Theta'$ s.t. $\Delta' \approx_{rbc} \Theta$
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The weak case: thoughts

Problem:

$\approx_{rbc}$ is not decomposable

The weak case: thoughts

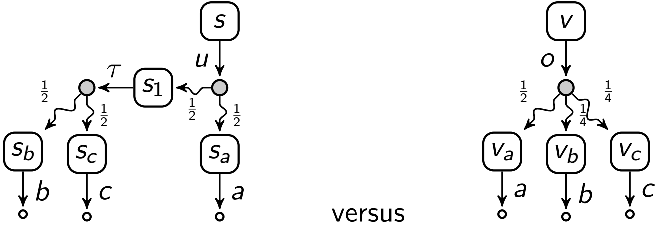
Problem:

$\approx_{rbc}$ is not decomposable

Consequence:

- Let $\approx_s$ be any relation in $S \times S$ eg a state-based weak bisimulation equivalence
- Then $\approx_{rbc}$ is NOT the same as $slift(\approx_s)$

$\approx_{rbc}$ is not decomposable



versus

$\overline{s} \approx_{rbc} \overline{v}$