On order equivalences between distance and similarity measures on sequences and trees

Martin Emms nd Hector-Hugo Franco-Penya

February 6, 2012

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- We will distinguish several distinct kinds of equivalence
- and show that while some kinds of equivalence hold, others do not

Outline

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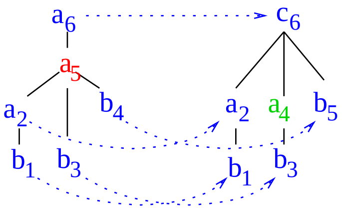

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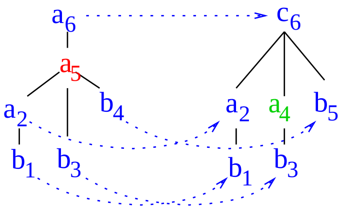

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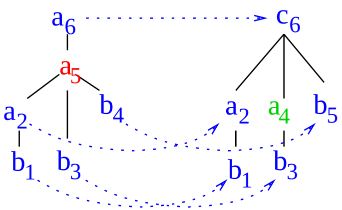

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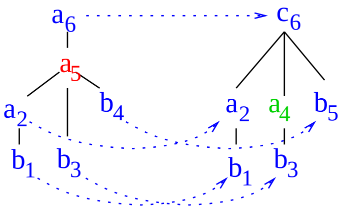

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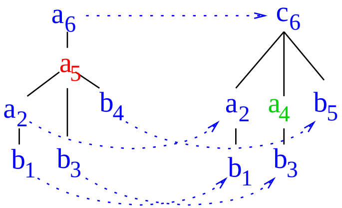

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$\mathcal{M}$ match/swaps: eg. a_6 goes to c_6
the $(i,j) \in \alpha$

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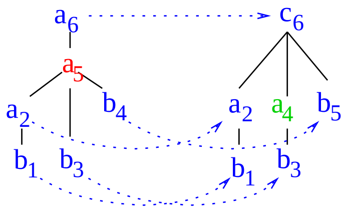

a 'cost' table C^{Δ} defines costs for members of $\mathcal{D}$, $\mathcal{I}$, $\mathcal{M}$

	λ	a	b	c
λ		•	1	•
a	1	0	•	1
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$$\Delta(\alpha : S \mapsto T) = \sum_{(i,j)\in\mathcal{M}} C^{\Delta}(i^{\gamma}, j^{\gamma}) + \sum_{i\in\mathcal{D}} C^{\Delta}(i^{\gamma}, \lambda) + \sum_{j\in\mathcal{I}} C^{\Delta}(\lambda, j^{\gamma})$$

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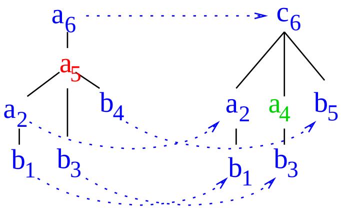

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on the example $\Delta(\alpha : S \mapsto T) = 3$

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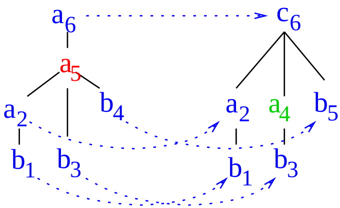

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- $\Delta(S, T)$ can be computed the Zhang/Shasha algorithm, and its correctness – ie. that it finds $min\{\Delta(\alpha : S \mapsto T)\}$ – does not require the cost-table C^{Δ} to satisfy any particular properties

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- this motivates the nearly universal adopted non-negativity assumption

$$\forall x, y \in \Sigma(C^\Delta(x, y) \geq 0, C^\Delta(x, \lambda) \geq 0, C^\Delta(\lambda, y) \geq 0) \qquad (1)$$

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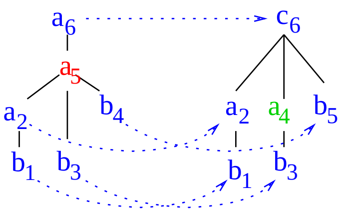

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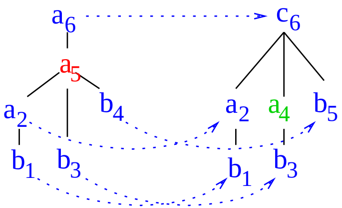

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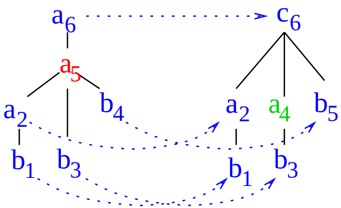

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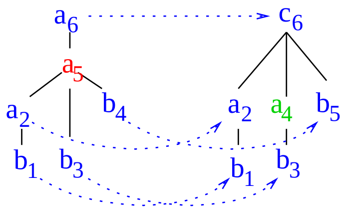

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eg. $\Theta(\alpha) = 9$

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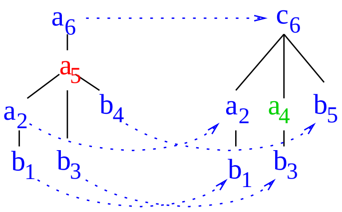

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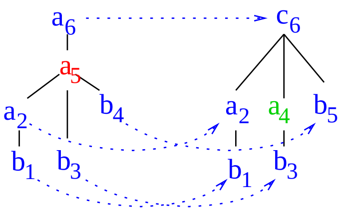

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- For the C^{Θ}-entries which are not deletions or insertions, it is quite common in biological sequence comparison to have both positive and negative entries

Summary

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same for a ’similarity’ Θ scoring

For each kind of ordering can ask whether an ordering by Δ can be replicated by Θ, and vice-versa

Definition (A-,N- and P-dual)

C^{Δ} and C^{Θ} are A-duals if the alignment orderings induced are the reverse of each other
C^{Δ} and C^{Θ} are N-duals if the neighbour orderings induced are the reverse of each other
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The natural question is: whether for *every* choice of C^{Δ}, there is a choice of $C^{\ominus}$ which is a A-dual, N-dual or P-dual, and vice-versa. More precisely we have the following

Order-relating Conjectures

$$\textbf{A-duality} \begin{cases} (i) & \forall C^{\Delta} \exists C^{\ominus} (C^{\Delta} \text{ and } C^{\ominus} \text{ are A-duals}) \\ (ii) & \forall C^{\ominus} \exists C^{\Delta} (C^{\Delta} \text{ and } C^{\ominus} \text{ are A-duals}) \end{cases}$$

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if these duality conjectures do not hold, then there are substantive difference, with the outcomes achievable by distances and similarities being distinct.

A-dualizing conversions

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For any C^{Δ}*, and* δ *s.t.*
$0 \leq \delta/2 \leq min(C^{\Delta}(\cdot,\lambda), C^{\Delta}(\lambda,\cdot))$
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For any $C^{\ominus}$, *and* δ *s.t.* $0 \leq \delta \geq max(C^{\ominus}(\cdot,\cdot))$ *derive* C^{Δ} *via (ii)*

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then in either case, for any $\alpha : S \mapsto T$

$$\Delta(\alpha) + \Theta(\alpha) = \delta/2 \times (\sum_{s \in S}(1) + \sum_{t \in T}(1)) \tag{3}$$

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Theorem

A-duality (i) and (ii) hold

- so distance and similarity are interchangeable?

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- but what about N duals ? (k-NN)
- and what about P duals ? (hierarchical clustering)

Outline

Dist to Sim: N- and P-duals

consider N-duality(i) and P-duality(i): can Neighbour- and Pair-orderings by Δ be replicated by Θ?

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Theorem

N-duality (i) and P-duality(i) hold

Outline

The Θ-to-Δ conversion of A-duality was

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so can't show N-duality(ii) and P-duality(ii) this way

Sim-to-Dist: P-duality(ii) fails

P-duality(ii) is stronger than *N-duality(ii)*. We can fairly easily show *P-duality(ii)* does not hold

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P-duality (ii) does not hold, that is, there are $C^{\ominus}$ such that there is no C^{Δ} such that $C^{\ominus}$ and C^{Δ} are P-duals.

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consider the equiv. classes

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eg. $C^{\Theta}(a,a)=1$, $C^{\Theta}(a,\lambda)=0 \Rightarrow$
$\Theta(a,a)=1 \ldots \Theta(a^n,a^n)=n$

in that case Δ-$\uparrow$ sequence cannot be equal to the Θ-$\downarrow$ sequence

Sim-to-Dist: N-duality(ii) fails

Theorem

There is $C^{\ominus}$ such that there is no C^{Δ} with $C^{\Delta}(x,\lambda) = C^{\Delta}(\lambda,x)$ such that $C^{\ominus}$ and C^{Δ} are N-duals

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Proof outline

Let $S = aa$, and *set* of neighbours be $\{a, aaa\}$

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can define $C^{\ominus}$ such that [*aaa*, *a*] = Θ-↓ neigbour ordering

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Let $S = aa$, and *set* of neighbours be $\{a, aaa\}$

can define $C^{\ominus}$ such that $[aaa,\ a]$ = Θ-$\downarrow$ neigbour ordering

cannot define C^{Δ} such that $[aaa,\ a]$ = Δ-$\uparrow$ neightbour ordering

further details

Let $C^{\ominus}(a,a) = x > 0, \quad C^{\ominus}(a,\lambda) = C^{\ominus}(\lambda,a) = y > 0$

further details

Let $C^{\Theta}(a,a) = x > 0, \quad C^{\Theta}(a,\lambda) = C^{\Theta}(\lambda,a) = y > 0$

$\alpha : aa \mapsto aaa$	$\Theta(\alpha)$	
2 a-matches	$2x - y$	

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2 a-matches	$2x - y$	
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2 a-matches	$2x - y$	(max)	
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So $(\Theta(aa,aaa) = 2x - y) > (\Theta(aa,a) = x - y)$

So [aaa, a] = Θ-$\downarrow$ neigbour ordering

Let $C^{\Delta}(a,a) = x'$, and $C^{\Delta}(a,\lambda) = C^{\Delta}(\lambda,a) = y'$.

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two cases: $\begin{cases} \text{(i) in-del < swap: } 2y' < x', \text{ so } x' = 2y' + \epsilon, \text{ for some } \epsilon > 0 \\ \text{(ii) in-del} \geq \text{swap: } 2y' \geq x', \text{ so } y' = x'/2 + \kappa, \text{ for some } \kappa \geq 0 \end{cases}$

Let $C^{\Delta}(a,a) = x'$, and $C^{\Delta}(a,\lambda) = C^{\Delta}(\lambda,a) = y'$.

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$\alpha : aa \mapsto aaa$	$\Delta(\alpha)$	(i)	(ii)
2 a-matches	$2x' + y'$		
1 a-matches	$x' + 3y'$		
0 a-matches	$5y'$		

Let $C^{\Delta}(a,a) = x'$, and $C^{\Delta}(a,\lambda) = C^{\Delta}(\lambda,a) = y'$.

two cases: $\begin{cases} \text{(i) in-del < swap: } 2y' < x', \text{ so } x' = 2y' + \epsilon, \text{ for some } \epsilon > 0 \\ \text{(ii) in-del} \geq \text{swap: } 2y' \geq x', \text{ so } y' = x'/2 + \kappa, \text{ for some } \kappa \geq 0 \end{cases}$

		(i)	(ii)
$\alpha : aa \mapsto aaa$	$\Delta(\alpha)$	$x' = 2y' + \epsilon$	
2 a-matches	$2x' + y'$	$5y' + 2\epsilon$	
1 a-matches	$x' + 3y'$	$5y' + \epsilon$	
0 a-matches	$5y'$	$5y'$ (min = $\Delta(aa, aaa)$)	

Let $C^{\Delta}(a,a) = x'$, and $C^{\Delta}(a,\lambda) = C^{\Delta}(\lambda,a) = y'$.

two cases: $\begin{cases} \text{(i) in-del < swap: } 2y' < x', \text{ so } x' = 2y' + \epsilon, \text{ for some } \epsilon > 0 \\ \text{(ii) in-del} \geq \text{swap: } 2y' \geq x', \text{ so } y' = x'/2 + \kappa, \text{ for some } \kappa \geq 0 \end{cases}$

$\alpha : aa \mapsto aaa$	$\Delta(\alpha)$	(i) $x' = 2y' + \epsilon$	(ii) $y' = x'/2 + \kappa$
2 a-matches	$2x' + y'$	$5y' + 2\epsilon$	$2.5x' + \kappa$ (eq. min = $\Delta(aa, aaa)$)
1 a-matches	$x' + 3y'$	$5y' + \epsilon$	$2.5x' + 3\kappa$
0 a-matches	$5y'$	$5y'$ (min = $\Delta(aa, aaa)$)	$2.5x' + 5\kappa$

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0 a-matches	$5y'$	$5y'$ (min = $\Delta(aa, aaa)$)	$2.5x' + 5\kappa$

$\alpha : aa \mapsto a$	$\Delta(\alpha)$
1 a-matches	$x' + y'$
0 a-matches	$3y'$

Let $C^{\Delta}(a,a) = x'$, and $C^{\Delta}(a,\lambda) = C^{\Delta}(\lambda,a) = y'$.

two cases: $\begin{cases} \text{(i) in-del < swap: } 2y' < x', \text{ so } x' = 2y' + \epsilon, \text{ for some } \epsilon > 0 \\ \text{(ii) in-del} \geq \text{swap: } 2y' \geq x', \text{ so } y' = x'/2 + \kappa, \text{ for some } \kappa \geq 0 \end{cases}$

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1 a-matches	$x' + 3y'$	$5y' + \epsilon$	$2.5x' + 3\kappa$
0 a-matches	$5y'$	$5y'$ (min = $\Delta(aa, aaa)$)	$2.5x' + 5\kappa$

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1 a-matches	$x' + y'$	$3y' + \epsilon$
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Let $C^{\Delta}(a,a) = x'$, and $C^{\Delta}(a,\lambda) = C^{\Delta}(\lambda,a) = y'$.

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$\alpha : aa \mapsto a$	$\Delta(\alpha)$	$x' = 2y' + \epsilon$	$y' = x'/2 + \kappa$
1 a-matches	$x' + y'$	$3y' + \epsilon$	$1.5x' + \kappa$ (eq. min = $\Delta(aa, a)$)
0 a-matches	$3y'$	$3y'$ (min = $\Delta(aa, a)$)	$1.5x' + 3\kappa$

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$\alpha : aa \mapsto a$	$\Delta(\alpha)$	$x' = 2y' + \epsilon$	$y' = x'/2 + \kappa$
1 a-matches	$x' + y'$	$3y' + \epsilon$	$1.5x' + \kappa$ (eq. min = $\Delta(aa, a)$)
0 a-matches	$3y'$	$3y'$ (min = $\Delta(aa, a)$)	$1.5x' + 3\kappa$

case (i): $(\Delta(aa, aaa) = 5y') > (\Delta(aa, a) = 3y')$
case (ii) $(\Delta(aa, aaa) = 2.5x' + \kappa) > (\Delta(aa, a) = 1.5x' + \kappa)$

Let $C^{\Delta}(a,a) = x'$, and $C^{\Delta}(a,\lambda) = C^{\Delta}(\lambda,a) = y'$.

two cases: $\begin{cases} \text{(i) in-del < swap: } 2y' < x', \text{ so } x' = 2y' + \epsilon, \text{ for some } \epsilon > 0 \\ \text{(ii) in-del} \geq \text{swap: } 2y' \geq x', \text{ so } y' = x'/2 + \kappa, \text{ for some } \kappa \geq 0 \end{cases}$

$\alpha : aa \mapsto aaa$	$\Delta(\alpha)$	(i) $x' = 2y' + \epsilon$	(ii) $y' = x'/2 + \kappa$
2 a-matches	$2x' + y'$	$5y' + 2\epsilon$	$2.5x' + \kappa$ (eq. min = $\Delta(aa, aaa)$)
1 a-matches	$x' + 3y'$	$5y' + \epsilon$	$2.5x' + 3\kappa$
0 a-matches	$5y'$	$5y'$ (min = $\Delta(aa, aaa)$)	$2.5x' + 5\kappa$

$\alpha : aa \mapsto a$	$\Delta(\alpha)$	$x' = 2y' + \epsilon$	$y' = x'/2 + \kappa$
1 a-matches	$x' + y'$	$3y' + \epsilon$	$1.5x' + \kappa$ (eq. min = $\Delta(aa, a)$)
0 a-matches	$3y'$	$3y'$ (min = $\Delta(aa, a)$)	$1.5x' + 3\kappa$

case (i): $(\Delta(aa, aaa) = 5y') > (\Delta(aa, a) = 3y')$
case (ii) $(\Delta(aa, aaa) = 2.5x' + \kappa) > (\Delta(aa, a) = 1.5x' + \kappa)$

in neither case do we get Δ-$\uparrow$ = $[aaa, a]$

the Order-relating Conjectures revisited

the Order-relating Conjectures revisited

A-duality

- (*i*) $\forall C^{\Delta} \exists C^{\ominus} (C^{\Delta}$ and $C^{\ominus}$ are A-duals) *TRUE*
- (*ii*) $\forall C^{\ominus} \exists C^{\Delta} (C^{\Delta}$ and $C^{\ominus}$ are A-duals) *TRUE*

N-duality

- (*i*) $\forall C^{\Delta} \exists C^{\ominus} (C^{\Delta}$ and $C^{\ominus}$ are N-duals)
- (*ii*) $\forall C^{\ominus} \exists C^{\Delta} (C^{\Delta}$ and $C^{\ominus}$ are N-duals)

P-duality

- (*i*) $\forall C^{\Delta} \exists C^{\ominus} (C^{\Delta}$ and $C^{\ominus}$ are P-duals)
- (*ii*) $\forall C^{\ominus} \exists C^{\Delta} (C^{\Delta}$ and $C^{\ominus}$ are P-duals)

the Order-relating Conjectures revisited

A-duality	(i)	$\forall C^{\Delta} \exists C^{\ominus}(C^{\Delta}$ and $C^{\ominus}$ are A-duals)	*TRUE*
	(ii)	$\forall C^{\ominus} \exists C^{\Delta}(C^{\Delta}$ and $C^{\ominus}$ are A-duals)	*TRUE*
N-duality	(i)	$\forall C^{\Delta} \exists C^{\ominus}(C^{\Delta}$ and $C^{\ominus}$ are N-duals)	*TRUE*
	(ii)	$\forall C^{\ominus} \exists C^{\Delta}(C^{\Delta}$ and $C^{\ominus}$ are N-duals)	
P-duality	(i)	$\forall C^{\Delta} \exists C^{\ominus}(C^{\Delta}$ and $C^{\ominus}$ are P-duals)	*TRUE*
	(ii)	$\forall C^{\ominus} \exists C^{\Delta}(C^{\Delta}$ and $C^{\ominus}$ are P-duals)	

the Order-relating Conjectures revisited

A-duality	(i)	$\forall C^{\Delta} \exists C^{\ominus}(C^{\Delta}$ and $C^{\ominus}$ are A-duals)	*TRUE*
	(ii)	$\forall C^{\ominus} \exists C^{\Delta}(C^{\Delta}$ and $C^{\ominus}$ are A-duals)	*TRUE*
N-duality	(i)	$\forall C^{\Delta} \exists C^{\ominus}(C^{\Delta}$ and $C^{\ominus}$ are N-duals)	*TRUE*
	(ii)	$\forall C^{\ominus} \exists C^{\Delta}(C^{\Delta}$ and $C^{\ominus}$ are N-duals)	*FALSE*
P-duality	(i)	$\forall C^{\Delta} \exists C^{\ominus}(C^{\Delta}$ and $C^{\ominus}$ are P-duals)	*TRUE*
	(ii)	$\forall C^{\ominus} \exists C^{\Delta}(C^{\Delta}$ and $C^{\ominus}$ are P-duals)	*FALSE*

the Order-relating Conjectures revisited

A-duality
- (i) $\forall C^{\Delta} \exists C^{\ominus} (C^{\Delta}$ and $C^{\ominus}$ are A-duals) *TRUE*
- (ii) $\forall C^{\ominus} \exists C^{\Delta} (C^{\Delta}$ and $C^{\ominus}$ are A-duals) *TRUE*

N-duality
- (i) $\forall C^{\Delta} \exists C^{\ominus} (C^{\Delta}$ and $C^{\ominus}$ are N-duals) *TRUE*
- (ii) $\forall C^{\ominus} \exists C^{\Delta} (C^{\Delta}$ and $C^{\ominus}$ are N-duals) *FALSE*

P-duality
- (i) $\forall C^{\Delta} \exists C^{\ominus} (C^{\Delta}$ and $C^{\ominus}$ are P-duals) *TRUE*
- (ii) $\forall C^{\ominus} \exists C^{\Delta} (C^{\Delta}$ and $C^{\ominus}$ are P-duals) *FALSE*

this means

the Order-relating Conjectures revisited

A-duality
- (*i*) $\forall C^{\Delta} \exists C^{\Theta} (C^{\Delta}$ and C^{Θ} are A-duals) *TRUE*
- (*ii*) $\forall C^{\Theta} \exists C^{\Delta} (C^{\Delta}$ and C^{Θ} are A-duals) *TRUE*

N-duality
- (*i*) $\forall C^{\Delta} \exists C^{\Theta} (C^{\Delta}$ and C^{Θ} are N-duals) *TRUE*
- (*ii*) $\forall C^{\Theta} \exists C^{\Delta} (C^{\Delta}$ and C^{Θ} are N-duals) *FALSE*

P-duality
- (*i*) $\forall C^{\Delta} \exists C^{\Theta} (C^{\Delta}$ and C^{Θ} are P-duals) *TRUE*
- (*ii*) $\forall C^{\Theta} \exists C^{\Delta} (C^{\Delta}$ and C^{Θ} are P-duals) *FALSE*

this means

- any hierarchical clustering outcome achieved via Δ can be replicated via Θ, but not vice-versa

the Order-relating Conjectures revisited

A-duality
- (*i*) $\forall C^{\Delta} \exists C^{\Theta} (C^{\Delta}$ and C^{Θ} are A-duals) *TRUE*
- (*ii*) $\forall C^{\Theta} \exists C^{\Delta} (C^{\Delta}$ and C^{Θ} are A-duals) *TRUE*

N-duality
- (*i*) $\forall C^{\Delta} \exists C^{\Theta} (C^{\Delta}$ and C^{Θ} are N-duals) *TRUE*
- (*ii*) $\forall C^{\Theta} \exists C^{\Delta} (C^{\Delta}$ and C^{Θ} are N-duals) *FALSE*

P-duality
- (*i*) $\forall C^{\Delta} \exists C^{\Theta} (C^{\Delta}$ and C^{Θ} are P-duals) *TRUE*
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this means

- any hierarchical clustering outcome achieved via Δ can be replicated via Θ, but not vice-versa
- any categorisation outcome using nearest-neighbours achieved via Δ can be replicated via Θ, but not vice-versa

the Order-relating Conjectures revisited

A-duality
- (i) $\forall C^{\Delta} \exists C^{\ominus} (C^{\Delta}$ and $C^{\ominus}$ are A-duals) *TRUE*
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N-duality
- (i) $\forall C^{\Delta} \exists C^{\ominus} (C^{\Delta}$ and $C^{\ominus}$ are N-duals) *TRUE*
- (ii) $\forall C^{\ominus} \exists C^{\Delta} (C^{\Delta}$ and $C^{\ominus}$ are N-duals) *FALSE*

P-duality
- (i) $\forall C^{\Delta} \exists C^{\ominus} (C^{\Delta}$ and $C^{\ominus}$ are P-duals) *TRUE*
- (ii) $\forall C^{\ominus} \exists C^{\Delta} (C^{\Delta}$ and $C^{\ominus}$ are P-duals) *FALSE*

this means

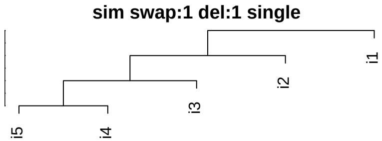
- any hierarchical clustering outcome achieved via Δ can be replicated via $\ominus$, but not vice-versa
- any categorisation outcome using nearest-neighbours achieved via Δ can be replicated via $\ominus$, but not vice-versa
- in this sense 'similarity' and 'distance' comparison measures on sequences and trees are not interchangeable.

Sim to Dist: unreproducible clustering

single-link clustering of $\{a^5, a^4, a^3, a^2, a^1\}$ using $C^{\ominus}(a,a) = 1, C^{\ominus}(a,\lambda) = 1$

Sim to Dist: unreproducible clustering

single-link clustering of $\{a^5, a^4, a^3, a^2, a^1\}$
using $C^{\ominus}(a,a) = 1, C^{\ominus}(a,\lambda) = 1$



using $C^{\Delta}(a,a) = 0, C^{\Delta}(a,\lambda) = 1$
all on the same level because $\Delta(a^m, a^{m+1}) = 1$

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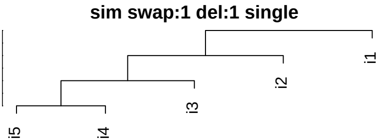

using $C^{\Delta}(a, a) = 0, C^{\Delta}(a, \lambda) = 1$
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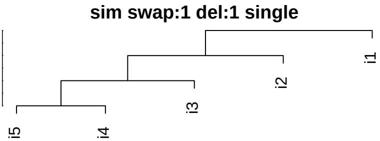
using $C^{\Delta}(a, a) = 1$
and $C^{\Delta}(a, \lambda)$
or $0.5 \leq C^{\Delta}(a, \lambda) \leq 5.5$
(so $2C^{\Delta}(a, \lambda) \geq C^{\Delta}(a, a)$)
or $0.1 \leq C^{\Delta}(a, \lambda) \leq 0.4$
(so $2C^{\Delta}(a, \lambda) < C^{\Delta}(a, a)$)

Sim to Dist: unreproducible clustering

single-link clustering of $\{a^5, a^4, a^3, a^2, a^1\}$ using $C^{\ominus}(a,a) = 1, C^{\ominus}(a,\lambda) = 1$



using $C^{\Delta}(a,a) = 0, C^{\Delta}(a,\lambda) = 1$ all on the same level because $\Delta(a^m, a^{m+1}) = 1$

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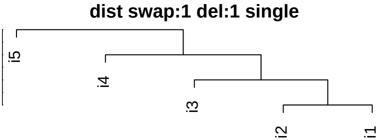

Dist to Sim: N-duality failures

There are conversions from Dist to Sim which make A-duals: to what degree does this make an N-duals?

Δ-to-Θ conversion for A-duality was

(i)

$$\begin{cases} C^{\Theta}(x,\lambda) = C^{\Delta}(x,\lambda) - \delta/2 \\ C^{\Theta}(\lambda,y) = C^{\Delta}(\lambda,y) - \delta/2 \\ C^{\Theta}(x,y) = \delta - C^{\Delta}(x,y) \end{cases}$$

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eg. derived C^{Θ} from unit-cost C^{Δ}

		A-dual C^{Θ} for varying δ						
	C^{Δ}	2	1.5	1	0.5	0.2	0.1	0
(x,λ)	1	0	0.25	0.5	0.75	0.9	0.95	1
(x,x)	0	2	1.5	1	0.5	0.2	0.1	0
(x,y)	1	1	0.5	0	-0.5	-0.8	-0.9	-1

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for a S, make Δ-$\uparrow$ neighb. ordering $N_{\Delta}(S)$

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(x,x)	0	2	1.5	1	0.5	0.2	0.1	0
(x,y)	1	1	0.5	0	-0.5	-0.8	-0.9	-1

for a S, make Δ-$\uparrow$ neighb. ordering $N_{\Delta}(S)$
for a S, make Θ-$\downarrow$ neighb. ordering $N_{\Theta}(S)$

Dist to Sim: N-duality failures

There are conversions from Dist to Sim which make A-duals: to what degree does this make an N-duals?

Δ-to-Θ conversion for A-duality was
(i)

$$\begin{cases} C^{\Theta}(x,\lambda) = C^{\Delta}(x,\lambda) - \delta/2 \\ C^{\Theta}(\lambda,y) = C^{\Delta}(\lambda,y) - \delta/2 \\ C^{\Theta}(x,y) = \delta - C^{\Delta}(x,y) \end{cases}$$

eg. derived C^{Θ} from unit-cost C^{Δ}

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	C^{Δ}	2	1.5	1	0.5	0.2	0.1	0
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(x,x)	0	2	1.5	1	0.5	0.2	0.1	0
(x,y)	1	1	0.5	0	-0.5	-0.8	-0.9	-1

for a S, make Δ-$\uparrow$ neighb. ordering $N_{\Delta}(S)$
for a S, make Θ-$\downarrow$ neighb. ordering $N_{\Theta}(S)$
$tau(N_{\Delta}(S), N_{\Theta}(S))$ = *kendall-tau* comparison of ordering

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Δ-to-Θ conversion for A-duality was

(i)

$$\begin{cases} C^{\Theta}(x,\lambda) = C^{\Delta}(x,\lambda) - \delta/2 \\ C^{\Theta}(\lambda,y) = C^{\Delta}(\lambda,y) - \delta/2 \\ C^{\Theta}(x,y) = \delta - C^{\Delta}(x,y) \end{cases}$$

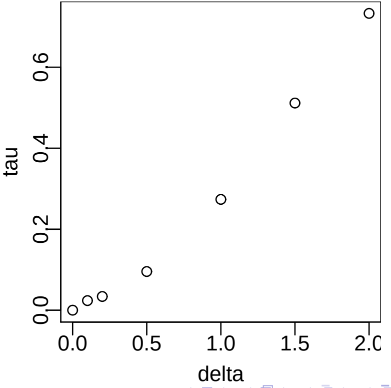
for a S, make Δ-$\uparrow$ neighb. ordering $N_{\Delta}(S)$

for a S, make Θ-$\downarrow$ neighb. ordering $N_{\Theta}(S)$

$tau(N_{\Delta}(S), N_{\Theta}(S))$ = *kendall-tau* comparison of ordering

eg. derived C^{Θ} from unit-cost C^{Δ}

		A-dual C^{Θ} for varying δ						
	C^{Δ}	2	1.5	1	0.5	0.2	0.1	0
(x,λ)	1	0	0.25	0.5	0.75	0.9	0.95	1
(x,x)	0	2	1.5	1	0.5	0.2	0.1	0
(x,y)	1	1	0.5	0	-0.5	-0.8	-0.9	-1



Sim-to-Dist: N-duality failures

There are conversions from Sim to Dist which make A-duals: to what degree does this make an N-duals

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Θ-to-Δ conversion for A-duality was

(ii)

$$\begin{cases} C^{\Delta}(x,\lambda) = C^{\Theta}(x,\lambda) + \delta/2 \\ C^{\Delta}(\lambda,y) = C^{\Theta}(\lambda,y) + \delta/2 \\ C^{\Delta}(x,y) = \delta - C^{\Theta}(x,y) \end{cases}$$

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A C^{Θ} and several A-dual C^{Δ}

		A-dual C^{Δ} for varying δ						
	C^{Θ}	1	1.5	2	2.5	3	3.5	4
(x,λ)	0.5	1	1.25	1.5	1.75	2	2.25	2.5
(x,x)	1	0	0.5	1	1.5	2	2.5	3
(x,y)	0	1	1.5	2	2.5	3	3.5	4

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	C^{Θ}	1	1.5	2	2.5	3	3.5	4
(x,λ)	0.5	1	1.25	1.5	1.75	2	2.25	2.5
(x,x)	1	0	0.5	1	1.5	2	2.5	3
(x,y)	0	1	1.5	2	2.5	3	3.5	4

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	C^{Θ}	1	1.5	2	2.5	3	3.5	4
(x,λ)	0.5	1	1.25	1.5	1.75	2	2.25	2.5
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(x,λ)	0.5	1	1.25	1.5	1.75	2	2.25	2.5
(x,x)	1	0	0.5	1	1.5	2	2.5	3
(x,y)	0	1	1.5	2	2.5	3	3.5	4

for a S, make Θ-$\downarrow$ neighb. ordering $N_{\Theta}(S)$

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(x,x)	1	0	0.5	1	1.5	2	2.5	3
(x,y)	0	1	1.5	2	2.5	3	3.5	4

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