Suppose you have some data on people concerning two possible variables $sea$, which is whether they live by the seaside, and $hip$ which is whether they have hip problems:

$$
\begin{array}{c|cc|c|cc}
& sea: + & sea: - \\
hip: + & 31 & 54 & \\
hip: - & 19 & 146 \\
\end{array}
$$

one of the formulations of independence is $P(X|Y) = P(X)$. Let's apply that to $sea$ and $hip$, in fact to the '+' settings of these variables

$$
p(hip: +) = (31 + 54)/250 = 0.34
$$

$$
p(hip: +|sea: +) = 31/(31 + 19) = 0.62
$$

so $hip: +$ and $sea: +$ are not independent; in fact sea-side living seems to increase the chance of hip problems, which seems weird.

Once you have a conditional independence it means that you can use the chain rule and use the conditional independence to simplify. We will see this in other examples; in the current case you could do this to get relatively simple formula for $p(old, sea, hip)$

$$
p(old, sea, hip) = p(hip|sea, old) \times p(sea|old) \times p(old) 
$$

(3) is just applying the chain rule and holds without any independence assumptions.

(4) is the simplification which is possibly by putting in the conditional independence that $p(hip|sea, old) = p(hip|old)$.

### Example 2

suppose that digging into the data a little further you find there was one other variable: $old$ for whether or not person was old. There were 50 old and 200 not old, and when the data is split into two sub-groups according to the value $old$ you find:

$$
\begin{array}{c|cc|c|cc}
old & sea: + & sea: - & \neg old & sea: + & sea: - \\
hip: + & 27 & 18 & hip: + & 4 & 36 \\
hip: - & 3 & 2 & hip: - & 16 & 144 \\
\end{array}
$$

- we can show that $hip: +$ is conditionally independent of $sea: +$ given $old: +$
  
  $$
p(hip: +|old: +) = 45/50 = 9/10
$$
  
  $$
p(hip: +|old: +, sea: +) = 27/30 = 9/10
$$

- we can show that $hip: +$ is conditionally independent of $sea: +$ given $old: -$
  
  $$
p(hip: +|old: -) = 40/200 = 1/5
$$
  
  $$
p(hip: +|old: -, sea: +) = 4/20 = 1/5
$$

- so zeroing in old people, sea-side living does not increase the chance of hip problems; zeroing in on young people, it doesn’t either