Suppose you have some data on people concerning two possible variables \( \text{sea} \), which is whether they live by the seaside, and \( \text{hip} \) which is whether they have hip problems:

\[
\begin{array}{c|cc}
\text{sea} & + & - \\
\text{hip} & + & 31 & 54 \\
& - & 19 & 146 \\
\end{array}
\]

one of the formulations of independence is \( P(X \mid Y) = P(X) \). Lets apply that to \( \text{sea} \) and \( \text{hip} \), in fact to the '+' settings of these variables

\[
p(\text{hip} : +) = (31 + 54)/250 = 0.34
\]
\[
p(\text{hip} : + \mid \text{sea} : +) = 31/(31 + 19) = 0.62
\]

so \( \text{hip} : + \) and \( \text{sea} : + \) are not independent; in fact sea-side living seems to increase the chance of hip problems, which seems weird.

Once you have a conditional independence it means that you can use the chain rule and use the conditional independence to simplify. We will see this in other examples; in the current case you could do this to get relatively simple formula for \( p(\text{old}, \text{sea}, \text{hip}) \)

\[
p(\text{old}, \text{sea}, \text{hip}) = p(\text{hip} \mid \text{sea}, \text{old}) \times p(\text{sea} \mid \text{old}) \times p(\text{old})
\]

(3) is just applying the chain rule and holds without any independence assumptions

(4) is the simplification which is possibly by putting in the conditional independence that \( p(\text{hip} \mid \text{sea}, \text{old}) = p(\text{hip} \mid \text{old}) \)