Prob Exercises

Martin Emms

October 6, 2017

HAND-IN: begin 12.00 lecture Fri Oct 13th

Answers can be handwritten.

1. show (i) \( P(A \land B) = P(A) \times P(B) \) implies (ii) \( P(A|B) = P(A) \) and \( P(B|A) = P(B) \), and vice-versa

2. A sound-bite may or may not have been produced by JedWard. A sound-bite may or may not report contain the word \textit{OMG}.

You hear \textit{OMG} and want to work out the probability that the speaker is Jedward

Formalize with 2 discrete variables

- discrete \textit{Speaker}, values in \{Jedward, Other\}
- discrete \textit{OMG}, values in \{true, false\}

Let \textit{jed} stand for \textit{Speaker} = Jedward, \textit{omg} stands for \textit{OMG} = true

(a) Work out which of \textit{jed} or \textit{¬jed} is likelier, given \textit{omg}, supposing the probabilities \( p(\textit{jed}) = 0.01, p(\textit{omg}|\textit{jed}) = 0.95, p(\textit{omg}|\textit{¬jed}) = 0.01 \)

(b) Do the same assuming \( p(\textit{jed}) = 0.15, p(\textit{omg}|\textit{jed}) = 0.95, p(\textit{omg}|\textit{¬jed}) = 0.01 \)

(c) Do the same assuming \( p(\textit{jed}) = 0.01, p(\textit{omg}|\textit{jed}) = 0.95, p(\textit{omg}|\textit{¬jed}) = 0.001 \)

3. Consider someone who lives in a basement flat. Sometimes it is quite noisy in the flat, and sometimes not. Sometimes it is rather cool in the flat, and sometimes not. Let \textit{noisy} be a variable indicating whether it is rather noisy or not, on a given day, and let \textit{cool} be a variable indicating whether it is rather cool or not.

Consider the frequency table

\begin{align*}
\text{noisy} : + & \quad \text{noisy} : - \\
\text{cool} : + & \quad 62 \quad 108 \\
\text{cool} : - & \quad 38 \quad 292
\end{align*} 

find \( p(\textit{cool} : +) \) and \( p(\textit{cool} : +|\textit{noisy} : +) \)

and thereby show that \textit{cool} : + is not independent of \textit{noisy} : +
4. Unknown to the occupant of the flat there is ventilator fixture in the wall which can be opened and shut to let air from the street outside in, or keep it out. Unknown to the occupant a pet cat plays about with this at night-time, sometimes leaving it open and sometimes leaving it shut. The table (1) concerns 500 days. The two tables below split these into a group of 100 days where the cat has left the ventilator open (2), and 400 days where the cat has left it shut (3)

\[
\begin{array}{ccc}
\text{open} & \text{noisy} & \text{noisy} \\
\text{cool} & 54 & 36 \\
\text{cool} & 6 & 4 \\
\end{array}
\]  \hspace{1cm} (2)

\[
\begin{array}{ccc}
\text{open} & \text{noisy} & \text{noisy} \\
\text{cool} & 8 & 72 \\
\text{cool} & 32 & 288 \\
\end{array}
\]  \hspace{1cm} (3)

With reference to the table (2), find \( p(\text{cool} : + | \text{open} : +) \) and \( p(\text{cool} : + | \text{open} : +, \text{noisy} : +) \) and thereby show that \( \text{cool} : + \) is conditionally independent of \( \text{noisy} : + \) given \( \text{open} : + \).

With reference to the table (3), find \( p(\text{cool} : + | \text{open} : -) \) and \( p(\text{cool} : + | \text{open} : -, \text{noisy} : +) \) and thereby show that \( \text{cool} : + \) is conditionally independent of \( \text{noisy} : + \) given \( \text{open} : - \)