



Fuzzy Logic and Fuzzy Systems – Knowledge Representation & Reasoning

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October 15th, 2013.**

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FUZZY LOGIC & FUZZY SYSTEMS

Knowledge Representation & Reasoning

Once we have found that the knowledge of a specialism can be expressed through linguistic variables and rules of thumb, that involve imprecise antecedents and consequents, then we have a basis of a *knowledge-base*.

- In this knowledge-base ‘facts’ are represented through linguistic variables and the rules follow *fuzzy logic*.
- In traditional expert systems facts are stated crisply and rules follow classical propositional logic.

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Knowledge Representation & Reasoning

But in the 'real' world some of our knowledge of facts is derived from the use of sensors – *quantity of heat* measured in degrees Centigrade; *length* measured in meters; *weight* – *the quantity of matter* – measured in grams.

This quantitative, and rather precise factual information has to be mapped onto the term-set of a linguistic variable – the process of *fuzzification*

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Knowledge Representation & Reasoning

Once mapped the rules within a knowledge base are invoked systematically to see which of the rules is fired and to what degree – the process of *inference*.

In traditional expert systems, only those rules fire whose antecedents are true → in fuzzy expert systems rules may fire to a certain degree: all rules may fire to a degree between zero and unity.

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Knowledge Representation & Reasoning

But, of course, we have to see what influence each rule has given the fuzzy input values.

An 'averaging' procedure is adopted to compute the effective contribution of each of the rules.

This is the process of *composition*.

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Knowledge Representation & Reasoning

And, finally, we have to convert the fuzzy values outputted by the inference procedure onto a crisp number that can be used in the 'real' world.

This process is called *defuzzification*.

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Knowledge Representation & Reasoning

The operation of a fuzzy expert system depends on the execution of **FOUR** major tasks:

**Fuzzification,
Inference,
Composition,
Defuzzification.**

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Knowledge Representation & Reasoning: The Air-conditioner Example

Recall that the rules governing the air-conditioner are as follows:

RULE#1: IF TEMP is COLD THEN SPEED is MINIMAL

RULE#2: IF TEMP is COOL THEN SPEED is SLOW

RULE#3: IF TEMP is PLEASANT THEN SPEED is
MEDIUM

RULE#4: IF TEMP is WARM THEN SPEED is FAST

RULE#5: IF TEMP is HOT THEN SPEED is BLAST

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Knowledge Representation & Reasoning: The Air-conditioner Example

**Consider the case
when we require
the air-conditioner
to operate at 16°C.**

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Knowledge Representation & Reasoning

The operation of a fuzzy expert system depends on the execution of FOUR major tasks:

Fuzzification: Crisp Value \rightarrow Linguistic Variables
(16C \rightarrow Cool/Pleasant)

Inference: Rules containing the linguistic variables
(Rules 2 & 3)

Composition: Create new membership function of the alpha levelled functions for Cool and Pleasant

Defuzzification: Examine the fuzzy sets of SLOW and MEDIUM and obtain a SPEED value in RPM

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Knowledge-Based Expert Systems

A computer program which, with its associated data, embodies organised knowledge concerning some specific area of human activity. Such a system is expected to perform competently, skilfully and in a cost-effective manner; it may be thought of as a computer program which mimics the performance of a human expert.

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Knowledge-Based Expert Systems

The Nature of Expertise

One view of human expertise is that some people have spent so much time solving problems in **one particular domain** that they 'know all there is to know' (nearly) and are able to see any problem as an instance of a class of problems with which they have been confronted before.

Once the expert has successfully classified or recognised a new problem as an instance of a previously experienced problem type, all the expert has to do is apply whatever **solution proved successful** in dealing with that type of problem in the past.

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Knowledge-Based Expert Systems

The Nature of Expertise

This model of human expertise that relies (a) on domain-specific knowledge and (b) experience-based recognition of solutions of problems, has served as the basis of numerous expert systems.

The idea that cognition is a ‘recognition’-based phenomenon. That is, when a medical doctor, say, examines a patient or hears what the patient has to say about his or her problem, the configuration of symptoms or signs suggests a particular illness with which the doctor is already completely familiar.

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Knowledge-Based Expert Systems

The Nature of Expertise

The ‘recognition-based’ phenomenon can be viewed as setting up a key goal in the problem solving process and then attempt to find out data that satisfies the goal. The goal is then broken up into sub-goals and data sought to prove each of the sub-goals. (The sub-goals can be broken into further sub-goals and so on and on).

While proving a sub-goal, the experts suspend the goal and try and satisfy individual goals. Once all sub-goals are satisfied then the key goal is deemed to be satisfied and the problem solved!

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Knowledge-Based Expert Systems

Pneumonia Expert System

Consider a small chunk of a medical experts knowledge about diagnosing pneumonia and fever

rule **pneumonia**

if 'the patient has chest pain' &
'the patient has a fever' &
'the patient produces purulent sputum'
then 'the patient has pneumonia'

rule **fever**

if 'the patient has a temperature above 100'
then 'the patient has a fever'.

The nurse has just come in a with a patient with the following symptoms:

'the patient has a chest pain' &
'the patient has a temperature above 100'&
'the patient produces purulent sputum'

And the expert has to deduce

whether or not the patient has pneumonia?

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Knowledge-Based Expert Systems

Pneumonia Expert System

The nurse has just come in a with a patient with the following symptoms:

'the patient has a chest pain' &
'the patient has a temperature above 100'&
'the patient produces purulent sputum'

And the expert has to deduce

whether or not the patient has pneumonia?

In order to prove whether or not the patient has pneumonia, the expert has to prove:

prove 'the patient has pneumonia'	← GOAL
prove 'the patient has a chest pain'	← SUB-GOAL
prove 'the patient has a fever'	← SUB-GOAL
prove 'the patient produces purulent sputum'	← SUB-GOAL

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Knowledge-Based Expert Systems

Clinical Practice: In medical diagnosis and treatment there is often uncertainty regarding attributes such as

- the *significance* the disease
 - the *efficacy* of a treatment
- or
- the diagnosis itself.

Medical Terminology: Medical (and other life and human sciences) terminology allows considerable scope for *vagueness, ambiguity, inexactness, imprecision* and/or *uncertainty*:

- "adequate" dosage of a drug
- "stable" condition of patient
- 'the patient is "feverish"'
- 'this is a "possible" case of meningitis'

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Knowledge-Based Expert Systems

The knowledge of a subject domain is *encoded* in its terminology.

Knowledge in an expert system is used for solving problems: a knowledge engineer is expected to engineer, or *decode*, this knowledge and use data structures to represent the results of the decoding process on a computer system.

And, if this knowledge contains descriptions which are vague, ambiguous, inexact or reflects uncertainty, then it is essential to model the vagueness such that the description can be engineered on a computer system.

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Knowledge-Based Expert Systems

Decision Making and Statistical Theories

Bayes' Theorem: A number of workers in medical sciences have used a variety of statistical methods and techniques for examining and utilizing evidence (e.g. clinical signs, patient history etc.) to select a diagnosis or to support a prescription. Bayes' Theorem has been used extensively in computer-based medical decision support systems.

For example, the medical diagnosis problem can be viewed as the assignment of probabilities to specific diagnosis after all the relevant data has been analyzed:



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Knowledge-Based Expert Systems

ξ \equiv sum of the relevant data
 d_i \equiv diagnosis of a disease i
 $P(d_i | \xi)$ = conditional probability that patient has disease d_i in the light of evidence ξ
 $P(\xi | d_i)$ = conditional prob. that patient has symptoms / signs ξ if suffering d_i
 $P(d_i)$ = unconditional prob. for disease d_i

Bayes' Theorem relates the prior probabilities $P(\xi|d_i)$ to a posterior probability $P(d_i | \xi)$ – a knowledge of all diseases i is required

$$P(d_i | \xi) = \frac{P(\xi | d_i) * P(d_i)}{\sum_i P(\xi | d_i) * P(d_i)}$$



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Knowledge-Based Expert Systems

- ◆ Intelligent beings perceive, reason and act.
- ◆ Intelligent beings are creative, learn from their mistakes.
- ◆ Intelligent beings can learn from their environment.
- ◆ Intelligent beings can learn with the help of tutors.
- ◆ Intelligent beings can work on their own/form groups.
- ◆ Intelligent beings have a value system, an exchange system.

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Representation:

Production Systems

- **Production Systems** are a modular knowledge representation scheme and are based on the notion of *condition-action pairs*, called **production rules** or just **productions**: "If this condition occurs, then do this action".
- The utility of the production system formalism comes from the fact that the conditions in which each rule is applicable are made **explicit** and, in theory at least, the interactions between rules are minimised in the sense that the rules do not 'call' each other.

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Representation: Production Systems

Consider a knowledge-base containing the following rules:

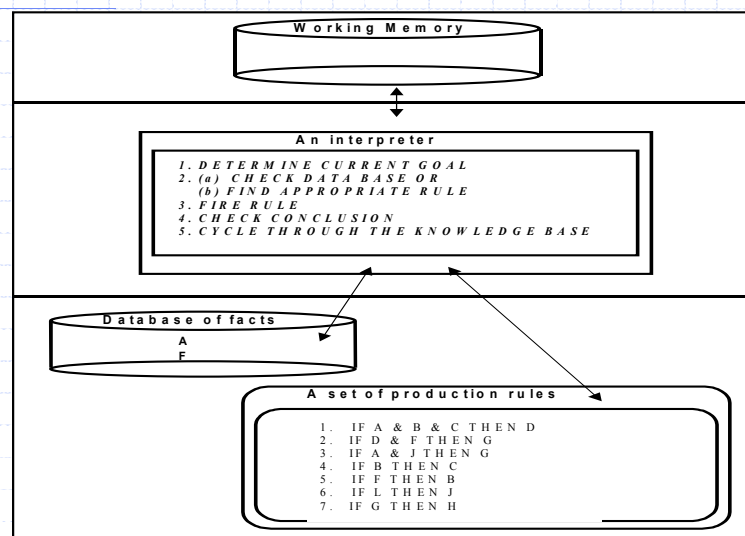
Rule#1:	<i>IF</i>	A&B&C	<i>THEN</i>	D
Rule#2:	<i>IF</i>	D&F	<i>THEN</i>	G
Rule#3:	<i>IF</i>	A&J	<i>THEN</i>	G
Rule#4:	<i>IF</i>	B	<i>THEN</i>	J
Rule#5:	<i>IF</i>	F	<i>THEN</i>	B
Rule#6:	<i>IF</i>	L	<i>THEN</i>	J
Rule#7:	<i>IF</i>	G	<i>THEN</i>	H

The knowledge-base also contains the following facts

Fact#1:	A.	<i>(‘A’ is known to be true)</i>
Fact#2:	E.	<i>(‘F’ is known to be true)</i>

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Representation: Production Systems



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Representation: Production Systems

An Example Problem: To prove that H is true?
Cycles of Production: FIRST CYCLE

1.	Current goal is <u>H</u>	
2.	Check Database	<u>H</u> is not in the database
3.	Find 'appropriate rule'	Rule 7 has <u>H</u> as an implication
4.	Fire rule 7	To prove that <u>G</u> is true
5.	Set <u>G</u> as the current goal	Store this information in the Working Memory (WM)
6.	Cycle through the KB	

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Representation: Production Systems

An Example Problem: To prove that H is true?
Cycles of Production: SECOND CYCLE

1.	Current goal is <u>G</u>	
2.	Check Database	<u>G</u> is not in the database
3.	Find 'appropriate rule'	Rule 2 has <u>G</u> as an implication
4.	Fire rule 2	To prove that <u>D & F</u> are true
5.	Set <u>D</u> as the current goal	Store <u>D (<-G<-H)</u> in the WM & <u>F</u> is TRUE
6.	Cycle through the KB	

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Representation: Production Systems

Consider a knowledge-base containing the following rules:

Rule#1:	<i>IF</i>	A&B&C	<i>THEN</i>	D
Rule#2:	<i>IF</i>	D&F	<i>THEN</i>	G
Rule#3:	<i>IF</i>	A&J	<i>THEN</i>	G
Rule#4:	<i>IF</i>	B	<i>THEN</i>	J
Rule#5:	<i>IF</i>	F	<i>THEN</i>	B
Rule#6:	<i>IF</i>	L	<i>THEN</i>	J
Rule#7:	<i>IF</i>	G	<i>THEN</i>	H.

The knowledge-base also contains the following facts

Fact#1:	A.	<i>(‘A’ is known to be true)</i>
Fact#2:	E.	<i>(‘F’ is known to be true)</i>

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Representation: Production Systems

An Example Problem: To prove that H is true?
Cycles of Production: **THIRD CYCLE**

1.	Current goal is <u>D</u>	
2.	Check Database	<u>D</u> is not in the database
3.	Find 'appropriate rule'	Rule 1 has <u>G</u> as an implication
4.	Fire rule 1	To prove that <u>A&B&C</u> are true
5.	Set <u>A</u> as the current goal	Store <u>A</u> in the WM & <u>B&C</u> are to be proven later on
6.	Cycle through the KB	

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Representation: Production Systems

An Example Problem: To prove that H is true?
Cycles of Production: FOURTH CYCLE

1.	Current goal is <u>A</u>	
2.	Check Database	<u>A</u> is in the database
3	Set <u>B</u> as the current goal	Store <u>B</u> in the WM
4	Cycle through the KB	

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Representation: Production Systems

Consider a knowledge-base containing the following rules:

Rule#1:	<i>IF</i>	A&B&C	<i>THEN</i>	D
Rule#2:	<i>IF</i>	D&F	<i>THEN</i>	G
Rule#3:	<i>IF</i>	A&J	<i>THEN</i>	G
Rule#4:	<i>IF</i>	B	<i>THEN</i>	J
Rule#5:	<i>IF</i>	F	<i>THEN</i>	B
Rule#6:	<i>IF</i>	L	<i>THEN</i>	J
Rule#7:	<i>IF</i>	G	<i>THEN</i>	H.

The knowledge-base also contains the following facts

Fact#1:	A.	(<i>'A' is known to be true</i>)
Fact#2:	F.	(<i>'F' is known to be true</i>)

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Representation: Production Systems

An Example Problem: To prove that H is true?
Cycles of Production: FIFTH CYCLE

1.	Current goal is <u>B</u>	
2.	Check Database	<u>B</u> is not in the database
3.	Find 'appropriate rule'	Rule 5 has B as an implication
4.	Fire rule 5	To prove that <u>F</u> is true, hence <u>B</u> is true
5.	Set <u>C</u> as the current goal	Store <u>C</u> in the WM
6.	Cycle through the KB	

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Representation: Production Systems

An Example Problem: To prove that H is true?
Cycles of Production: SIXTH CYCLE

1.	Current goal is <u>B</u>	
2.	Check Database	<u>C</u> is not in the database
3.	Fire rule 4	To prove that <u>B</u> is true, hence <u>C</u> is true
4.	Goal D is satisfied	Hence goal <u>G</u> , and therefore goal <u>H</u> is satisfied
5.	STOP	

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Knowledge Representation & Reasoning

A fuzzy knowledge-based system (KBS) is a KBS that performs approximate reasoning. Typically a fuzzy KBS uses knowledge representation and reasoning in systems that are based on the application of Fuzzy Set Theory. A fuzzy knowledge base comprises vague facts and vague rules of the form:

KB Entity	Fuzzy KB	Crisp KB
Fact	X is μ_X	X is TRUE or X is NOT TRUE
Rule	IF X is μ_X THEN Y is μ_Y	IF X THEN Y

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Knowledge Representation

There are two challenges:

- (a) How to interpret and how to represent vague rules with the help of appropriate fuzzy sets?
- (b) How to find an inference mechanism that is founded on well-defined semantics and that permits approximate reasoning by means of a conjunctive general system of vague rules and case-specific vague facts?

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Knowledge Representation

The fuzzy rules, or vague rules, are also called linguistic *rules* comprising an antecedent/premise (the IF part) and a consequent/conclusion (the THEN part).

The antecedent/premise describes an object or event or state in the form of a fuzzy specification of a measured value.

The consequent/conclusion specifies an appropriate fuzzy value.

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Knowledge Representation

Linguistic Variables

Informally, a linguistic variable is a variable whose values are words or sentences in a natural or artificial language.

For example, if **age** is interpreted as a linguistic variable, then its **term-set**, $T(\text{age})$, that is, the set of its linguistic values, might be

$$T(\text{age}) = \text{young} + \text{old} + \text{very young} + \text{not young} + \text{very old} + \text{very very young} + \text{rather young} + \text{more or less young} + \dots$$

where each of the terms in $T(\text{age})$ is a label of a fuzzy subset of a universe of discourse, say $U = [0,100]$.

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Knowledge Representation

Linguistic Variables

A linguistic variable is associated with two rules:

- (a) A **syntactic rule**, which defines the well-formed sentences in $T()$; and
- (b) a **semantic rule**, by which the meaning of the terms in $T()$ may be determined. If X is a term in $T()$, then its meaning (in a denotational sense) is a subset of U . A primary fuzzy set, that is, a term whose meaning must be defined a priori, and serves as a basis for the computation of the meaning of the nonprimary terms in $T()$.

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Knowledge Representation

Linguistic Variables

- In knowledge representation attempts are made to represent complex concepts which are rich in meaning.
- What appears to us as simple concepts, say *height*, *heat* or *colour*, concepts for which we have physical, perceptible, and standardised measurements, for instance *metre*, *degrees Celsius*, *wavelength in Angstroms (A°)*, are given meaning-rich and ambiguous linguistic assignments: For example, for

***height* we have {short, medium, tall};**

***heat* we have { cold, warm, hot};**

and for

***colour* we have {red, blue, green}.**

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Knowledge Representation

Linguistic Variables

- This mapping from the perceptible, measurable and standardisable data to a cognitive, vague, and ambiguous linguistic description involves the mapping of the raw data onto a fuzzy set.
- **The linguistic description –variable- can be modified to make the description more diffuse or highly focussed with the help of the fuzzy modifiers**
- It is the combination of the raw data values with the linguistic feature fuzzy sets, that helps to capture the meaning encapsulated in a given concept through fuzzy logic concepts of *fuzzy sets*, *membership functions*, and *fuzzy logic operations* including conjunction and disjunction, quantification, and negation.

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Knowledge Representation

Linguistic Variables

For example, the primary terms in the equation above are **young** and **old**, whose meaning might be defined by their respective compatibility functions μ_{young} and μ_{old} . From these, then, the meaning - or, equivalently, the compatibility functions - of the non-primary terms in equation (1) above may be computed by the application of a semantic rule.

$$\mu_{\text{very young}} = (\mu_{\text{young}})^2$$

$$\mu_{\text{more or less old}} = (\mu_{\text{old}})^{1/2}$$

$$\mu_{\text{not very young}} = 1 - (\mu_{\text{young}})^2$$

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FUZZY LOGIC & FUZZY SYSTEMS

Knowledge Representation & Reasoning: The Air-conditioner Example

Consider the problem of controlling an air-conditioner (again). The rules that are used to control the air-conditioner can be expressed as a cross product:

$$\underline{CONTROL} = \underline{TEMP} \times \underline{SPEED}$$

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FUZZY LOGIC & FUZZY SYSTEMS

Knowledge Representation & Reasoning: The Air-conditioner Example

The rules can be expressed as a cross product of two term sets:

Temperature and Speed.

$$\underline{CONTROL} = \underline{TEMP} \times \underline{SPEED}$$

Where the set of linguistic values of the term sets is given as

$$\begin{aligned} \underline{TEMP} &= \underline{COLD} + \underline{COOL} + \underline{PLEASANT} + \underline{WARM} + \underline{HOT} \\ \underline{SPEED} &= \underline{MINIMAL} + \underline{SLOW} + \underline{MEDIUM} + \underline{FAST} + \underline{BLAST} \end{aligned}$$

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Knowledge Representation & Reasoning: The Air-conditioner Example

Recall that the rules governing the air-conditioner are as follows:

RULE#1: IF TEMP is COLD THEN SPEED is MINIMAL

RULE#2: IF TEMP is COOL THEN SPEED is SLOW

RULE#3: IF TEMP is PLEASANT THEN SPEED is MEDIUM

RULE#4: IF TEMP is WARM THEN SPEED is FAST

RULE#5: IF TEMP is HOT THEN SPEED is BLAST

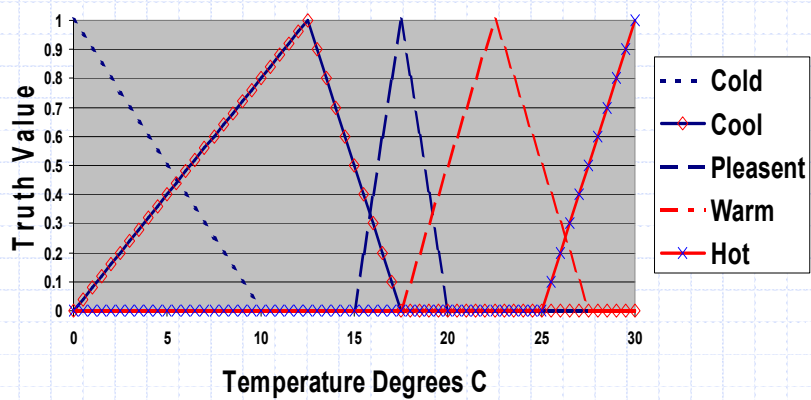
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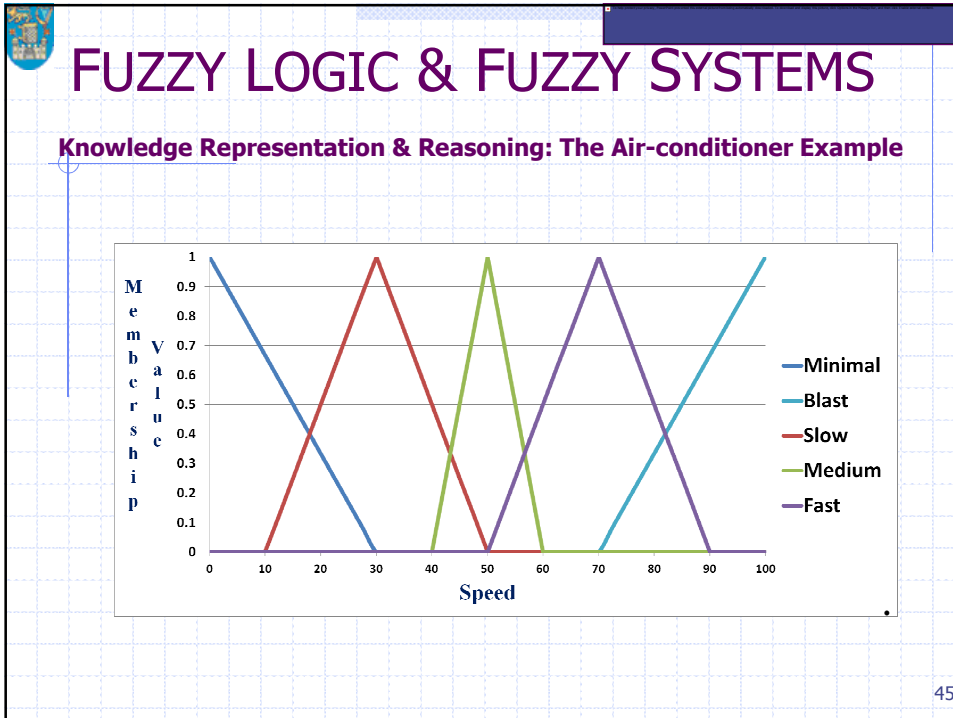
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Knowledge Representation & Reasoning: The Air-conditioner Example

Temperature Fuzzy Sets



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Fuzzy Relationships

EXAMPLE (CONTD.): The analytically expressed membership for the reference fuzzy subsets for speed are:

Term	Membership function	a	b	c
MINIMAL	$\mu_{MINIMAL}(V) = \min\left(\frac{c-V}{a}, 1\right)$	30		1
SLOW	$\mu_{SLOW}(V) = \max\left(\min\left(\frac{V-a}{b-a}, \frac{c-V}{c-b}\right), 0\right)$	10	30	50
MEDIUM	$\mu_{MEDIUM}(V) = \max\left(\min\left(\frac{V-a}{b-a}, \frac{c-V}{c-b}\right), 0\right)$	40	50	60
FAST	$\mu_{FAST}(V) = \max\left(\min\left(\frac{V-a}{b-a}, \frac{c-V}{c-b}\right), 0\right)$	50	70	90
BLAST	$\mu_{BLAST}(V) = \min\left(\frac{V-c}{a}, 1\right)$	30		70

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Fuzzy Relationships

EXAMPLE (CONTD.): The analytically expressed membership for the reference fuzzy subsets for speed are:

Term	Membership function	a	b	c
COLD	$\mu_{COLD}(T) = \min\left(\frac{c-T}{a}, 1\right)$	10		10
COOL	$\mu_{COOL}(V) = \max\left(\min\left(\frac{T-a}{b-a}, \frac{c-T}{c-b}\right), 0\right)$	0	12.5	17.5
PLEASANT	$\mu_{PLEASANT}(V) = \max\left(\min\left(\frac{T-a}{b-a}, \frac{c-T}{c-b}\right), 0\right)$	15	22.5	20
WARM	$\mu_{WARM}(V) = \max\left(\min\left(\frac{T-a}{b-a}, \frac{c-T}{c-b}\right), 0\right)$	17.5	22.5	27.5
HOT	$\mu_{HOT}(T) = \min\left(\frac{T-c}{a}, 1\right)$	5		25

FUZZY LOGIC & FUZZY SYSTEMS

Knowledge Representation & Reasoning: The Air-conditioner Example

FUZZIFICATION: Consider that the temperature is 16°C and we want our knowledge base to compute the speed. The fuzzification of the the crisp temperature gives the following membership for the Temperature fuzzy set:

	μ_{COLD}	μ_{COOL}	$\mu_{PLEASANT}$	μ_{WARM}	μ_{HOT}
Temp=16°C	0	0.3	0.4	0	0
Fire Rule (#)	(#1)	(#2)	(#3)	(#4)	(#5)
yes/no	no	yes	yes	no	no



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Fuzzy Relationships

EXAMPLE (CONTD.): A sample computation of the SLOW membership function as a triangular membership function:

Speed (V)	$\left(\frac{V-a}{b-a}\right)$	$\left(\frac{c-V}{c-b}\right)$	$\mu_{SLOW}(V) = \max\left(\min\left(\frac{V-a}{b-a}, \frac{c-V}{c-b}\right), 0\right)$
10	0	2	0
15	0.25	1.75	0.25
20	0.5	1.5	0.5
25	0.75	1.25	0.75
30	1	1	1
35	1.25	0.75	0.75
40	1.5	0.5	0.5
45	1.75	0.25	0.25
50	2	0	0
55	2.25	-0.25	0

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Knowledge Representation & Reasoning: The Air-conditioner Example

INFERENCE: Consider that the temperature is 16°C and we want our knowledge base to compute the speed. Rule #2 & 3 are firing and are essentially the fuzzy patches made out of the cross products of

COOL x SLOW
PLEASANT x MEDIUM

The COOL and PLEASANT sets have an output of 0.3 and 0.4 respectively. The fuzzy sets for SLOW and MEDIUM have to be given an α -level cut for these output values respectively:

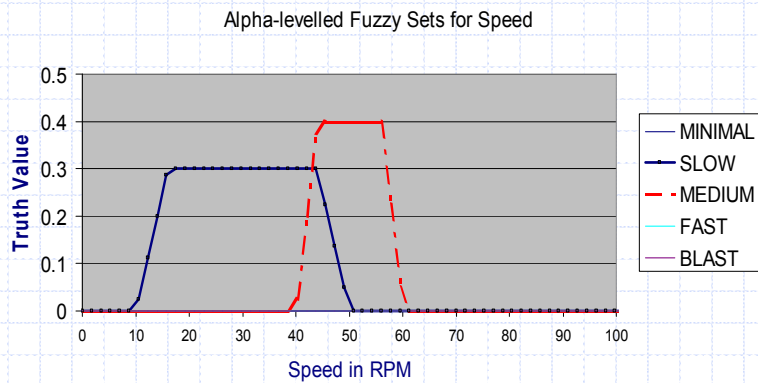
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Knowledge Representation & Reasoning: The Air-conditioner Example

COMPOSITION: The COOL and PLEASANT sets have an output of 0.3 and 0.4 respectively. The fuzzy sets for SLOW and MEDIUM have to be given an alpha-level cut for these output values respectively:



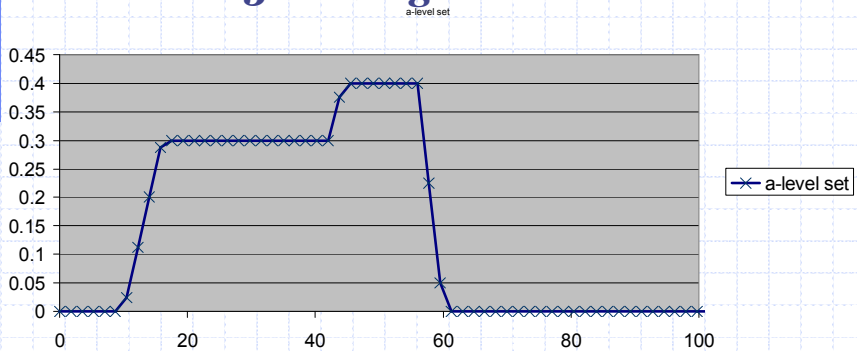
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Knowledge Representation & Reasoning: The Air-conditioner Example

COMPOSITION: The fuzzy output from Rules 2 and 3 can be given as:



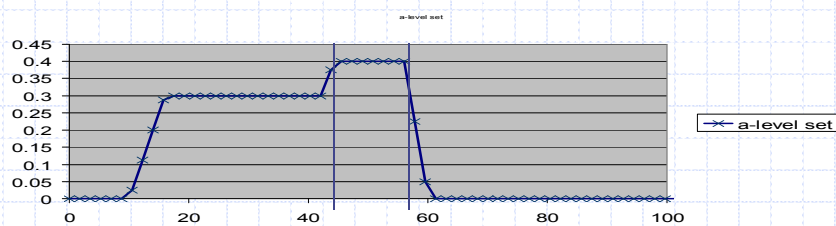
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Knowledge Representation & Reasoning: The Air-conditioner Example

DEFUZZIFICATION: Now we have to find a way to obtain one single number from the curve below – one number corresponding to the speed of the air-conditioner's motor.



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FUZZY LOGIC & FUZZY SYSTEMS

Knowledge Representation & Reasoning: The Air-conditioner Example

DEFUZZIFICATION:
Two popular defuzzification techniques are the
CENTROID
&
MAXIMUM

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FUZZY LOGIC & FUZZY SYSTEMS

Knowledge Representation & Reasoning: The Air-conditioner Example

DEFUZZIFICATION: First we can compute the 'Centre of Gravity (COG) or 'Centre of Area' (COA) of the output of the rules: The COG involves the computation of the weighted sum of the Speed and the corresponding membership function of the output fuzzy set and the weighted sum of the membership function.

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FUZZY LOGIC & FUZZY SYSTEMS

Knowledge Representation & Reasoning: The Air-conditioner Example

DEFUZZIFICATION: The 'Centre of Gravity' (COG) of the output of the rules: Formally, the crisp value is the value located under the centre of gravity of the area that is given by the function

$$\eta = \frac{1}{\int_{y \in Y} \mu_{x_1, \dots, x_n}^{output}(y) dy} \int_{y \in Y} y \mu_{x_1, \dots, x_n}^{output}(y) dy$$

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FUZZY LOGIC & FUZZY SYSTEMS

Knowledge Representation & Reasoning: The Air-conditioner Example

DEFUZZIFICATION: The centre of gravity approach attempts to take the rules into consideration according to their degree of applicability. If a rule dominates during a certain interval then its dominance is discounted in other intervals.

There are problems with the notion of COG & COA.



FUZZY LOGIC & FUZZY SYSTEMS

Knowledge Representation & Reasoning: The Air-conditioner Example

DEFUZZIFICATION: The crisp value h can be obtained by approximating the integral with a sum

$$\eta = \frac{1}{\sum \mu_{x_1, \dots, x_n}^{output}(y)} \sum y \mu_{x_1, \dots, x_n}^{output}(y)$$

The centre of gravity approach attempts to take the rules into consideration according to their degree of applicability. If a rule dominates during a certain interval then its dominance is discounted in other intervals.



FUZZY LOGIC & FUZZY SYSTEMS

Knowledge Representation & Reasoning: The Air-conditioner Example:
DEFUZZIFICATION: CENTRE OF AREA COMPUTATION

DEFUZZIFICATION: The following is a small excerpt of the computation:

SPEED	MINIMAL	SLOW	OUTPUT OF 2 RULES	WEIGHTED SPEED
12.5	0.125	0	0.125	1.5625
15	0.25	0	0.25	3.75
17.5	0.3	0	0.3	5.25
20	0.3	0	0.3	6
22.5	0.3	0	0.3	6.75
25	0.3	0	0.3	7.5
27.5	0.3	0	0.3	8.25
30	0.3	0	0.3	9
32.5	0.3	0	0.3	9.75
35	0.3	0	0.3	10.5
37.5	0.3	0	0.3	11.25
40	0.3	0	0.3	12
42.5	0.3	0.25	0.3	12.75
45	0.25	0.4	0.4	18
47.5	0.125	0.4	0.4	19
50	0	0.4	0.4	20
52.5	0	0.4	0.4	21
55	0	0.4	0.4	22
57.5	0	0.25	0.25	14.375
SUM			5.925	218.6875

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FUZZY LOGIC & FUZZY SYSTEMS

Knowledge Representation & Reasoning: The Air-conditioner Example

DEFUZZIFICATION: The computation leads to a SINGLE VALUE for the speed – an average value computed with respect to the centre of gravity of the output fuzzy set: And the computed speed is 36.91 RPM (=218.6875/5.925).

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FUZZY LOGIC & FUZZY SYSTEMS

Knowledge Representation & Reasoning: The Air-conditioner Example

DEFUZZIFICATION: Another method of defuzzification is that of Mean of Maxima (MOM) Method. Here again the weighted sum and weighted membership are worked out, except that the membership function is given another alpha level cut corresponding to the **maximum value** of the output fuzzy set. The crisp value for MOM method is given as:

$$\eta = \frac{1}{\left| \text{Max}(\mu_{x_1, \dots, x_n}^{\text{output}}) \right|} \sum_{y \in \text{Max}(\mu_{x_1, \dots, x_n}^{\text{output}})} y$$

where $\text{Max}(\mu_{x_1, \dots, x_n}^{\text{output}})$ denotes the set of all outputs y , with

$$\mu_{x_1, \dots, x_n}^{\text{output}}(y) = \text{Max}(\mu_{x_1, \dots, x_n}^{\text{output}})$$

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FUZZY LOGIC & FUZZY SYSTEMS

Knowledge Representation & Reasoning: The Air-conditioner Example

DEFUZZIFICATION: Another method of defuzzification is that of Mean of Maxima Method. Here again the weighted sum and weighted membership are worked out, except that the membership function is given another alpha level cut corresponding to the **maximum value** of the output fuzzy set. This value is 0.4 and the corresponding speed value is 50 RPM (=100/2).

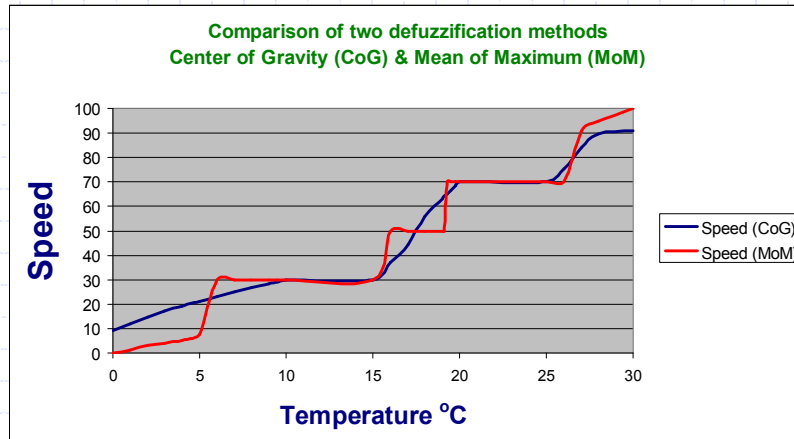
SPEED	MINIMAL	SLOW	OUTPUT OF 2 RULES	WEIGHTED OUTPUT
45	0.25	0.4	0.4	18
47.5	0.125	0.4	0.4	19
50	0	0.4	0.4	20
52.5	0	0.4	0.4	21
55	0	0.4	0.4	22
SUM			2	100

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FUZZY LOGIC & FUZZY SYSTEMS

Knowledge Representation & Reasoning: The Air-conditioner Example

DEFUZZIFICATION: Two popular defuzzification techniques are the CENTROID & MAXIMUM



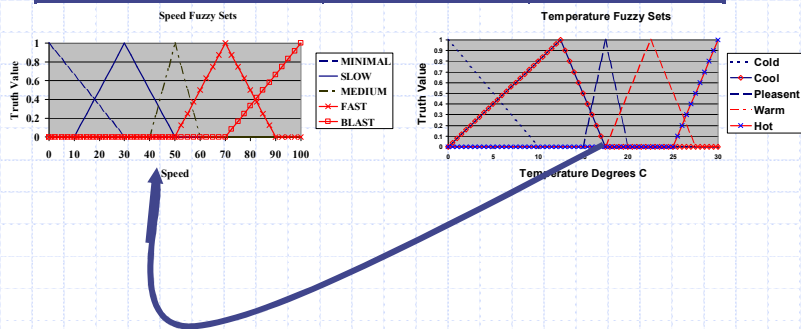
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FUZZY LOGIC & FUZZY SYSTEMS

Knowledge Representation & Reasoning: The Air-conditioner Example

DEFUZZIFICATION: The speed at which the fan will operate will vary due to the method chosen:

Method	Centre of Gravity	Mean of Maxima
Fan Speed for 16 degrees Centigrade	36.91	50



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FUZZY LOGIC & FUZZY SYSTEMS

Knowledge Representation & Reasoning

RECAPITULATE

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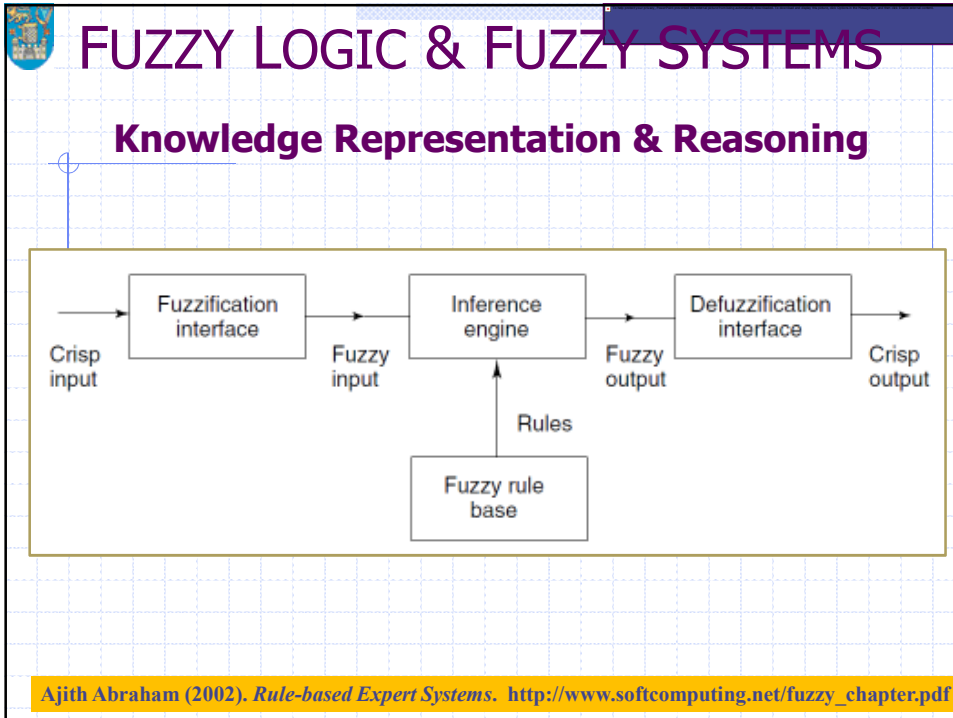
FUZZY LOGIC & FUZZY SYSTEMS

Knowledge Representation & Reasoning

The operation of a fuzzy expert system depends on the execution of FOUR major tasks:

**Fuzzification,
Inference,
Composition,
Defuzzification.**

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FUZZY LOGIC & FUZZY SYSTEMS

Knowledge Representation & Reasoning

***Fuzzification* involves the choice of variables, fuzzy input and output variables and defuzzified output variable(s), definition of membership functions for the input variables and the description of fuzzy rules.**

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FUZZY LOGIC & FUZZY SYSTEMS

Knowledge Representation & Reasoning

Fuzzification :

The membership functions defined on the **input variables** are applied to their actual values to determine the **degree of truth for each rule premise**.

The **degree of truth for a rule's premise** is sometimes referred to as its **α (alpha) value**. If a rule's premise has a non-zero degree of truth, that is if the rule applies at all, then the rule is said to fire.

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FUZZY LOGIC & FUZZY SYSTEMS

Knowledge Representation & Reasoning

Inference: The truth-value for the premise of each rule is computed and the conclusion applied to each part of the rule. This results in one fuzzy subset assigned to each output variable for each rule.

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FUZZY LOGIC & FUZZY SYSTEMS

Knowledge Representation & Reasoning

Inference: MIN and PRODUCT are two inference methods.

1. In MIN inferencing the output membership function is clipped off at a height corresponding to the computed degree of truth of a rule's premise. This corresponds to the traditional interpretation of the fuzzy logic's AND operation.
2. In PRODUCT inferencing the output membership function is scaled by the premise's computed degree of truth.

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FUZZY LOGIC & FUZZY SYSTEMS

Knowledge Representation & Reasoning

Composition: All the fuzzy subsets assigned to each output variable are combined together to form a single fuzzy subset for each output variable.

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FUZZY LOGIC & FUZZY SYSTEMS

Knowledge Representation & Reasoning

Composition: MAX and SUM are two composition rules:

1. In MAX composition, the combined fuzzy subset is constructed by taking the pointwise maximum over all the fuzzy subsets assigned to the output variable by the inference rule.
2. The SUM composition, the combined output fuzzy subset is constructed by taking the pointwise sum over all the fuzzy subsets assigned to output variable by their inference rule. (Note that this can result in truth values greater than 1).

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FUZZY LOGIC & FUZZY SYSTEMS

Knowledge Representation & Reasoning

Defuzzification: The fuzzy value produced at the composition stage needs to be converted to a single number or a *crisp value*.

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FUZZY LOGIC & FUZZY SYSTEMS

Knowledge Representation & Reasoning

Defuzzification: The crisp value is essentially the centre of the area under the curve of the new fuzzy subset derived from the composition stage. Such a computation takes into account the effect of each rule in a proportionate manner. Sometimes, however, it is important to take only into account those rules that have the maximum impact.

Hence there are different methods of *defuzzification*.

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FUZZY LOGIC & FUZZY SYSTEMS

Knowledge Representation & Reasoning

Defuzzification: Two popular defuzzification techniques are the CENTROID and MAXIMUM techniques.

1. The use of CENTROID technique relies on using the centre of gravity of the membership function to calculate the crisp value of the output variable.
2. The MAXIMUM techniques, and there are a number of them, broadly speaking, use one of the variable values at which the fuzzy subset has its maximum truth value to compute the crisp value.

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FUZZY LOGIC & FUZZY SYSTEMS

Knowledge Representation & Reasoning: The Air-conditioner Example

DEFUZZIFICATION: The ‘Centre of Gravity’ (COG) of the output of the rules: Formally, the crisp value is the value located under the centre of gravity of the area that is given by the function

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FUZZY LOGIC & FUZZY SYSTEMS

Knowledge Representation & Reasoning: The Air-conditioner Example

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The centre of gravity approach attempts to take the rules into consideration according to their degree of applicability. If a rule dominates during a certain interval then its dominance is discounted in other intervals.

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FUZZY LOGIC & FUZZY SYSTEMS

Knowledge Representation & Reasoning: The Air-conditioner Example

DEFUZZIFICATION: Another method of defuzzification is that of Mean of Maxima (MOM) Method. Here again the weighted sum and weighted membership are worked out, except that the membership function is given another alpha level cut corresponding to the maximum value of the output fuzzy set. The crisp value for MOM method is given as:

$$\eta = \frac{1}{|Max(\mu_{x1, \dots, xn}^{output})|} \sum_{y=Max(\mu_{x1, \dots, xn}^{output})} y$$

where $Max(\mu_{x1, \dots, xn}^{output})$ denotes the set of all outputs y , with

$$\mu_{x1, \dots, xn}^{output}(y) = Max(\mu_{x1, \dots, xn}^{output})$$



FUZZY LOGIC & FUZZY SYSTEMS

Knowledge Representation & Reasoning

What kind of fuzzy logic we have been discussing?

Mamdani calculus where membership functions of both antecedant and consequent variables are to be considered at the composition stage.

Mamdani calculus involves computation of the consequent fuzzy variables. This is not always possible –for real-time systems for example running at high throughput rates- or not always desirable on the basis of Occam’s logic; things to be kept simple wherever possible. So if you can approximate a function with a single variable then this is better than having a function; when possible the approximation of a constant is better than having a variable.



FUZZY LOGIC & FUZZY SYSTEMS

Knowledge Representation & Reasoning

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FUZZY LOGIC & FUZZY SYSTEMS

A worked example of the use of Mamdani Calculus

The Shamrock Phone Company sells mobile phones using the following rules:

**IF salary is excellent OR debts are small
IF salary is good AND debts are large
IF salary is poor**

**THEN risk is Low
THEN risk is Medium
THEN risk is High**

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FUZZY LOGIC & FUZZY SYSTEMS

A worked example of the use of Mamdani Calculus

The membership functions for the linguistic variables SALARY and DEBTS are given in units of 1,000 Euros (€), or 1K€, and the RISK in percentage terms; in the following (values of) membership functions relating to salaries and debts, the kilo symbol 'K' has been omitted.



FUZZY LOGIC & FUZZY SYSTEMS

A worked example of the use of Mamdani Calculus

All membership functions are piecewise linear with

$$\mu_{Salary}^{Excellent}(x) = 0, \forall x \leq 90; \quad \mu_{Salary}^{Excellent}(x) = 1, x \geq 120;$$

$$\mu_{Salary}^{Good}(x) = 0, \forall x \leq 50 \text{ \& } \forall x \geq 100; \quad \mu_{Salary}^{Good}(x) = 1, x = 75;$$

$$\mu_{Salary}^{Poor}(x) = 0, \forall x \geq 60; \quad \mu_{Salary}^{Poor}(x) = 1, x \leq 10.$$

$$\mu_{Debt}^{Small}(x) = 0, \forall x \geq 50; \quad \mu_{Debt}^{Small}(x) = 1, x \leq 10;$$

$$\mu_{Debt}^{Large}(x) = 0, \forall x \leq 15; \quad \mu_{Debt}^{Large}(x) = 1, x \geq 60.$$

$$\mu_{Risk}^{Low}(x) = 0, \forall x \geq 40\%; \quad \mu_{Risk}^{Low}(x) = 1, x \leq 20\%;$$

$$\mu_{Risk}^{Medium}(x) = 0, \forall x \leq 20\% \text{ \& } \forall x \geq 80\%; \quad \mu_{Risk}^{Medium}(x) = 1, 40\% \leq x \leq 60\%;$$

$$\mu_{Risk}^{High}(x) = 0, \forall x \leq 60\%; \quad \mu_{Risk}^{High}(x) = 1, x \geq 80\%.$$



FUZZY LOGIC & FUZZY SYSTEMS

A worked example of the use of Mamdani Calculus

Jim has applied for a mobile telephone: his salary is €140K and his debts amount to €60K. Use the above rule base to compute the risk associated with Jim using the *mean of maxima* in the Defuzzification task.

Let us look at each of the four tasks involved in finding the level of risk associated with Jim.

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FUZZY LOGIC & FUZZY SYSTEMS

A worked example of the use of Mamdani Calculus

FUZZIFICATION:

Salary: 140K Euros

$$\mu_{\text{Excellent}}(140) = 1$$

$$\mu_{\text{Good}}(140) = 0$$

$$\mu_{\text{Poor}}(140) = 0$$

Debts: 60k Euros

$$\mu_{\text{Small}}(60) = 0$$

$$\mu_{\text{Large}}(60) = 1$$

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FUZZY LOGIC & FUZZY SYSTEMS

A worked example of the use of Mamdani Calculus

INFERENCE: Risk Evaluation

$$Rule\ 1 : \mu_{Salary}^{Excellent}(x) \text{ OR } \mu_{Debts}^{Small} = \max \left[\mu_{Salary}^{Excellent}, \mu_{Debts}^{Small} \right]$$

$$\mu_{Risk}^{Low} = \max[1, 0] = 1$$

$$Rule\ 2 : \mu_{Salary}^{Good}(x) \text{ AND } \mu_{Debts}^{Large}(x) = \min \left[\mu_{Salary}^{Good}, \mu_{Debts}^{Large} \right]$$

$$\mu_{Risk}^{Medium} = \min[0, 1] = 0$$

$$Rule\ 3 : \mu_{Salary}^{Poor} = 0 \qquad \mu_{Risk}^{High} = 0$$

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FUZZY LOGIC & FUZZY SYSTEMS

A worked example of the use of Mamdani Calculus

AGGREGATION & COMPOSITION

Only one rule fires and aggregation/composition is unnecessary

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FUZZY LOGIC & FUZZY SYSTEMS

A worked example of the use of Mamdani Calculus

DEFUZZIFICATION:

In Mean of Maxima (MOM) Method the weighted sum and weighted membership are worked out, except that the membership function is given by another alpha level cut corresponding to the maximum value of the output fuzzy set. The crisp value for MOM method is given as:

$$\eta = \frac{\sum_{\mu_{Risk}(x) \equiv \text{Max}(\mu_{Risk})} \mu_{Risk}(x) * x}{\sum_{\mu_{Risk}(x) \equiv \text{Max}(\mu_{Risk})} \mu_{Risk}(x)}$$

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FUZZY LOGIC & FUZZY SYSTEMS

A worked example of the use of Mamdani Calculus

DEFUZZIFICATION:

In Mean of Maxima (MOM) Method the weighted sum and weighted membership are worked out, except that the membership function is given another alpha level cut corresponding to the maximum value of the output fuzzy set. The crisp value for MOM method is given as:

Rule 1: $\mu=1$
Rule 2: $\mu=0$
Rule 3: $\mu=0$

$$\eta = \frac{(0 + 10 + 20) * 1}{3 * 1} \%$$
$$\eta = \frac{30}{3} = 10\%$$

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FUZZY LOGIC & FUZZY SYSTEMS

A worked example of the use of Mamdani Calculus

DEFUZZIFICATION:

The more widely used centre of area (or gravity) method requires a weighted average over all the values of the variables x, rather than just those where the membership function is the highest:

$$COG = \frac{\sum_{x=0\%}^{100\%} \mu_{Risk}(x)x}{\sum_{x=0\%}^{100\%} \mu_{Risk}}$$

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FUZZY LOGIC & FUZZY SYSTEMS

A worked example of the use of Mamdani Calculus

DEFUZZIFICATION:

COG: The more widely used centre of area (or gravity) method requires a weighted average over all the values of the variables x, rather than just those where the membership function is the highest:

$$\frac{(0 + 10 + 20) \times 1 + 0.5 * 30}{(1 + 1 + 1 + 0.5)} \% = \frac{45}{3.5} = 12.857\%$$

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FUZZY LOGIC & FUZZY SYSTEMS

A worked example of the use of Mamdani Calculus

DEFUZZIFICATION:

MOM:	10%
COG:	12.857%

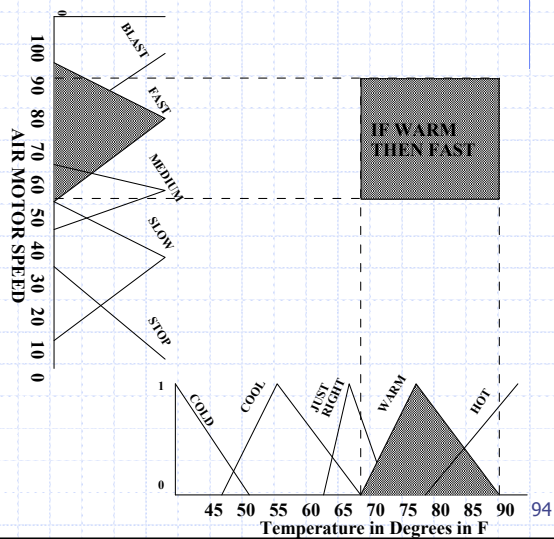
COG usually gives higher results than MOM.

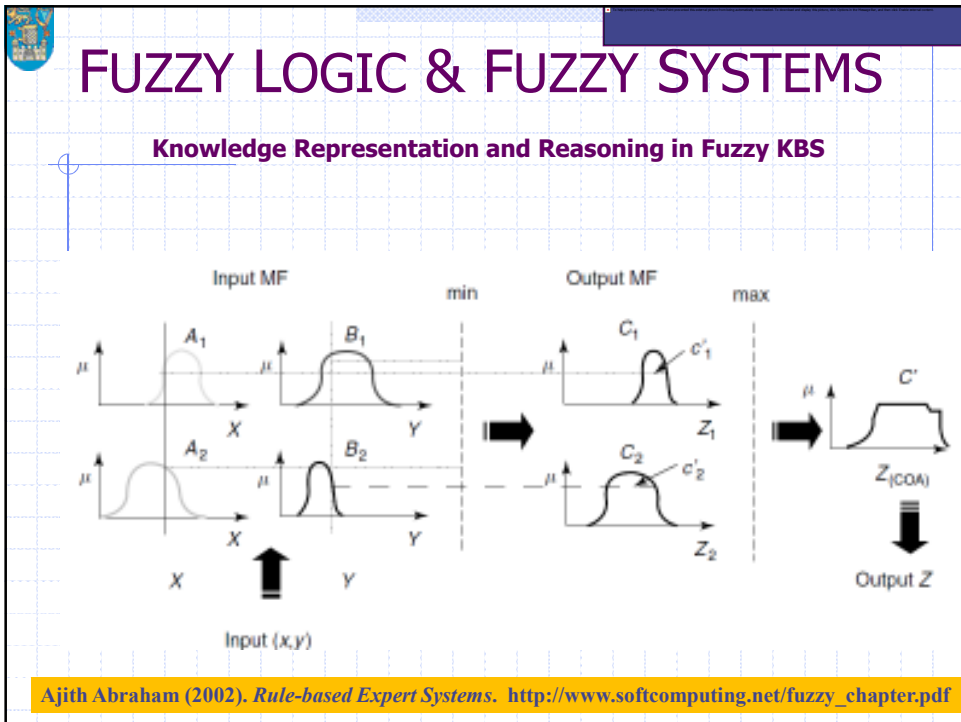
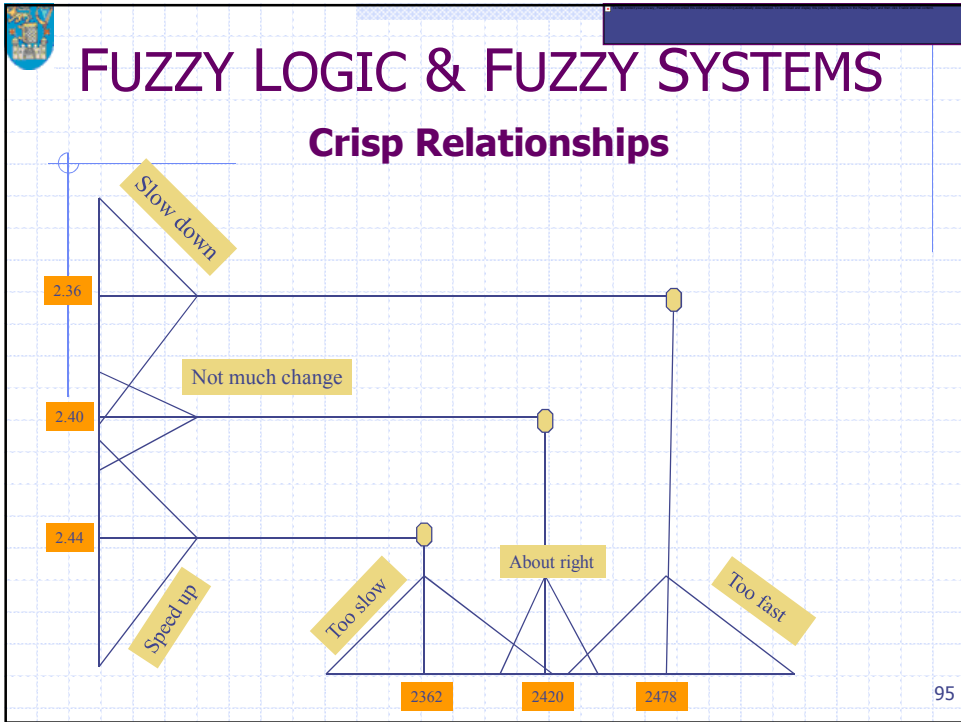


FUZZY LOGIC & FUZZY SYSTEMS

Fuzzy Relationships

A fuzzy patch is defined by a fuzzy rule: a patch is a mapping of two membership functions, it is a product of two geometrical objects, line segments, triangles, squares etc.







FUZZY LOGIC & FUZZY SYSTEMS

Knowledge Representation & Reasoning: Umbilical Cord Blood

Analysis of Umbilical Cord Blood: ‘An assessment of neonatal outcome may be obtained from analysis of blood in the umbilical cord [UC] of an infant immediately after delivery [..]. The umbilical cord vein carries blood from the placenta to the fetus and the two smaller cord arteries return blood from the fetus. The blood from the placenta has been freshly oxygenated, and has a relatively high partial pressure of oxygen (pO_2) and low partial pressure of carbon dioxide (pCO_2). Oxygen in the blood fuels *aerobic* cell metabolism, with carbon dioxide produced as ‘waste’.

Garibaldi JM, Ifeachor EC. The development of a fuzzy expert system for the analysis of umbilical cord blood. In: Szczepaniak PS, Lisboa PJG, Kacprzyk J, editors. Chapter in fuzzy systems in medicine. Berlin: Springer, Physica-Verlag, 2000. p. 652:668. <http://www.cs.mffl.ac.uk/~jmg/papers/fsm-00.pdf>

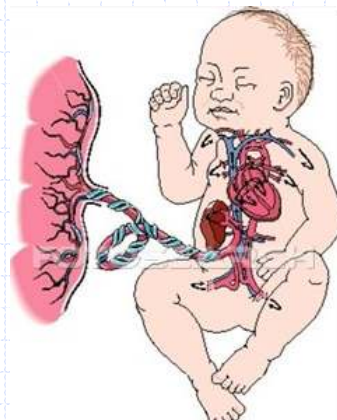
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FUZZY LOGIC & FUZZY SYSTEMS

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sa201028 www.fotosearch.com

<http://www.fotosearch.com/LF113/sa201028/>

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FUZZY LOGIC & FUZZY SYSTEMS

Knowledge Representation & Reasoning: Umbilical Cord Blood

Analysis of Umbilical Cord Blood: The blood returning from the fetus has relatively low oxygen and high carbon dioxide content. Some carbon dioxide dissociates to form carbonic acid in the blood, which increases the acidity (lowers the pH). If oxygen supplies are too low, *anaerobic* (without oxygen) metabolism can supplement aerobic metabolism to maintain essential cell function, but this produces lactic acid as 'waste'. This further acidifies the blood, and can indicate serious problems for the fetus.'

Garibaldi JM, Ifeachor EC. The development of a fuzzy expert system for the analysis of umbilical cord blood. In: Szczepaniak PS, Lisboa PJG, Kacprzyk J, editors. Chapter in fuzzy systems in medicine. Berlin: Springer, Physica-Verlag, 2000. p. 652:668. <http://www.cs.mcgill.ca/~jmg/papers/fsm-00.pdf>

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FUZZY LOGIC & FUZZY SYSTEMS

Knowledge Representation & Reasoning: Umbilical Cord Blood

Blood samples are taken from blood vessels in the (clamped) umbilical cord. The chemical composition of blood is examined and machines are used to measure (a) the acidity/basicity of the blood (*pH*) and (b) the dissolved oxygen (*pO₂*) and carbon dioxide (*pCO₂*). The measurements are used to compute the so-called *base deficit of extracellular fluid (BDecf)*. *BDecf* can help us to find whether a low pH is caused **either** by *Respiratory acidosis* → short term accumulation of *CO₂* **or** by *Metabolic acidosis* → lactic acid from a longer-term oxygen deficiency.

Garibaldi JM, Ifeachor EC. The development of a fuzzy expert system for the analysis of umbilical cord blood. In: Szczepaniak PS, Lisboa PJG, Kacprzyk J, editors. Chapter in fuzzy systems in medicine. Berlin: Springer, Physica-Verlag, 2000. p. 652:668. <http://www.cs.mcgill.ca/~jmg/papers/fsm-00.pdf>

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FUZZY LOGIC & FUZZY SYSTEMS

Knowledge Representation & Reasoning: Umbilical Cord Blood

Carbon dioxide is the result of energy production and cell component production → Metabolism

Condition	Increase in the blood of	Decrease in the blood of
Hypocapnia		Carbon dioxide
Hypercapnia	Carbon dioxide	
Acidemia	Hydrogen Ion	
Alkalemia		Hydrogen Ion

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FUZZY LOGIC & FUZZY SYSTEMS

Knowledge Representation & Reasoning: Umbilical Cord Blood

Ranges for acid-base parameters for two clinics (Plymouth and Exeter)

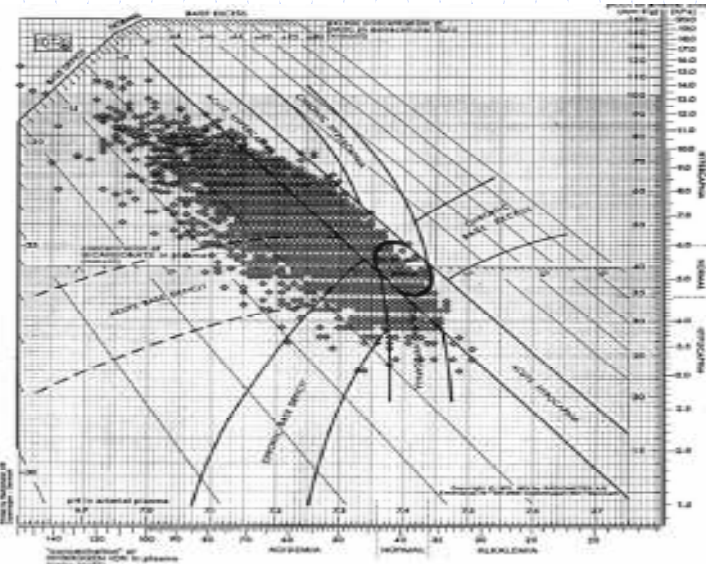
Plymouth (n=2684)								
Vessel	pH		pCO ₂		pO ₂		Bdecf	
	Min	Max	Min	Max	Min	Max	Min	Max
Artery	7.06	7.40	4.5	9.7	0.8	3.7	-0.9	11.0
Vein	7.16	7.49	3.3	7.6	1.9	5.3	-0.5	9.4
Exeter (n=1222)								
Vessel	pH		pCO ₂		pO ₂		Bdecf	
	Min	Max	Min	Max	Min	Max	Min	Max
Artery	7.03	7.36	4.6	10.3	0.9	4.3	-0.4	12.2
Vein	7.15	7.46	3.4	7.9	2.1	6.1	-0.6	10.3

Garibaldi J.M., Westgate, J.A., and Heachor E.C. (1999). The evaluation of an expert system for the analysis of umbilical cord blood. *Artificial Intelligence in Medicine*. Vol. 17, pp 109-130

FUZZY LOGIC & FUZZY SYSTEMS

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FUZZY LOGIC & FUZZY SYSTEMS

Knowledge Representation & Reasoning: Umbilical Cord Blood

Problems with umbilical acid-base analysis:

- (1) The samples can result in two samples from the same vessel or mixed samples, whilst blood in the syringe can alter due to exposure to air.
- (2) Blood gas analysis machines require regular calibration and quality control checks to ensure continuing performance to the manufacturer's specifications
- (3) Careful retrospective analysis of the acid-base results obtained during a trial on electronic fetal monitoring [...] highlighted a 25% failure rate to obtain arterial and venous paired samples with all parameters [...]
- (4) **Considerable expertise was required to reliably recognize these errors and accurately interpret the results.**

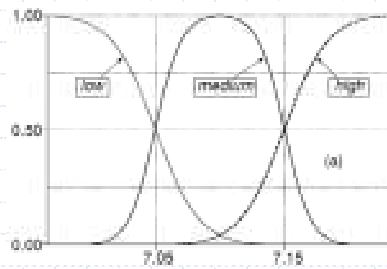
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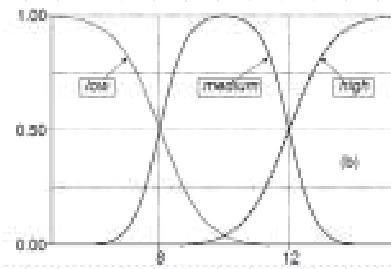


FUZZY LOGIC & FUZZY SYSTEMS

Knowledge Representation & Reasoning: Umbilical Cord Blood



Term set for arterial pH



Term set for arterial BD_{cef}

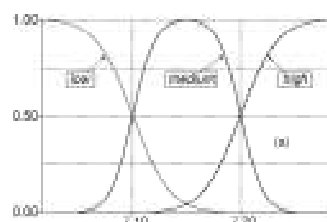
Garibaldi JM, Ifeachor EC. The development of a fuzzy expert system for the analysis of umbilical cord blood. In: Szczepaniak PS, Lisboa PJG, Kacprzyk J, editors. Chapter in fuzzy systems in medicine. Berlin: Springer, Physica-Verlag, 2000. p. 652:668. http://www.cs.moff.ac.uk/~jmg/papers/fsm_00.pdf

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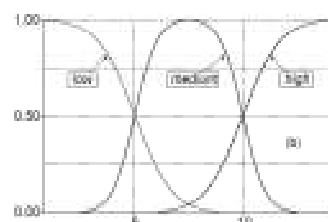


FUZZY LOGIC & FUZZY SYSTEMS

Knowledge Representation & Reasoning: Umbilical Cord Blood



Term set for venous pH



Term set for venous BD_{cef}

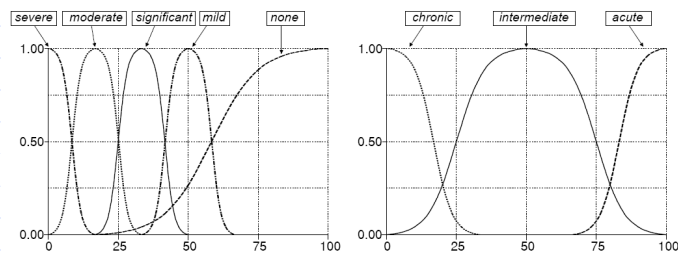
Garibaldi JM, Ifeachor EC. The development of a fuzzy expert system for the analysis of umbilical cord blood. In: Szczepaniak PS, Lisboa PJG, Kacprzyk J, editors. Chapter in fuzzy systems in medicine. Berlin: Springer, Physica-Verlag, 2000. p. 652:668. http://www.cs.moff.ac.uk/~jmg/papers/fsm_00.pdf

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Term set for venous *acidemia*

Term set for venous *duration*

Garibaldi JM, Ifeachor EC. The development of a fuzzy expert system for the analysis of umbilical cord blood. In: Szczepaniak PS, Lisboa PJG, Kacprzyk J, editors. Chapter in fuzzy systems in medicine. Berlin: Springer, Physica-Verlag, 2000. p. 652:668. <http://www.cs.nott.ac.uk/~img/papers/fsm-00.pdf>

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FUZZY LOGIC & FUZZY SYSTEMS

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IF (venous – arterial pH) ≥ 0.06 AND (arterial – venous $p\text{CO}_2$) < 4 mmHg
THEN mark the arterial and venous $p\text{CO}_2$ parameters as errors

If the pH and $p\text{CO}_2$ values for a sample are accepted as valid, the base deficit of the extracellular fluid (BDefc) is calculated by equations from Siggaard-Andersen [19]:

$$\text{BDefc} = - (1 - 0.023\text{Hb})(\text{HCO}_3^- - 24.1 + (2.30\text{Hb} + 7.7)(\text{pH} - 7.40))$$

where $\text{HCO}_3^- = 0.23p\text{CO}_2 \log^{-1}$ and $\text{Hb} = 3.7$ mmol/l

Garibaldi J.M., Westgate, J. A., Ifeachor E. C., and Greene, K.R. (1997). The development and implementation of an expert system for the analysis of umbilical cord blood. *Artificial Intelligence in Medicine*. Vol 10, pp 129-144. <http://www.cs.nott.ac.uk/~img/papers/fsm-00.pdf>

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FUZZY LOGIC & FUZZY SYSTEMS

Knowledge Representation & Reasoning: Umbilical Cord Blood

The different combinations of realistic values of pH, pCO₂ and BDecf results in 54 different interpretations varying from 'normal' to 'severe metabolic acidemia'. The diagnosticians can separate 'respiratory acidosis' from 'metabolic acidosis'.

The differences between blood taken from veins and arteries, if available, is used to confirm the results thus obtained.

Garibaldi J.M., Westgate, J. A., Ifeachor E. C., and Greene, K.R. (1997). The development and implementation of an expert system for the analysis of umbilical cord blood. *Artificial Intelligence in Medicine*. Vol 10, pp 129-144 (<http://www.cs.nott.ac.uk/~img/papers/ism-97.pdf>)

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FUZZY LOGIC & FUZZY SYSTEMS

Output from an expert system: Umbilical Cord Blood

Patient Data
 Surname: [Smith] First Name: [Andrew] ID: [00000000] Multiple:
 Artery: 00355 Vein: 00356
 pH: 7.35 pH: 7.38
 pCO₂ (mmHg): 38 pCO₂ (mmHg): 39
 pO₂ (mmHg): 99 pO₂ (mmHg): 98
 HCO₃ (ACT): 22.8 HCO₃ (ACT): 23.1
 BD (FCF): 12.8 BD (FCF): 10.0
 Date: 27/01/1994 Date: 27/01/1994
 Time: 16:52 Time: 16:52
 Interpretation: [1]
 Moderate acidemia.
 Arterial and venous metabolic acidemia.
 Possibility of neonatal hypoglycemia and perhaps cardiac pulmonary sequelae.

Garibaldi J.M., Westgate, J. A., Ifeachor E. C., and Greene, K.R. (1997). The development and implementation of an expert system for the analysis of umbilical cord blood. *Artificial Intelligence in Medicine*. Vol 10, pp 129-144 (<http://www.cs.nott.ac.uk/~img/papers/ism-97.pdf>)

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