BA Mod. (Computer Science) Trinity Term 2008
SS Examination

# Solutions to
# 4BA13 Fuzzy Logic and Fuzzy Control Systems

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Q1. **This question is about *fuzzy sets*.**

Q1 a. **Can you use a *crisp set* to describe the contents of Bertie's *mammals*? Give at least ONE reason to support your answer.**

Answer 1a: Crisp sets are useful only when we have mutually exclusive sets where membership of objects under consideration is restricted to one single set. The categorization of monotermes and sea mammals, and that of non-flighted birds, will benefit when we have elastic membership – an object may weakly belong to one or more sets.

[15 Marks]

Q1 b. **What will be advantage of using fuzzy sub-sets to describe Bertie's mammals and Bertie's birds?**

Answer 1b. Advantage of fuzzy sub-sets is that membership can be described in whole range between 0 and 1 (not only by 0 and 1). Thus we can set that monotermes are mammals to 0.5 membership degree and whales and dolphins are mammals to 0.1 membership degree.

[5 Marks]

Q1 c. **Using the convention of listing the members of a fuzzy sub-set, i.e the *belongingness* value and the name of the member of the sub-set, describe Bertie's mammals and birds as two sub-sets $A^{mammals}$ and $A^{birds}$.**

Answer 1c.

$A^{mammals}$ ={0.5/platypus, 0.5/ echidnas, 1/horses, 1/zebras, 1/tigers, 0.1/whales, 0.1/dolphins, 0/sparrows, 0/robins, 0/crows, 0/penguins and 0/kiwis}

$A^{birds}$ ={0/platypus, 0/ echidnas, 0/horses, 0/zebras, 0/tigers, 0/whales, 0/dolphins, 1/sparrows, 1/robins, 1/crows, 0.5/penguins and 0.5/kiwis}

[21 Marks]

Q1 d. **Construct a subset $E^{mammals}$ which is a *fuzzy intersection* of $A^{mammals}$ and its complement $\tilde{A}^{mammals}$.**

Answer 1d

$$A^{mammals} \cap \tilde{A}^{mammals} = E^{mammals}$$

A′ mammals: $\mu_{A'mammals}(x) = 1 - \mu_{A\,mammals}(x)$

A′ mammals ={0.5/platypus, 0.5/ echidnas, 0/horses, 0/zebras, 0/tigers, 0.9/whales, 0.9/dolphins, 1/sparrows, 1/robins, 1/crows, 1/penguins and 1/kiwis}

$$A^{mammals} \cap \tilde{A}^{mammals} : \mu_{A\,mammals \cap \tilde{A}\,mammals}(x) = \min(\mu_{A\,mammals}, \mu_{\tilde{A}\,mammals}).$$

$E^{mammals}$ ={0.5/platypus, 0.5/ echidnas, 0/horses, 0/zebras, 0/tigers, 0.1/whales, 0.1/dolphins, 0/sparrows, 0/robins, 0/crows, 0/penguins and 0/kiwis}

[21 Marks]

Q1 e. **To what extent, the subset $A^{mammals}$ obeys the well known laws of crisp set theory: the Law of the Excluded Middle and the Law of Contradiction?**

Answer 1e: Law of Contradiction states that

$$A \cap \tilde{A} = \varnothing$$

But $E^{mammals} \neq \varnothing$ and thus $A^{mammals}$ violates the Law of Contradiction for crisp sets.

Law of excluded middle sates that

$$A \cup \tilde{A} = X$$

The union

$$A^{mammals} \cup \widetilde{A}^{mammals} : \mu_{A\,mammals \cup \widetilde{A}\,mammals}(x) = \max(\mu_{A\,mammals}, \mu_{\widetilde{A}\,mammals}).$$

We have

$A \cup \widetilde{A} =$ {0.5/platypus, 0.5/ echidnas, 1/horses, 1/zebras, 1/tigers, 0.9/whales, 0.9/dolphins, 1/sparrows, 1/robins, 1/crows, 1/penguins and 1/kiwis}.

Thus in this case the Law of excluded middle is not satisfied.

[28 Marks]

## Q2 **This question is about aggregation operations on fuzzy sets**.

**Q2 a. (i) Describe the conditions of *commutativity*, *associativity*, *monotonicity*, and '*respective identities*' that are used to characterize the t-norm operator (*T*) and the t-conorm operator (*S*).**

Answer 2a (i)

Commutativity: O(x,y) = O(y,x) for all x,y $\in$ [0,1].

➔ Implies that the order in which the inputs are processed is not important.

Associativity: O(x,O(y,z)) = O(O(x,y),z) for all x,y,z $\in$ [0,1].

➔ Implies that when three inputs are processed the result is given uniquely.

Monotonicity: O(x,y) ≤ O(x,z) iff y ≤ z for all x,y,z $\in$ [0,1].

➔ Implies that the bigger inputs will produce bigger outputs.

Identity: For T we have T(1,x) = x.

➔ Implies that (together with commutativity) that 1 is a neutral element.

For S we have S(0,x) = x.

➔ Implies that (together with commutativity) that 0 is a neutral element.

[24 Marks]

**Q2a (ii) How will you distinguish between the operators *T* and *S*?**

Answer 2a (ii)

(ii) The difference between T and S is in the neutral element, which is 1 for T (T(1,x)=x) and 0 for S (S(0,x)=x). Operators T – t-norms are used to model conjunction and operators S t-conorms are used to model disjunction.

[12 Marks]

Q2 b. **Using some of the four conditions listed above, show how the fuzzy intersection operator *T* reduces to the crisp intersection on {0,1}.**

Identity implies T(1,0) = 0 and T(1,1) =1.
Commutativity implies T(0,1) = T(1,0) =0.
Monotonicity implies 0 ≤ T(0,0) ≤ T(1,0) = 0 and
thus T(0,0)=0.

[21 Marks]

Q2 c. **Some of the commonly encountered *t-norm* and *t-conorm duals* in fuzzy modelling and control literature are:**

| ***t-norm*** | ***t-conorm*** | ***Name*** |
|---|---|---|
| **Min ( *a* , *b*)** | **Max (*a* , *b*)** | **Min/Max** |
| ***ab*** | ***a + b - ab*** | **Product/Probablisitc Sum** |
| **Max ( 0, *a* + *b* - 1)** | **Min (1, *a* + *b*)** | **Bold Union/Bounded Sum** |

**Give reasons as to why *Min* and *Max* are most often used as 'a choice for implementation of *AND* (intersection) [ *OR* (union)] operators' respectively?**

Answer 2c

We have $T(x,y) \le \min(x,y)$ for all operators T and all $x,y \in [0,1]$. This is the reason why the Min is the strongest AND operator. Similarly, $\max(x,y) \le S(x,y)$ for all operators S and all $x,y \in [0,1]$. This is the reason why the Max is the weakest OR operator. The aggregation of more inputs by T usually yields output closer to 0. The use of Min as an AND operator avoids getting closer to zero when more inputs are aggregated.

Similarly the use of Max prevents the aggregation or more inputs to tend to 1.

[21 Marks]

Q2 d. **There are 6 numbers to be aggregated by a quasi-arithmetic mean $M_f$ generated by $f(x)=x^2$ on [0,1]. However, assume that you are not aware of the first three numbers, but only know the mean of the three:**

**$M_f$(x1,x2,x3)=0.4.**

**Compute the quasi-arithmetic mean $M_f$ of all 6 numbers, where the last three numbers are**
***0.1*, *0.5*, and *0.8*.**

Answer 2d: The property of decomposability ensures that the answer is as follows:

$$\text{Mf}(0.4,0.4,0.4,0.1,0.5,0.8) = \sqrt{\frac{0.4^2 + 0.4^2 + 0.4^2 + 0.1^2 + 0.5^2 + 0.8^2}{6}}$$

$$= \sqrt{\frac{0.16 + 0.16 + 0.16 + 0.01 + 0.25 + 0.64}{6}}$$

$$= \sqrt{\frac{1.38}{6}} = \sqrt{0.23} = 0.4796$$

[22 Marks]

## Q3. **This question is about fuzzy rule based systems.**

**You have been asked to design an environmental-protection information system for protecting the city of Lilliput especially against the oil-spillage and contamination of the water systems that form the aquatic environment of the city.**

**<u>Location of Containment</u>: The aquatic environment for Lilliput where oil spillage can take place comprises: the open waters of the Degul Sea (DS) together with its 3-mile ($z_3$) and 12-mile ($z_{12}$) zones regarded as Lilliput aquatic territory under international law. Furthermore, oil spills can also take place in the various canal systems (LCS) in Liiliput, in the Lilliput harbour (LH) and the refuelling (RD) and loading docks (LD) in the harbour.**

**<u>Agencies Involved</u>: The protection of the environment involves multi-agency overlapping jurisdictions. The Environmental Protection Agency (EPA) has the responsibility for the open sea and the 12 mile zone. The Port of Lilliput Authority (PLA) has the sole responsibility for the Harbour and the docks; neither the EPA nor the Lilliput Police (LP) can enforce environmental protection laws in the Harbour and the docks. The Lilliput City Council (LCC) is responsible for the canals. The Fire Brigades Authority (FBA) is obliged to put out fires and remove chemical hazards, including oil spills, but has no equipment to work on the open seas.**

**<u>Remedial Actions</u>: The environmental protection laws have the following civil and criminal remedies for the plaintiffs, usually one of the five authorities mentioned above:**
**(i) the polluter can be issued with a warning by the EPA;**
**(ii) the polluter pays for the clean-up operation as levied by the FBA, the Port of Lilliput Authority, and the Lilliput Police;**
**(iii) the polluter is fined by the FBA;**
**(iv) the polluter can be given a prison sentence if and only if the Lilliput Police registers a case against the polluter and successfully prosecutes the polluter.**

**Q3 a. The three *domains* that will be used in modelling the aquatic environment protection system are: *location-of-containment* ($\Omega_{Loc}$), *agencies-involved* ($\Omega_{Age}$), and *remedial actions* ($\Omega_{Rem}$).**
**Describe the members of the term-sets related to each of the three domains based on the description of the aquatic environment protection system for oil spills.**

Answer 3a:

Location ={DS,z3,z12,LCS,LH,RD,LD}

Agencies={EPA,PLA,LP,LCC,FBA}

Remedial actions={warning, pay for clean-up operation, fine, imprisonment}

[9 marks]

**Q3 b. The universe of discourse for the oil-spill related environmental protection system is**

$$\Omega = \Omega Loc \times \Omega Age \times \Omega Rem.$$

**Describe how many states or elementary events that are permitted a priori.**

Answer 3b:

The number of *a priori* possible states are

card(Location)*card(Agencies)*card(Remedial actions), where card(L) refers to the number of the term-sets related to domain L. Thus there are 7*5*4=140 possible states.

[15 marks]

**Q3 c. (i) Extract at least 5 rules from the description of the system comprising two domains at a time.**

Answer 3c (i) Rules on $\Omega_{Loc}$ x $\Omega_{Age}$:

**If** Loc. is DS **Then** Age. is EPA
**If** Loc. is z12 **Then** Age. is EPA
**If** Loc. is LH **Then** Age. is PLA
**If** Loc. is RD **Then** Age. is PLA
**If** Loc. is LD **Then** Age. is PLA
**If** Loc. is LCS **Then** Age. is LCC
**If** Loc. is LCS **Then** Age. is FBA
**If** Loc. is LH **Then** Age. is FBA
**If** Loc. is RD **Then** Age. is FBA
**If** Loc. is LD **Then** Age. is FBA
**If** Loc. is LCS **Then** Age. is LP
**If** Loc. is DS **Then** Age. is LP
**If** Loc. is z3 **Then** Age. is LP
**If** Loc. is z12 **Then** Age. is LP

Rules on $\Omega_{Age}$ x $\Omega_{Rem}$:

**If** Age. is EPA **Then** Rem. is warning
**If** Age. is FBA **Then** Rem. is pay for clean-up operation
**If** Age. is PLA **Then** Rem. is pay for clean-up operation
**If** Age. is LP **Then** Rem. is pay for clean-up operation
**If** Age. is FBA **Then** Rem. is fine
**If** Age. is LP **Then** Rem. is prison
**If** Age. is LCC **Then** Rem. is fine

[16 marks]

**Q3 c. (ii) Following on from the description and your derived rule set show a tabular relationship between**

$$\Omega_{Loc} \text{ x } \Omega_{Age}$$

**and a tabular relationship between**

$$\Omega_{Age} \text{ x } \Omega_{Rem}$$

Answer 3c (ii) Given the description given in the preamble to Question 2, we have $\Omega_{Loc}$ x $\Omega_{Age}$ in tabular form

| AGE\Loc | DS | $Z_3$ | $Z_{12}$ | LCS | LH | RD | LD |
|---|---|---|---|---|---|---|---|
| EPA | Y | N | Y | N | N | N | N |
| PLA | N | N | N | N | Y | Y | Y |
| LP | Y | Y | Y | Y | N | N | N |
| LCC | N | N | N | Y | N | N | N |
| FBA | N | N | N | Y | Y | Y | Y |

In tabular form $\Omega_{Age}$ x $\Omega_{Rem}$

| Rem\Age | EPA | PLA | LP | LCC | FBA |
|---|---|---|---|---|---|
| Warning | Y | N | N | N | N |
| Pay | N | Y | Y | N | Y |
| Fine | N | N | N | Y | Y |
| Prison | N | N | Y | N | N |

where Y means that the two linguistic variables have a relationship and N means that there is no relationship.

[30 marks]

**Q3 d. Using your rule base and the relationships above, what inference can you draw from the rather sketchy reports of two incidents:**

**(i) There was an oil slick that originated in one of the canals or at one of the refuelling docks in the Lilliput harbour. The Lilliput Police was not involved in the case.**

Answer 3d: (i) Here Loc = {LCS,RD} thus from the first table we see that if LP is not involved we have Age ={LCC,FBA,PLA}. From second table we see that Rem={Pay,Fine}. This means that contamination happened in the canal system or in the refuelling docks. The Lilliput City Council or Fire Brigades Authority or Port Lilliput Authority can be involved, and the remedial action will be paying for clean-up operation and/or imposing a fine.

[15 marks]

**(ii) A Borbdignag oil tanker was found listing and lost its cargo of diesel oil somewhere between the open seas and the 12-mile zone, and was towed into Lilliput Port by the Lilliput Port Authority.**

Answer 3d (ii) Here Loc = {DS,z12}. From first table we see that Age = {EPA,LP}. Now from second table we see that Rem={warning, pay, prison}. This means that contamination happened on the open sea or in the 12-mile zone. The Environmental Protection Agency or Lilliput Police will be involved, and the remedial action will comprise a criminal penalty or issuing with warning or in paying for clean-up operation.

[15 marks]

# Q4. **This question is about Takagi-Sugeno-Kang controllers**

**(i) The number of vehicles per hour was computed as an aggregate of a variety of vehicles (cars, motor bikes, and heavy goods vehicles) per hour:**

$$n_{eq} = n_{cars} + 3*n_{motor\_bikes} + 6*n_{heavy\_goods\_vehicles}$$

**(ii) The so-called *equivalent continuous noise level* (*$L_{AeqT}$*) is defined as a function of the number of vehicles per hour (*$n_{eq}$*), the average height *$\hat{h}$*, of the buildings along a road that has an average width *$\hat{w}$*:**

$$L_{AeqT} = f(n_{eq}, \hat{h}, \hat{w}).$$

**Fichera et al defined that the term set *$n_{eq}$* as comprising the linguistic variables *small* and *large*. They assume that the number of equivalent vehicles is definitely *small* if the vehicle count is less than or equal to 923 and the *$n_{eq}$* is definitely not *small* if the number is over 10489. Contrarily perhaps *$n_{eq}$* is definitely *large* if the number of equivalent vehicles is greater than or equal to 8944; if the vehicle count falls below 924 then the number of equivalent vehicles is definitely not *large*.**

**The height of the building term set *h* comprises *tall* and *low* buildings. A building that is below 12.7 metres is definitely a *low* building and it is definitely not *low* if the height is greater than 31.88 meters. Contrariwise, a *tall* building should measure more than 12.44 meters and a building with a height greater than or equal to 34.49 meters is definitely a *tall* building.**

**The fuzzy rule set for computing the noise level is given as:**

$$R_1 : IF\ n_{eq} is\ small\ \&\ h\ \ is\ low\ THEN\ L_{eq} = -148.6 - 0.087 n_{eq} + 24.38h + 0.24w$$
$$R_2 : IF\ n_{eq} is\ small\ \&\ h\ \ is\ tall\ THEN\ L_{eq} = -894.2 + 0.087 n_{eq} + 26.53h - 0.09w$$
$$R_3 : IF\ n_{eq} is\ large\ \&\ h\ \ is\ low\ THEN\ L_{eq} = 1180 - 0.071 n_{eq} - 26.99h - 0.61w$$
$$R_4 : IF\ n_{eq} is\ large\ \&\ h\ \ is\ tall\ THEN\ L_{eq} = 413.99 + 0.072 n_{eq} - 30.97h - 1.99w$$

Q4 a. Compute the membership functions for the linguistic variables *small, large, tall* and *low.*

Answer 4a The membership functions are as follows:

***small***:

$$\mu(x) = 1\ \ if\ n_{eq} \le 923$$
$$\mu(x) = 0\ \ if\ n_{eq} \ge 10489$$
$$\mu(x) = \frac{10489 - x}{9566}\ \ if\ 923 \le n_{eq} \le 10489$$

***large:***

$$\mu(x) = 1 \;\; if \; n_{eq} \geq 8944$$
$$\mu(x) = 0 \;\; if \; n_{eq} \leq 924$$
$$\mu(x) = \frac{x - 924}{8020} \;\; if \; 924 \leq n_{eq} \leq 8944$$

***tall***

$$\mu(x) = 1 \;\; if \; h \geq 34.49$$
$$\mu(x) = 0 \;\; if \; h \leq 12.44$$
$$\mu(x) = \frac{x - 12.44}{22.05} \;\; if \; 12.44 \leq h \leq 34.49$$

***low***

$$\mu(x) = 1 \;\; if \; h \leq 12.7$$
$$\mu(x) = 0 \;\; if \; h \geq 31.88$$
$$\mu(x) = \frac{31.88 - x}{19.18} \;\; if \; 12.7 \leq h \leq 31.88$$

[25 Marks]

**Q4 b. Using a Takagi-Sugeno-Kang fuzzy control system, compute the average equivalent noise level $L_{AeqT}$ based on equivalent noise levels $L_{eq}$ from the following observations:**

**The average number of vehicles observed in one hour was 5000 on a road with an average width of 30 metres and buildings along the way have an average height of 15 metres.**

**Clearly show all the stages of computation: fuzzification, composition and inference for these observations.**

[60 marks]

Answer 4b .Fuzzification: Neq =5000; h=15

| ***Term*** | **Linguistic Variable** | **Membership Value** |
|---|---|---|
| **Neq** | **Small (Neq)** | **0.57** |
| | **Large (Neq)** | **0.51** |
| **h** | **Tall (h)** | **0.12** |
| | **Low (h)** | **0.88** |

Inference + Composition

Neq=5000; h=15.0 M, w=30.0M

| | Leq | Union Neq & *h* | Weight | Product |
|---|---|---|---|---|
| **Rule 1** | **-210.7** | **Small +Low** | **0.57** | **-120.9** |
| **Rule 2** | **-63.95** | **Small+Tall** | **0.12** | **-7.425** |
| **Rule 3** | **401.85** | **Large+Low** | **0.51** | **204.23** |
| **Rule 4** | **249.74** | **Large+Tall** | **0.12** | **28.995** |
| **SUM** | | | **1.32** | **104.90** |

**OUTPUT:**

**LAeqt = 104.90/1.32= 79.82**

**Q4 c. Could you have carried out this task using Mamdani's formulation of a fuzzy control system?**

This situation can be modelled by Mamdani model as well provided we could define the fuzzy sets for $L_{eq}$ together with fuzzy sets for the width of the road (w). A new rule set has to be derived as well.

[15 marks]

Q5. **This question is about neuro-fuzzy systems**

Q5 a. **Consider the following perceptron learning algorithm that can be used to train the perceptron to learn the behaviour of two logic gates: *AND* & *OR* gates.**

| STEP | Description | Equations |
|---|---|---|
| 1 | Set initial weights $w_1$ and $w_2$ and the bias θ to set of random numbers | |
| 2 | Compute the weighted sum (Σ) | $\Sigma = x_1*w_1+x_2*w_2+\theta$ |
| 3 | Calculate the output using a δ-function | $y(i)=\delta(x_1*w_1+x_2*w_2+\theta)$; $\delta(x)=1$, if $x > 0$; $\delta(x)=0$, if $x \le 0$ |
| 4 | Compute the error (e) which is the difference between the actual output (y)and desired output ($y_{desired}$) | $e(i)= y_{desired} - y(i)$ |
| 5 (a) | IF the errors during a training epoch are all zero THEN STOP | |
| 5 (b) | IF the errors during a training epoch are not all zero THEN UPDATE weights | $w_j(i+1)=w_j(i)+\alpha*x_j*e(i)$, j=1,2; i:=i+1; |
| 5 (c ) | GO TO STEP 2 | |

**Consider the OR-gate with two inputs and one output:**

| X1 | X2 | Y |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 1 | 1 |

**You are expected to train a perceptron, with two inputs and one output, to learn to behave like an *OR* gate. Assume that the learning constant $\alpha$=0.2, bias θ=-0.3 and input weights are $w_1$=0.3 and $w_2$=-0.1. Train the perceptron for at least 4 epochs. Tabulate the inputs and outputs (actual and desired) together with weight changes.**

Answer 5a The training shows convergence in Epoch 4.

Epoch 1

| X1 | X2 | d | w1 | w2 | sum | out | err | w1 | w2 |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0.3 | -0.1 | -0.3 | 0 | 0 | 0.3 | -0.1 |
| 1 | 0 | 1 | 0.3 | -0.1 | 0 | 0 | 1 | 0.5 | -0.1 |
| 0 | 1 | 1 | 0.5 | -0.1 | -0.4 | 0 | 1 | 0.5 | 0.1 |
| 1 | 1 | 1 | 0.5 | 0.1 | 0.3 | 1 | 0 | 0.5 | 0.1 |

Epoch 2

| X1 | X2 | d | w1 | w2 | sum | out | err | w1 | w2 |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0.5 | 0.1 | -0.3 | 0 | 0 | 0.5 | 0.1 |
| 1 | 0 | 1 | 0.5 | 0.1 | 0.2 | 1 | 0 | 0.5 | 0.1 |
| 0 | 1 | 1 | 0.5 | 0.1 | -0.2 | 0 | 1 | 0.5 | 0.3 |
| 1 | 1 | 1 | 0.5 | 0.3 | 0.5 | 1 | 0 | 0.5 | 0.3 |

Epoch 3

| X1 | X2 | d | w1 | w2 | sum | out | err | w1 | w2 |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0.5 | 0.3 | -0.3 | 0 | 0 | 0.5 | 0.3 |
| 1 | 0 | 1 | 0.5 | 0.3 | 0.2 | 1 | 0 | 0.5 | 0.3 |
| 0 | 1 | 1 | 0.5 | 0.3 | 0 | 0 | 1 | 0.5 | 0.5 |
| 1 | 1 | 1 | 0.5 | 0.5 | 0.7 | 1 | 0 | 0.5 | 0.5 |

Epoch 4

| X1 | X2 | d | w1 | w2 | sum | out | err | w1 | w2 |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0.5 | 0.5 | -0.3 | 0 | 0 | 0.5 | 0.5 |
| 1 | 0 | 1 | 0.5 | 0.5 | 0.2 | 1 | 0 | 0.5 | 0.5 |
| 0 | 1 | 1 | 0.5 | 0.5 | 0.2 | 1 | 0 | 0.5 | 0.5 |
| 1 | 1 | 1 | 0.5 | 0.5 | 0.7 | 1 | 0 | 0.5 | 0.5 |

[60 Marks]

**Q5 b. Consider the XOR gate:**

| X1 | X2 | Y |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 1 | 0 |

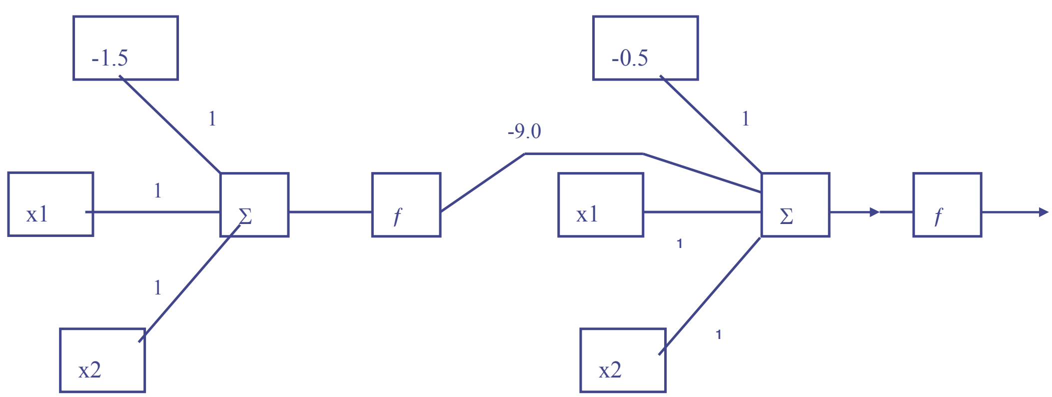
**Can you use the perceptron learning algorithm for learning the behaviour of *XOR* gate? Give your reasons as to whether you can or cannot?**

Answer 5b. XOR gate cannot be modelled by simple perceptron since it doesn't belong to the set of linearly separable problems. Single perceptron can represent a line in an input space where all inputs below this line have one output value and all inputs above this line have another value. Since input space of XOR problem cannot be divided by line to get all inputs related to 0 outputs on the one side and all inputs related to 1 outputs to the other side of the line, single perceptron can never model a XOR gate.

[25 Marks]

**Q5 c. Describe the architecture of an adaptive neuro-fuzzy system that can learn the behaviour described in the above rule base.**

Answer 5c. In order to model XOR gate we need to design a neuro-fuzzy system with one hidden layer. In this system, first layer would comprise inputs x1 and x2 and a bias. Two input nodes together with bias would all be connected to one neuron in the hidden layer. Another neurons in the hidden layer would be the bias and original inputs. All neurons from the hidden layer are connected to the one neuron in the output layer.




[15 Marks]