MODEL ANSWERS for Fuzzy logic and systems CS 2011-2012

Q1. This question is about fuzzy sets and fuzzy taxonomy.

Phylogeny of mammals is the study of the connections between all groups of mammals as understood by ancestor/descendant relationships. Genome sequence data bases comprising data about primates, ranging from humans to lemurs, are being used to understand relationships between different primates at the molecular level. The table below shows three *clades*, a clade is group consisting of a species (extinct or extant) and all its descendants, used by the researchers to describe the relationships between the primates. *Strepsirrhini*, or 'turning nose primates', *Platyrrhini*, or 'New World Monkeys', *Catarrhini*, or 'drooping nose primates'.

According to evolutionary scientists species evolve and each subsequent generation has different properties from its predecessors – there is an intergenerational 'distance' of sorts. For each descendant the table has a number in parentheses which shows the evolutionary stage of the given descendant. For example, *galago monkeys* are the least evolved primates, as indicated by the number '2'; and *humans, chimps* and *gorillas* are among the most evolved of the primates which is indicated by the number '5'

Q1 a. Design fuzzy subsets for (i) *Strepsirrhini*, (ii) *Platyrrhini*, (iii) *Catarrhini* and (iv) *Not Orangutan*. You may use the numerical information about the evolutionary stage to compute the belongingness.

[40 Marks]

Answer 1 a (Continues on the next sheet)

Primata (0)										
Strepsirrhini (1)→ 'turning nose primates'		Platyrrhini (1) → 'New World Monkeys'			Catarrhini (1) → 'drooping nose primates'					
	Lemur (2)		Not Dusky titi (2)		Cercopithecidae (2) → 'Old World Monkeys'			Non- Cercopithecidae	Not Gibbon (3)	
				Not owl monkey (3)		Not Colobus (3)				Not Orangutan (4)
							Not Verveet (4)			
Galago (2)	Mouse lemur (3); Right tailed lemur (3)	Dusky titi (2)	Owl monkey (3)	Squirrel monkey (4); Marmoset (4)	Colobus Monkey (3)	Verveet (4)	Baboon (5); Macaque (5)	Gibbon (3)	Orangutan (4)	Humans (5); chimps (5); gorilla (5)

Intergenerational distance is 1/sqrt (descendent postion −predecessor's position)

$$Clade^{\text{Strepsirrhini}} = \{1.0 \text{ alago, } 0.71 \text{ Mouse lemur, } 0.7 \text{ Right Tailed Lemur}\}$$

$$Clade^{\text{Platyrrhini}} = \{1.0 \text{ Dusky titi, } 0.7 \text{ Owl monkey, } 0.71 \text{ Right Tailed Lemur, } 0.58 \text{ Squirrel Monkey, } 0.58 \text{ Marmoset}\}$$

$$Clade^{\text{Catarrhini}} = \{1.0 \text{ Cercopithecidae, } 0.71 \text{ Gibbon, } 0.58 \text{ Orangutan, } 0.5 \text{ Humans, } 0.5 \text{ Chimpanzee, } 0.5 \text{ Gorilla}\}$$

$$Clade^{\text{Not Orangutan}} = \{1.0 \text{ Humans, } 1.0 \text{ Chimapnzee, } 1.0 \text{ Gorilla}\}$$

(10 Marks for each clade).

Q1 b. Compute the cardinality of each of the fuzzy subsets above.

[40 Marks]

Clade	Cardinality (15 Marks)	Core (10 Marks)	Support (15 Marks
Strepsirrhini	2.142	Galago	Galago, Mouse lemur, Right Tailed Lemur
Platyrrhini	3.32	Dusky titi	Dusky titi, squirrel monkey, owl monkey, marmoset
Catarrhini	3.78	Cercopithecidae ={Colobus Monkey}	Colobus Monkey,Verveet, Baboon, Macaque
Not Orangutan	3	Humans, Chimpanzees, Gorillas	Humans, Chimpanzees, Gorillas

Q1 c. Describe in your own words the advantages of using fuzzy sets for describing an evolving taxonomy.

[20 Marks]

The student is expected to note that the evolutionary *classification* is a good example of fuzzy taxonomy where belongingness of an organism to a *species*, or indeed to *zoological family* or *kingdom*, cannot be defined exactly or *crisply*. Fuzzy descriptions handle uncertainty, in biology due to the evolution of a species for example.

Q2. **This question is about fuzzy rule based systems.**

You have been asked to design an environmental-protection information system for protecting the city of Lilliput especially against oil-spillage and the contamination of the water systems that form the aquatic environment of the city.

Location of Containment: The aquatic environment for Lilliput where oil spillage can take place comprises: the open waters of the Degul Sea (DS) together with its 3-mile (z_3) and 12-mile (z_{12}) zones regarded as Lilliput aquatic territory under international law. Furthermore, oil spills can also take place in the various canal systems in Lilliput (LCS), in the Lilliput harbour (LH), and in the refuelling (RD) and loading docks (LD) in the harbour.

Agencies Involved: The protection of the environment involves multi-agency overlapping jurisdictions. The Environmental Protection Agency (EPA) has the responsibility for the open sea and the 12 mile zone. The Port of Lilliput Authority (PLA) has the sole responsibility for the Harbour and the docks; neither the EPA nor the Lilliput Police (LP) can enforce environmental protection laws in the Harbour and the docks. The Lilliput City Council (LCC) is responsible for the canals. The Fire Brigades Authority (FBA) is obliged to put out fires and remove chemical hazards, including oil spills, but has no equipment to work on the open seas.

Remedial Actions: The environmental protection laws have the following civil and criminal remedies for the plaintiffs, usually one of the five authorities mentioned above:

  (i)   the polluter can be issued with a warning by the EPA;

  (ii) the polluter pays for the clean-up operation as levied by the FBA, the Port of Lilliput Authority, and the Lilliput Police;

  (iii) the polluter is fined by the FBA;

(iv) the polluter can be given a prison sentence if and only if the Lilliput Police registers a case against the polluter and successfully prosecutes the polluter.

Q2 a. The three *domains* that will be used in modelling the aquatic environment protection system are: *location-of-containment (Ω_{Loc})*, *agencies-involved (Ω_{Age}),* and *remedial actions (Ω_{Rem})*.

Describe the members of the term-sets related to each of the three domains based on the description of the aquatic environment protection system for oil spills.

[20 marks]

Answer 2a

Location ={DS,z3,z12,LCS,LH,RD,LD}

Agencies={EPA,PLA,LP,LCC,FBA}

Remedial actions={warning, pay for clean-up operation, fine, imprisonment}

Q2 b. The universe of discourse for the oil-spill related environmental protection system is

$$\Omega=\Omega_{Loc} \times \Omega_{Age} \times \Omega_{Rem.}$$

Describe how many states or elementary events that are permitted *a priori.*

[20 marks]

Answer 2b

The number of *a priori* possible states are

card(Location)*card(Agencies)*card(Remedial actions),

where card(L) refers to the number of the term-sets related to domain L. Thus there are 7*5*4=140 possible states.

Q2 c.

(i) Extract at least **5** rules from the description of the system comprising

two domains at a time.

[20 marks]

Answer 2 c(i):

(i) Rules on Ω_{Loc} x Ω_{Age}:

If Loc. is DS **Then** Age. is EPA

If Loc. is z12 **Then** Age. is EPA

If Loc. is LH **Then** Age. is PLA

If Loc. is RD **Then** Age. is PLA

If Loc. is LD **Then** Age. is PLA

If Loc. is LCS **Then** Age. is LCC

If Loc. is LCS **Then** Age. is FBA

If Loc. is LH **Then** Age. is FBA

If Loc. is RD **Then** Age. is FBA

If Loc. is LD **Then** Age. is FBA

If Loc. is LCS **Then** Age. is LP

If Loc. is DS **Then** Age. is LP

If Loc. is z3 **Then** Age. is LP

If Loc. is z12 **Then** Age. is LP

Rules on Ω_{Age} x Ω_{Rem}:

If Age. is EPA **Then** Rem. is warning

If Age. is FBA **Then** Rem. is pay for clean-up operation

If Age. is PLA **Then** Rem. is pay for clean-up operation

If Age. is LP **Then** Rem. is pay for clean-up operation

If Age. is FBA **Then** Rem. is fine

If Age. is LP **Then** Rem. is prison

If Age. is LCC **Then** Rem. is fine

Q2 (c)   (ii) Following on from the description and your derived rule set show a tabular relationship between

$$\Omega_{Loc} \times \Omega_{Age}$$

and a tabular relationship between

$$\Omega_{Age} \times \Omega_{Rem}$$

[20 marks]

Answer 2 c(ii):
We have $\Omega Loc \times \Omega Age$ in tabular form

AGE\Loc	DS	Z3	Z12	LCS	LH	RD	LD
EPA	Y	N	Y	N	N	N	N
PLA	N	N	N	N	Y	Y	Y
LP	Y	Y	Y	Y	N	N	N
LCC	N	N	N	Y	N	N	N
FBA	N	N	N	Y	Y	Y	Y

In tabular form $\Omega Age \times \Omega Rem$

Rem\Age	EPA	PLA	LP	LCC	FBA
Warning	Y	N	N	N	N
Pay	N	Y	Y	N	Y
Fine	N	N	N	Y	Y
Prison	N	N	Y	N	N

where Y means that the two linguistic variables have a relationship and N means that there is no relationship.

Q2 d. Using your rule base and the relationships above, what inference can you draw from the rather sketchy report of the following incident: *There was an oil slick that originated in one of the canals or at one of the refuelling docks in Lilliput harbour. The Lilliput Police were not involved in the case.*

[20 marks]

Answer 2 (d)

Here Loc = {LCS,RD} thus from the first table we see that if LP is not involved we have Age ={LCC,FBA,PLA}. From second table we see that Rem={Pay,Fine}.

This means that contamination happened in the canal system or in the refuelling docks. The Lilliput City Council or Fire Brigades Authority or Port Lilliput Authority can be involved, and the remedial action will be paying for clean-up operation and/or imposing a fine.

Q3. **This question is about fuzzy control systems.**

Q3 a. Describe the key difference between a Mamdani controller and a Takagi-Sugeno-Kang Controller.

[20 Marks]

Answer 3 (a). The key difference is that the consequent of rules in a TKS system is a linear function of some or all the variables used in the antecedents the rules; the dependence on each of the variables is weighted. The weights in linear dependence can be derived, or data-mined, from the input-output data of the control system. The computation is considerably simplified for two reasons: (i) the TSK system designer is not required to find the membership functions of the term set for each linguistic variable; (ii) the computation time is considerably reduced because the composition and de-fuzzication stage are merged into one.

The Mamdani controllers are, in one sense, more sophisticated than the TSK controllers in that the consequents are not derived from a linear regression. However, the Mamdani controller is less parsimonious both in terms system design (Item i above) and in terms of computational efficiency (Item 2).

Q3 b. Consider a rule base for a 2-Input-1-Output Mamdani controller: The input variables, $\lambda 1$ and $\lambda 2$ and η is the output variable. The variables belong to the universes of discourse X1, X2 and Y respectively, such that $\lambda 1 \in X1$ = [-1,1], $\lambda 2 \in X2$ = [-1,1] and $\eta \in Y$ = [-1,1]. The rule base for the controller is given as:

	λ1		
λ2	**Neg**	**Zero**	**Pos**
Neg	Neg		Zero
Zero		Zero	
Pos	Zero		Pos

The fuzzy partitions of the sets X1, X2 and Y are given as

$$\chi_{Neg}(x) = \begin{cases} 0 & if\ x \geq 0, \\ 1 & if\ x = -1, \\ \alpha_1 + \beta_1\ x & otherwise. \end{cases}$$

$$\chi_{Pos}(x) = \begin{cases} 0 & if\ x \leq 0, \\ 1 & if\ x = 1 \\ \alpha_2 + \beta_2\ x & otherwise. \end{cases}$$

$$\chi_{zero}(x) = \max\left(\min(-(x+1), 1-x), 0\right)$$

Compute the control values η for the following values of

$$(\lambda_1, \lambda_2) \in \{(0,0), (1,1), (1,0), (-1,0), (0.25,0.5), (-0.25,0.5), (0.75,0.75)\}$$

Show all the four stages of the computation: fuzzification, inference, composition and defuzzification.

[80 Marks]

Answer 3 b

λ$_1$				
λ$_2$		Neg	Zero	Pos
	Neg	Neg		Zero
	Zero		Zero	
	Pos	Zero		Pos

The rules are:

1. If λ1 is Neg and λ2 is Neg then η is Neg
2. If λ1 is Neg and λ2 is Pos then η is Zero
3. If λ1 is Zero and λ2 is Zero then η is Zero
4. If λ1 is Pos and λ2 is Neg then η is Zero
5. If λ1 is Pos and λ2 is Pos then η is Pos

The student is expected to ,produce the following table:

Input	Fuzzification	Inference	Composition	Defuzzification
		Rules Fired		Weighted η
{0,0}	λ1 is Zero and λ2 is Zero	3	η is Zero	1
{1,1}	If λ1 is Pos and λ2 is Pos	5	η is Pos	1

Each input-output computation carries 10 marks.

Q4. **This question is about Takagi-Sugeno Kang fuzzy control systems**

Noise pollution in major conurbations leads to a general degradation of quality of life. Fichera and colleagues have simulated noise pollution in a small town in Italy using fuzzy rule based systems. They have found that the noise level can be predicted using:

(i) The average number of vehicles per hour was computed as an aggregate of a variety of vehicles (cars, motor bikes, and heavy goods vehicles) per hour:

$$n_{eq} = n_{cars} + 3 * n_{motor\_bikes} + 6 * n_{heavy\_goods\_vehicles}$$

(ii) The so-called *equivalent continuous noise level (LAeqT)* is defined as a function of the number of vehicles per hour (*neq*), the average height $\hat{h}$, of the buildings along a road that has an average width $\hat{w}$:

$$L_{AeqT} = f(n_{eq}, \hat{h}, \hat{w}).$$

Fichera et al defined that the term set n_{eq} as comprising the linguistic variables *small* and *large*. They assume that the number of equivalent vehicles is definitely *small* if the vehicle count is less than or equal to 923, and n_{eq} is definitely not *small* if the number is over 10489. Contrarily, n_{eq} is definitely *large* if the number of equivalent vehicles is greater than or equal to 8944; if the vehicle count falls below 924 then the number of equivalent vehicles is definitely not *large*.

The height of the building term set, *h,* comprises *tall* and *low* buildings. A building that is below 12.7 metres is definitely a *low* building and it is definitely not *low* if the height is greater than 31.88 meters. Contrariwise, a *tall* building should measure more than 12.44 meters and a building with a height greater than or equal to 34.49 meters is definitely a *tall* building. The fuzzy rule set for computing the noise level is given as:

$$R_1 : IF\ n_{eq}\ is\ small\ \&\ h\ is\ low\ THEN\ L_{eq} = -148.6 - 0.087n_{eq} + 24.38h + 0.24w$$

$$R_2 : IF\ n_{eq}\ is\ small\ \&\ h\ is\ tall\ THEN\ L_{eq} = -894.2 + 0.087n_{eq} + 26.53h - 0.09w$$

$$R_3 : IF\ n_{eq}\ is\ l\arg e\ \&\ h\ is\ low\ THEN\ L_{eq} = 1180 - 0.071n_{eq} - 26.99h - 0.61w$$

$$R_4 : IF\ n_{eq}\ is\ l\arg e\ \&\ h\ is\ tall\ THEN\ L_{eq} = 413.99 + 0.072n_{eq} - 30.97h - 1.99w$$

Q4 a.   Derive the membership functions for the linguistic variables *small, large, tall* and *low* from the description given above*.*

[20 Marks]

Answer 4a The membership functions are as follows:

small:

$$\mu(x)=1 \;\; if \; n_{eq} \leq 923$$
$$\mu(x)=0 \;\; if \; n_{eq} \geq 10489$$
$$\mu(x)=\frac{10489-x}{9566} \;\; if \; 923 \leq n_{eq} \leq 10489$$

large:

$$\mu(x)=1 \;\; if \; n_{eq} \geq 8944$$
$$\mu(x)=0 \;\; if \; n_{eq} \leq 924$$
$$\mu(x)=\frac{x-924}{8020} \;\; if \; 924 \leq n_{eq} \leq 8944$$

tall

$$\mu(x)=1 \;\; if \; h \geq 34.49$$
$$\mu(x)=0 \;\; if \; h \leq 12.44$$
$$\mu(x)=\frac{x-12.44}{22.05} \;\; if \; 12.44 \leq h \leq 34.49$$

low

$$\mu(x)=1 \;\; if \; h \leq 12.7$$
$$\mu(x)=0 \;\; if \; h \geq 31.88$$
$$\mu(x)=\frac{31.88-x}{19.18} \;\; if \; 12.7 \leq h \leq 31.88$$

Q4 b. Using a Takagi-Sugeno-Kang fuzzy control system, compute the average equivalent noise level L_{AeqT} based on equivalent noise levels L_{eq} from the following observations:

The average number of vehicles observed in one hour was 5000 on a road with an average width of 30 metres and buildings along the way have an average height of 15 metres.

Clearly show all the stages of computation: fuzzification, composition and inference for these observations.

[80 marks]

Answer 4b .Fuzzification: n_{eq} =5000, h=15, w=30

Term	Linguistic Variable	Membership Value
n_{eq}	Small (n_{eq})	0.57
	Large (n_{eq})	0.51
h	Tall (h)	0.12
	Low (h)	0.88

Inference + Composition

n_{eq} =5000; h=15, w=30

	L_{eq}	Union n_{eq} & h	Weight	Product
Rule 1	-210.7	Small +Low	0.57	-120.9
Rule 2	-63.95	Small+Tall	0.12	-7.425
Rule 3	401.85	Large+Low	0.51	204.23
Rule 4	249.74	Large+Tall	0.12	28.995
SUM			1.32	104.90

OUTPUT:

$$L_{Aeqt} = 104.90/1.32 = 79.82$$

Question – 5: This question is about the aggregation functions

Q 5 (a). List those of the following properties that are satisfied by multiplication on *[0,1]*.

1. Monotonicity, 2. Boundary conditions, 3. Continuity, 4. Associativity, 5. Symmetry, 6. Neutral element, 7. Annihilator, 8. Idempotency, 9. Archimeden property, 10. Self identity property.

[20 marks]

Answer 5 (a)
Assume any natural *n*. Then we have to focus to multiplication just as *n*-ary aggregation operator and:
1. Monotonicity: if $x_i \leq y_i$ and $x_1 \cdot x_2 \cdot \ldots \cdot x_{i-1} \cdot x_{i+1} \cdot \ldots \cdot x_n = k$ then $x_1 \cdot x_2 \cdot \ldots \cdot x_i \cdot \ldots \cdot x_n = k \cdot x_i$ and $x_1 \cdot x_2 \cdot \ldots \cdot y_i \cdot \ldots \cdot x_n = k \cdot y_i$. Since we work on *[0,1]* there is $k \geq 0$ and thus $k \cdot x_i \leq k \cdot y_i$ and therefore $x_1 \cdot x_2 \cdot \ldots \cdot x_i \cdot \ldots \cdot x_n \leq x_1 \cdot x_2 \cdot \ldots \cdot y_i \cdot \ldots \cdot x_n$
2. Boundary conditions: $0 \cdot \ldots \cdot 0 = 0$ and $1 \cdot \ldots \cdot 1 = 1$
3. Continuity: denoting $x_1 \cdot x_2 \cdot \ldots \cdot x_{i-1} \cdot x_{i+1} \cdot \ldots \cdot x_n = k$ if we restrict multiplication to *i*-th coordinate we get function $f(x_i) = k \cdot x_i$ with *k* being a constant and such a function is evidently continuous.
4. Associativity: if we divide inputs in groups, multiply elements within groups and then multiply the group results the output of the aggregation will be the same as $x_1 \cdot x_2 \cdot \ldots \cdot x_i \cdot \ldots \cdot x_n$ and therefore multiplication on *[0,1]* is associative. Formally, $x \cdot (y \cdot z) = (x \cdot y) \cdot z$ and $x_1 \cdot x_2 \cdot \ldots \cdot x_i \cdot \ldots \cdot x_n = (x_1 \cdot (x_2 \cdot \ldots \cdot (x_i \cdot \ldots (x_{n-1} \cdot x_n))))$
5. Symmetry – the order of inputs is not important when multiplication is applied
6. *1* is the neutral element of multiplication on *[0,1]*
7. *0* is the annihilator of multiplication on *[0,1]*
8. Not: For example *0.5·0.5=0.25* and therefore multiplication on *[0,1]* is not idempotent
9. Since $\lim x^n \to 0$ when $n \to \infty$ for all x from *]0,1[* and multiplication is continuous, the Archimedean property is satisfied.
10. Not: again since *0.5·0.5=0.25* and *0.5·0.5.0.25 = 0.0625* the self identity property is not satisfied.

Q5 b. Compute OWA (Ordered Weighted Average) of values

$$x_1=0.6, \; x_2=0.06, \; x_3 = 0.4$$

with weights

$$w_1=1/3, \; w_2=1/2, \; w_3=1/6.$$

[20 marks]

Answer 5(b)

First we have to reorder inputs into non-decreasing order: *0.06, 0.4,0.6*.
Then apply weights:

$$1/3·0.06+1/2·0.4+1/6·0.6 = 0.02+0.2+0.1 = 0.32$$

Q5 c. Let *T* be a t-norm generated by additive generator $t(x)=(1-x)^2$. Find a t-conorm *C* that is dual to *T*. Apply isomorphism $f(x)=x^{1/2}$ ($f(x) = \sqrt{x}$) to the t-conorm *C* in order to obtain C_f and use this t-conorm C_f to aggregate values *0.7* and *0.5*.
   Hint: inverse function of $f(x)=(1-x)^2$ is $f^{-1}(x)=1-x^{1/2}$.

[60 marks]

Answer 5(c)
 T-norm generated by $t(x)=(1-x)^2$ is the Yager t-norm with parameter 2 since

$$T(x,y) = t^{-1}\big(\min(t(0), t(x) + t(y))\big) = 1 - (\min(1, (1 - x)^2 + (1 - y)^2))^{\frac{1}{2}} =$$

$$= 1 - \min\left(1^{\frac{1}{2}}, ((1 - x)^2 + (1 - y)^2)^{\frac{1}{2}}\right) = \max\left(1 - 1, 1 - ((1 - x)^2 + (1 - y)^2)^{\frac{1}{2}}\right) =$$

$$\max\left(0, 1 - ((1 - x)^2 + (1 - y)^2)^{\frac{1}{2}}\right)$$

Dual t-conorm is given by

$$C(x,y) = 1 - T(1 - x, 1 - y) = 1 - \max\left(0, 1 - ((x)^2 + (y)^2)^{\frac{1}{2}}\right) = \min\left(1, (x^2 + y^2)^{\frac{1}{2}}\right)$$

Now we apply the isomorphism $f(x)=x^{1/2}$

$$C_f(x,y) = f^{-1}\left(C\big(f(x), f(y)\big)\right) = \min\left(1, ((x^{\frac{1}{2}})^2 + (y^{\frac{1}{2}})^2)^{\frac{1}{2}}\right)^2 = \min\left(1, x + y\right)$$

i.e., C_f is the bounded sum. Finally $C_f(0.7, 0.5) = min(1, 1.2) = 1$.