BA Mod. (Computer Science) Trinity Term 2011
SS Examination

# Solutions to
# CS4001 Fuzzy Logic and Fuzzy Control Systems

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Q1 (a). Design fuzzy subsets, for each of the three ancestors and the parent, that have the term set comprising the following daughter species:

{**Pacific Lamprey, Arctic Lamprey, Atlantic Salmon, Pacific Salmon, Blue Shark, White Shark, Non-snake Lizard, Snake-like Lizards**}

Use the fuzzy subset notation of ordered pairs $\{(x, \mu_A(x))\}$ where $x \in X$, X being the universe of discourse and A is a fuzzy (sub)-set of X:.

$$\mu_A : X \rightarrow [0, 1] \text{ the membership function of } A$$

$\mu_A(x) \in [0, 1]$ is grade of membership of x in A.

**<u>Answer 1a:</u>** In evolution of species there is continuous change of properties of an organism. So the there I have assumed that the membership function of a fuzzy set can be computed using an 'inter-generational distance'- the distance between immediate successor generations is assumed to be one and use the inverse of the exponential of half the distance as the membership function.

| | Parent/ Daughter | Grandparent/ Daughter | Great Grandparent/ Daughter | Great great-grandparent/ Daughter |
|---|---|---|---|---|
| Distance | 1 | 2 | 3 | 4 |
| Belongingness | 0.61 | 0.37 | 0.22 | 0.14 |

**$A^{great\text{-}great\text{-}grandparents}$ ={0.14/Pacific Lamprey, 0.14/Arctic Lamprey, 0.14/Atlantic Salmon, 0.14/Pacific Salmon, 0.14/Blue Shark, 0.14/White Shark, 0.14/Non-snake Lizard, 0.14/Snake-like Lizards }**

**$A^{great\text{-}grandparents}$ ={0.22/Atlantic Salmon, 0.22/Pacific Salmon, 0.22/Blue Shark, 0.22/White Shark, 0.22/Non-snake Lizard, 0.22/Snake-like Lizards }**

**$A^{grandparent\text{-}daughter}$ ={0.37/Atlantic Salmon, 0.37/Pacific Salmon, 0.37/Non-snake Lizard, 0.37/Snake-like Lizards }**

**$A^{salmon\text{-}daughter}$ ={0.61/Atlantic Salmon, 0.61/Pacific Salmon}**

**$A^{lizard\text{-}daughter}$ ={0.61/Non-snake Lizard, 0.61/Snake-like Lizards }**

**[70 Marks]**

Q1 (b) Compute the *core, support* and *cardinality* of the fuzzy sub-set of the three ancestors and the two parents.

## Answer 1b

(i) Core of all these fuzzy sets is the null set $\varphi$, as no members have the membership function value equal to unity.

[10 Marks]

(ii) Support of all the fuzzy sets comprises all the listed members as members with zero belongingness have been omitted.

[10 Marks]

(iii) The cardinality of the sets is as follows:

**Card($A^{great\text{-}great\text{-}grandparents}$) = 8*0.14= 1.12**

**Card($A^{great\text{-}grandparents}$) = 6 *0.22 = 1.32**

**Card($A^{grandparent\text{-}daughter}$) =4 *0.37 = 1.28**

**Card ($A^{salmon\text{-}daughter}$)** **=2 * 0.61= 1.32**

**Card($A^{lizard\text{-}daughter}$)=** **=2 * 0.61= 1.32**

**[10 Marks]**

**Q2 (a)** In conventional knowledge based systems, relationships between domain objects, say A and B, can be expressed as

## Answer 2(a)

| | Relationships | |
|---|---|---|
| | Simple or **Conditional and Relational Statements between domain objects *A*, *B*:** | Complex or **Ordered sequences of instructions comprising** |
| **Crisp Statements** | **IF *A* THEN B;**<br><br>***A* is-a-part-of *B***<br><br>***A* weighs 5KG** | ***A*=5;**<br><br>**IF *A* < *5* THEN *B*=*A*+*5***<br>**………** |
| **Fuzzy Statements** | **IF *A* ($\Psi_A$) THEN *B* ($\Psi_B$)**<br><br>**A is a sub-set of B, i.e. For all x, $\Psi_A(x) \leq \Psi_B(x)$**<br><br>***A* weighs *about* 5KG** | ***A* IS-SMALL; $\Psi_{small}(5)=1$;**<br>**$\Psi_{small}(x)=0 \rightarrow x<4$, or $x>6$.**<br><br>**IF *A* ($\Psi_A$) IS_SMALL THEN *B* = A+5** |

**[40 Marks]**

**Q2 (b)**

Construct a fuzzy rule-base, where income and risk are in a conditional relationship with income as antecedent and risk as consequent, by creating fuzzy patches.

**The student is expected to discuss the Cartesian product of two fuzzy subsets – income and risk:**

**The *5artesian* or *cross product* of fuzzy subsets *A* and *B,* of sets *X* and *Y* respectively is denoted as**

$$A \times B$$

**This cross product relationship *T* on the set $X \times Y$ is denoted as**

$$T = A \times B$$

**This will lead to**

$$\mu_T(x, y) = MIN\,[(\mu_A(x), \mu_B(y))]$$

**The fuzzy sets can be derived from the following definitions:**

$$\mu_{Income}^{Excellent}(x) = 0, \forall x \le 90; \quad \mu_{Income}^{Excellent}(x) = 1, x \ge 120;$$
$$\mu_{Income}^{Good}(x) = 0, \forall x \le 50 \ \& \ \forall x \ge 100; \quad \mu_{Income}^{Good}(x) = 1, x = 75;$$
$$\mu_{Income}^{Poor}(x) = 0, \forall x \ge 60; \quad \mu_{Income}^{Poor}(x) = 1, x \le 10.$$

$$\mu_{Risk}^{Low}(x) = 0, \forall x \ge 40\%; \quad \mu_{Risk}^{Low}(x) = 1, x \le 20\%;$$
$$\mu_{Risk}^{Medium}(x) = 0, \forall x \le 20\% \ \& \ \forall x \ge 80\%; \quad \mu_{Risk}^{Medium}(x) = 1, 40\% \le x \le 60\%;$$
$$\mu_{Risk}^{High}(x) = 0, \forall x \le 60\%; \quad \mu_{Risk}^{High}(x) = 1, x \ge 80\%;$$

**Answer 2(b)**

The computation of the fuzzy membership functions for terms (sub-)set {Excellent, Poor} for the linguistic variable *income* can be calculated using a linear function with a threshold value; similarly for the two terms {Low, High} of the *risk* variable. The membership function for the term *Good Income* can be computed using a triangular membership function:

$$trimf(x; a, b, c) = \max\left(\min\left(\frac{x-a}{b-a}, \frac{c-x}{c-b}\right), 0\right),$$

where a=20, b=40 and c=80.

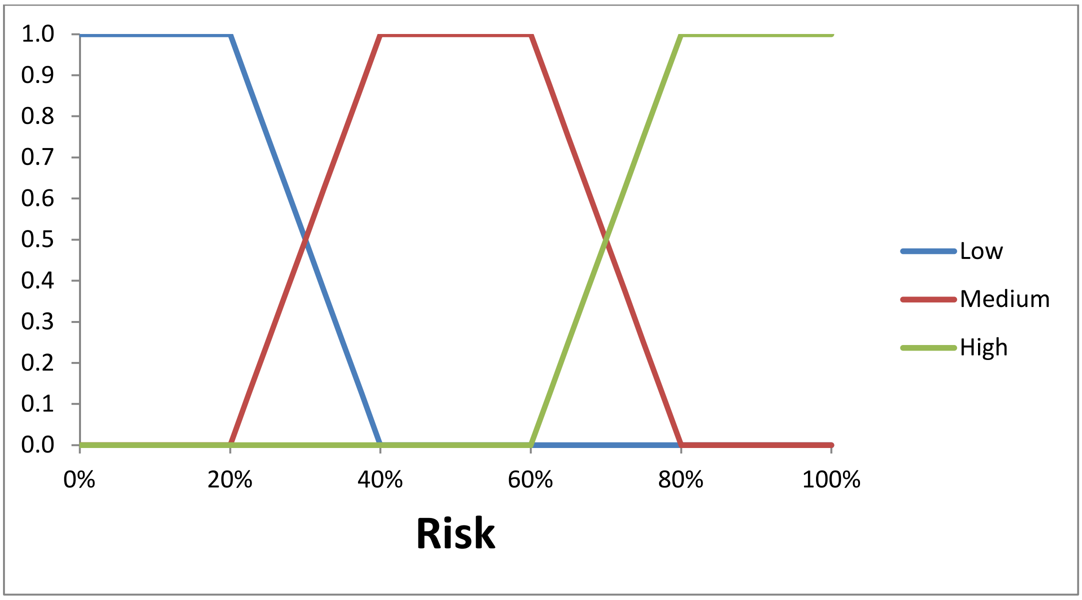
And, the term *Medium Risk* can be computed using a trapezoidal membership function:

$$trapmf(x; a, b, c, d) = \max\left(\min\left(\frac{x-a}{b-a}, 1, \frac{d-x}{d-c}\right), 0\right),$$

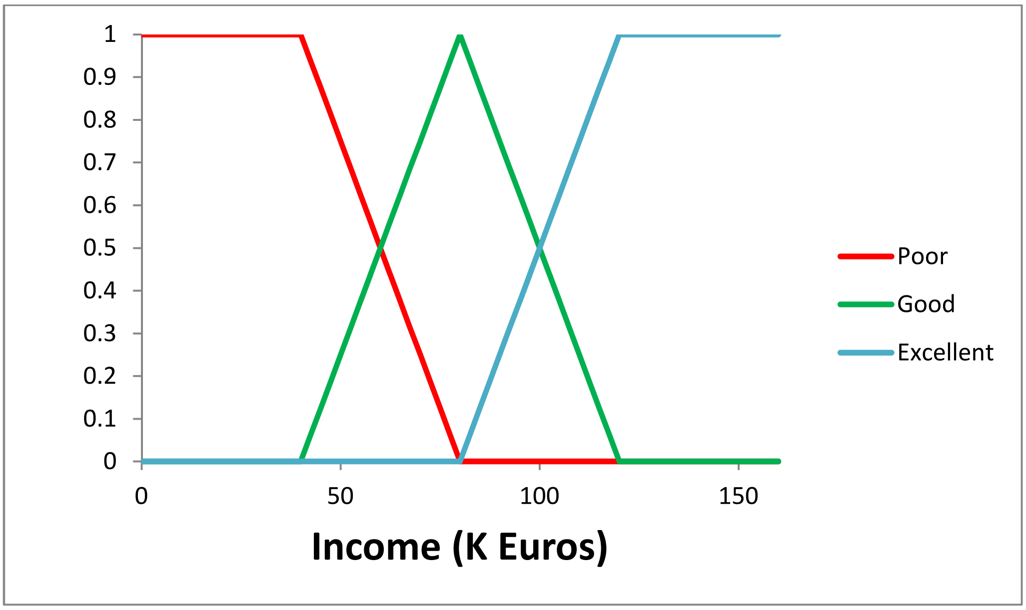
where the value of the constants is as follows:

a=20%, b=40%, c=60% and d=80%

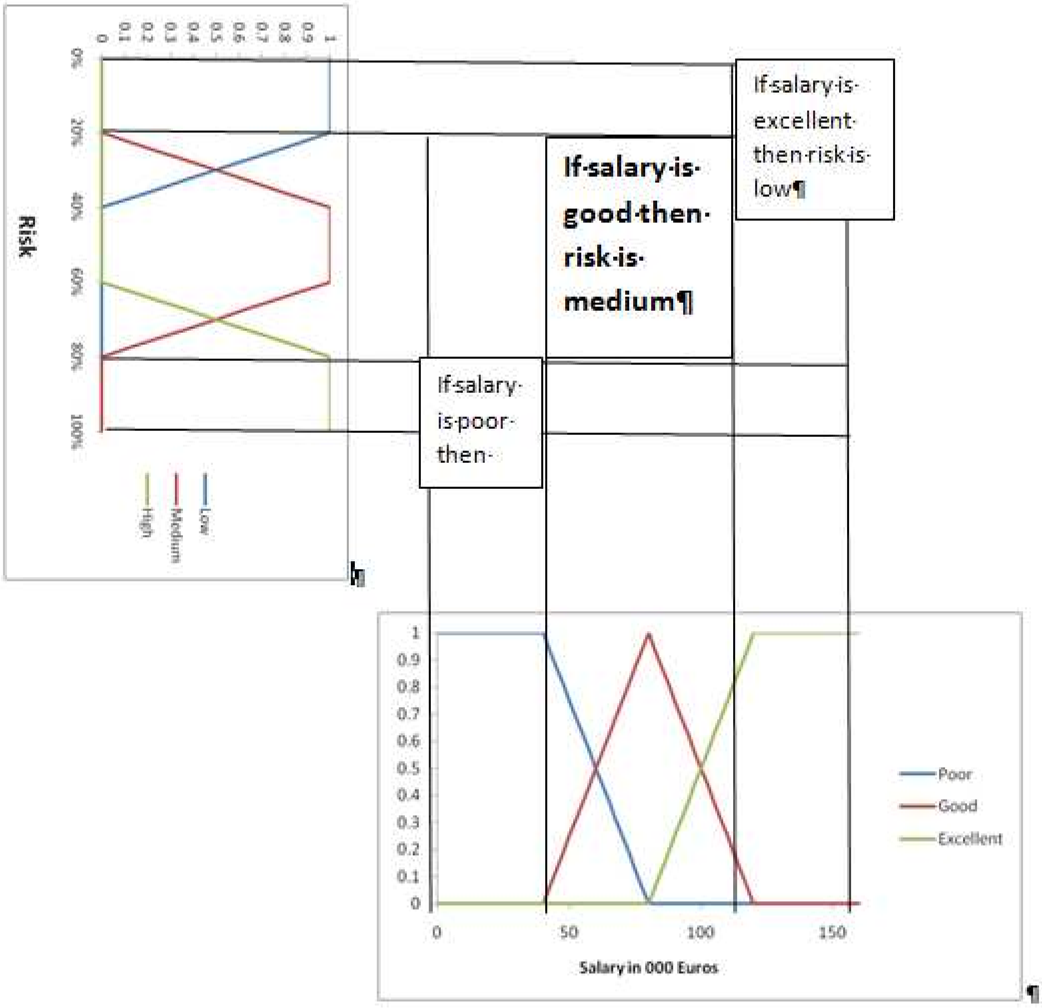
The student is expected to draw the two graphs:


1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0%
20%
40%
60%
80%
100%
Risk
Low
Medium
High

**And**


1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
50
100
150
Income (K Euros)
Poor
Good
Excellent

Then the fuzzy patches can be drawn as follows:




[60 Marks]

Q3 (a). Use the algebraic relationship between the input and the output, for computing the value of the constants $\boldsymbol{p_{01}}$, $\boldsymbol{p_{11}}$, $\boldsymbol{p_{02}}$, $\boldsymbol{p_{12}}$.

$$Rule^1: \quad x \text{ is } \quad \mu_1(x) \quad THEN \ y = p_{01} + p_{11}x$$
$$Rule^2: \quad x \text{ is } \quad \mu_2(x) \quad THEN \ y = p_{02} + p_{12}x$$

## Answer 3a

The composition phase of fuzzy inference will yield:

$$
\begin{aligned}
&COMPOSITION\\
&y = \frac{\mu_1(x) * [p_{01} + p_{11}x] + \mu_2(x) * [p_{02} + p_{12}x]}{\mu_1(x) + \mu_2(x)}\\
&\hat{\mu}_1(x) = \frac{\mu_1(x)}{\mu_1(x) + \mu_2(x)}\\
&\hat{\mu}_2(x) = \frac{\mu_2(x)}{\mu_1(x) + \mu_2(x)}\\
&\therefore y = \hat{\mu}_1(x) * [p_{01} + p_{11}x] + \hat{\mu}_2(x) * [p_{02} + p_{12}x]
\end{aligned}
$$

$$
\begin{aligned}
&y = \hat{\mu}_1(x) * [p_{01} + p_{11}x] + \hat{\mu}_2(x) * [p_{02} + p_{12}x]\\
&Four\ values\ of\ \ x = x_1, x_2, x_3, x_4\ and\ \ for\ each\ x, will\ have a value\\
&y_1 = \hat{\mu}_1(x_1) * p_{01} + \hat{\mu}_2(x_1) * p_{02} + \hat{\mu}_1(x_1) * x_1 * p_{11} + \hat{\mu}_2(x_1) * x_1 * p_{11}
\end{aligned}
$$

$$
\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} =
\begin{bmatrix}
\hat{\mu}_1(x_1) & \hat{\mu}_2(x_1) & \hat{\mu}_2(x_1) & \hat{\mu}_1(x_1) * x_1 + \hat{\mu}_2(x_1) * x_1 \\
\hat{\mu}_1(x_2) & \hat{\mu}_2(x_2) & \hat{\mu}_2(x_2) & \hat{\mu}_1(x_2) * x_1 + \hat{\mu}_2(x_2) * x_2 \\
\hat{\mu}_1(x_3) & \hat{\mu}_2(x_3) & \hat{\mu}_2(x_3) & \hat{\mu}_1(x_3) * x_3 + \hat{\mu}_2(x_3) * x_3 \\
\hat{\mu}_1(x_4) & \hat{\mu}_2(x_4) & \hat{\mu}_2(x_4) & \hat{\mu}_1(x_4) * x_4 + \hat{\mu}_2(x_4) * x_4
\end{bmatrix}
\begin{bmatrix} p_{01} \\ p_{02} \\ p_{11} \\ p_{21} \end{bmatrix}
$$

[30 Marks]

Q3 (b) Describe in your own words the importance of Takagi and Sugeno's insight in using the algebraic relationship in being able to derive a fuzzy control system's parameter.

## Answer 3(b)

In order to determine the values of the parameters *p* in the consequents, one solves the LINEAR system of algebraic equations and tries to minimize the difference between the ACTUAL output of the system (*Y*) and the simulation *[X]$^T$[P]* :

$$[Y]-[X]^T[P]\leq \varepsilon$$

where ε is the error term.

This result suggests that the parameters, *p*, in the consequent can be derived from a subset of input-output relationships: Takagi and Sugeno claimed that 'this method of identification enables us to obtain just the same parameters as the original system, if we have a sufficient number of noiseless output data for identification'.

What is equally important here is that Takagi and Sugeno have method to an *n*-rule, *m*-parameter system.

[20 Marks]

Q3 (c) Consider the following fuzzy implications (or rules) $R_1$, $R_2$, and $R_3$ used in the design of a Takagi-Sugeno controller:

**$R_1$→ If $x_1$ is *small* & $x_2$ is *medium* then $y = x_1+x_2$**

**$R_2$→ If $x_1$ is *big* then $y = 2x_1$**

**$R_3$→ If $x_2$ is *critical* then $y = 3x_2$**

where y (i) refers to the consequent variable for each rule labelled $\boldsymbol{R_i}$ and $\boldsymbol{x_1}$ and $\boldsymbol{x_2}$ refer to the input variables that appear in premise of the rules.

The membership functions are defined as follows:

| x | *small* | *medium* | *big* | *critical* |
|---|---|---|---|---|
| **5** | 0.69 | 0.37 | 0 | 0.37 |
| 12 | 0.25 | 0 | 0.20 | 1 |

Compute the output of the 3-rule TS controller for input values 5 and 12. Show all three steps of the computation, fuzzification, inference, and composition for each of the input values.

50 Marks [25 marks for each of the two inputs]

**<u>Answer 3(c)</u>** The student should breakdown his/her calculation in the three stages of computation in a Takagi-Sigeno fuzzy knowledge based system:

**<u>FUZZUFICATION:</u>** We have the following values of the membership functions for the two values $x_1$ = 12 & $x_2$ = 5:

| | **Membership Functions** | | | |
|---|---|---|---|---|
| | Small | Medium | Big | Critical |
| **$x_1$=5** | 0.69 | 0.37 | 0 | 0.37 |
| **$x_2$=12** | 0.25 | 0 | 0.2 | 1 |

**<u>INFERENCE & CONSEQUENCE</u>**

| Rule | Premise 1 | Premise 2 | Consequence | Truth Value (Min(Premise1 & Premise 2) |
|---|---|---|---|---|
| $R_1$ | small($x_1$)=0.25 | medium ($x_2$)=0.37 | $y=x_1+x_2=12+5=17$ | min(0.25,0.37)=0.25 |
| $R_2$ | big($x_1$)=0.2 | | $y=2 * x_1=24$ | 0.2 |
| $R_3$ | critical($x_2$)=0.37 | | $Y=3* x_2=15$ | 0.37 |

**<u>AGGREGATION (& DEFUZZIFICATION)</u>**

The aggregation can be performed by using a centre of area computation:

$$= \frac{\sum_{i=1,3} \left| y = y^{(i)} \right| * y^{(i)}}{\sum_{i=1,3} \left| y = y^{(i)} \right|}$$

$$y = \frac{0.25*17+0.2*24+0.375*15}{0.25+0.2+0.375} = 17.8$$

**<u>The system output is 17.8 which is quite close to the arithmetic average of the three outputs {17,24,15}, 18.6.</u>**

[50 Marks]

## Q4

Compute Enda's speed of rotation (η) for the following cases: Indicate clearly all the processes of fuzzy inference in your computation.

| Distance from the | | Direction |
|---|---|---|
| **beginning of the bend (cms)** | **inner barrier (cms)** | **θ** |
| 9.95 | 30 | $0^\circ$ |
| 65.1 | 30 | $-30^\circ$ |

## ANSWER 4.

**FUZZIFICATION:**

Input Variables: $\chi_1 = 10$; $\chi_2 = 30$; and $\chi_3 = 0$

| Linguistic Variable | Distance from the beginning of the bend | Distance from the inner bend | Direction of Rotation |
|---|---|---|---|
| Small | $\chi_1^{small}(10) = -\frac{10}{50} + 1 = \frac{4}{5} = 0.8$ | $\chi_2^{small}(30) = -\frac{30}{40} + 1 = \frac{1}{4} = 0.25$ | NOT APPLICABLE |
| Moderate | 0 | NOT APPLICABLE | NOT APPLICABLE |
| Longway | 0 | $\chi_2^{longway}(30) = \frac{30}{60} - \frac{1}{3} = \frac{1}{2} - \frac{1}{3} = 0.16$ | NOT APPLICABLE |
| Forward | NOT APPLICABLE | NOT APPLICABLE | $\chi_3^{forward}(0) = -\frac{0}{30} + 1 = 1$ |
| Inward | NOT APPLICABLE | NOT APPLICABLE | 0 |
| Outward | NOT APPLICABLE | NOT APPLICABLE | 0 |

[20 Marks]

## **Answer 4 (Continued)**

**FUZZIFICATION:**

| Input Variables: $\chi_1 = 65$; $\chi_2 = 30$; and $\chi_3 = -30$ | | | |
|---|---|---|---|
| Linguistic Variable | Distance from the beginning of the bend | Distance from the inner bend | Direction of Rotation |
| Small | | $\chi_2^{small}(30) = -\frac{30}{40} + 1$ $= \frac{1}{4} = 0.25$ | NOT APPLICABLE |
| Moderate | $\chi_1^{moderate}(x) = -\frac{x}{30} + 3$ $= -\frac{65}{30} + 3$ $= \frac{5}{6} = 0.83$ | NOT APPLICABLE | NOT APPLICABLE |
| Longway | 0 | $\chi_2^{longway}(30) = \frac{30}{60} - \frac{1}{3}$ $= \frac{1}{2} - \frac{1}{3} = 0.16$ | NOT APPLICABLE |
| Forward | NOT APPLICABLE | NOT APPLICABLE | 0 |
| Inward | NOT APPLICABLE | NOT APPLICABLE | 0 |
| Outward | NOT APPLICABLE | NOT APPLICABLE | $\chi_3^{outwards}(-30) = -\frac{(-30)}{40} - \frac{1}{2}$ $= \frac{3}{4} - \frac{1}{2} = \frac{1}{4}$ |

[20 Marks]

## Answer 4 (Continued)

## *Composition (and Defuzzification)*

Input Variables: $\chi_1 = 10$; $\chi_2 = 30$; and $\chi_3 = 0$

| | X1 | X2 | X3 |
|---|---|---|---|
| | 9.95 | 30 | 0 |
| **Linguistic Variable** | **Distance from the beginning of the bend** | **Distance from the inner bend** | **Direction of Rotation** |
| Small | 0.801 | 0.25 | NA |
| Moderate | 0 | NA | NA |
| Longway | 0 | 0.167 | NA |
| Forward | NA | NA | 1 |
| Inward | NA | NA | 0 |
| Outward | NA | NA | 0 |

| Rule | α | η | Output=α*η |
|---|---|---|---|
| **Rule** *1* : *If* $\chi_1$ is *small* and $\chi_2$ is *small* and $\eta = 0.25 - 0.02\,\chi_1 + 0.06\,\chi_2 - 0.05\,\chi_3$ | 0.250 | 1.851 | *0.463* |
| | | | |
| **Rule** *2* : *If* $\chi_1$ is *small* and $\chi_2$ is *small* and then $\eta = -0.025\,\chi 1 + 0.07\,\chi 2 - 0.075\,\chi 3$ | 0.000 | 1.851 | *0.000* |
| | | | |
| **Rule** *3* : *If* $\chi_1$ is *small* and $\chi_2$ is *long way* $\eta = 3 - 0.02\,\chi_1 - 0.02\,\chi_3$ | 0.167 | *2.801* | *0.467* |
| | | | |
| **Rule** *4*: *If* $\chi_1$ is *moderate* and $\chi_2$ is *small* $\eta = 3 - 0.02\,\chi_1 + 0.01\,\chi_2 - 0.04\,\chi_3$ | 0.000 | *3.101* | *0.000* |
| | | | |
| **Rule** *5* : *If* $\chi_1$ is *long way* and $\chi_2$ is *small* $\eta = 0.5 - 0.005\,\chi_3$ | 0.000 | *0.500* | *0.000* |
| **Total** | **0.417** | | **0.930** |

$$\eta(\chi_1, \chi_2, \chi_3) \equiv$$

$$\eta(10,30,0) = \frac{\sum_{i=1}^{5} \alpha_i * \eta_i}{\sum_{i=1}^{5} \alpha_i} = \frac{0.0.8}{0.4166} = 2.2298\,\text{deg/sec}$$

[30 Marks]

## Answer 4 (Concluded)

| Input Variables: $\chi_1 = 65.1$; $\chi_2 = 30$; and $\chi_3 = -30$ |
|---|

| | X1<br>65.1 | X2<br>30 | X3<br>-30 |
|---|---|---|---|
| **Linguistic Variable** | **Distance from the beginning of the bend** | **Distance from the inner bend** | **Direction of Rotation** |
| Small | 0 | 0.25 | NA |
| Moderate | 0.83 | NA | NA |
| Longway | 0 | 0.167 | NA |
| Forward | NA | NA | 0 |
| Inward | NA | NA | 0 |
| Outward | NA | NA | 0.25 |

| Rule | α | η | Output=α*η |
|---|---|---|---|
| **Rule** *1* **:** *If* $\chi_1$ **is** ***small*** **and** $\chi_2$ **is** ***small*** **and** $\eta = 0.25 - 0.02\,\chi_1 + 0.06\,\chi_2 - 0.05\,\chi_3$ | 0.000 | 2.248 | *0.000* |
| | | | |
| **Rule** *2* **:** *If* $\chi_1$ **is** ***small*** **and** $\chi_2$ **is** ***small*** **and** $\eta = -0.025\,\chi 1 + 0.07\,\chi 2 - 0.075\,\chi 3$ | 0.000 | 2.723 | *0.000* |
| | | | |
| **Rule** *3* **:** *If* $\chi_1$ **is** ***small*** **and** $\chi_2$ **is** ***long way*** $\eta = 3 - 0.02\,\chi_1 - 0.02\,\chi_3$ | 0.000 | *2.298* | *0.000* |
| | | | |
| **Rule** *4:* *If* $\chi_1$ **is** ***moderate*** **and** $\chi_2$ **is** ***small*** $\eta = 3 - 0.02\,\chi_1 + 0.01\,\chi_2 - 0.04\,\chi_3$ | 0.250 | *3.198* | *0.800* |
| | | | |
| **Rule** *5* **:** *If* $\chi_1$ **is** ***long way*** **and** $\chi_2$ **is** ***small*** $\eta = 0.5 - 0.005\,\chi_3$ | 0.000 | *0.650* | *0.000* |

| | | | |
|---|---|---|---|
| **Total** | **0.250** | | **0.800** |

$$\eta(\chi_1, \chi_2, \chi_3) \equiv$$

$$\eta(65.1, 30, -30) = \frac{\sum_{i=1}^{5} \alpha_i * \eta_i}{\sum_{i=1}^{5} \alpha_i} = \frac{0.8}{0.250} = 3.198\ \text{deg/sec}$$

[30 Marks]

Q5

Q5(a). Consider the OR-gate with two inputs and one output:

| X1 | X2 | Y |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 1 | 1 |

You are expected to train a perceptron, with two inputs and one output, to learn to behave like an *OR* gate. Assume that the learning constant $\alpha$=0.2, bias $\theta$=-0.3 and input weights are w1=0.3 and w2=-0.1. Train the perceptron for at least 4 epochs. Tabulate the inputs and outputs (actual and desired) together with weight changes.

**Answer 5a** The training shows convergence in Epoch 4.

Epoch 1

| X1 | X2 | d | w1 | w2 | sum | out | err | w1 | w2 |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0.3 | -0.1 | -0.3 | 0 | 0 | 0.3 | -0.1 |
| 1 | 0 | 1 | 0.3 | -0.1 | 0 | 0 | 1 | 0.5 | -0.1 |
| 0 | 1 | 1 | 0.5 | -0.1 | -0.4 | 0 | 1 | 0.5 | 0.1 |
| 1 | 1 | 1 | 0.5 | 0.1 | 0.3 | 1 | 0 | 0.5 | 0.1 |

Epoch 2

| X1 | X2 | d | w1 | w2 | sum | out | err | w1 | w2 |
|---|---|---|---|---|---|---|---|---|---|

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0.5 | 0.1 | -0.3 | 0 | 0 | 0.5 | 0.1 |
| 1 | 0 | 1 | 0.5 | 0.1 | 0.2 | 1 | 0 | 0.5 | 0.1 |
| 0 | 1 | 1 | 0.5 | 0.1 | -0.2 | 0 | 1 | 0.5 | 0.3 |
| 1 | 1 | 1 | 0.5 | 0.3 | 0.5 | 1 | 0 | 0.5 | 0.3 |

Epoch 3

| X1 | X2 | d | w1 | w2 | sum | out | err | w1 | w2 |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0.5 | 0.3 | -0.3 | 0 | 0 | 0.5 | 0.3 |
| 1 | 0 | 1 | 0.5 | 0.3 | 0.2 | 1 | 0 | 0.5 | 0.3 |
| 0 | 1 | 1 | 0.5 | 0.3 | 0 | 0 | 1 | 0.5 | 0.5 |
| 1 | 1 | 1 | 0.5 | 0.5 | 0.7 | 1 | 0 | 0.5 | 0.5 |

Epoch 4

| X1 | X2 | d | w1 | w2 | sum | out | err | w1 | w2 |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0.5 | 0.5 | -0.3 | 0 | 0 | 0.5 | 0.5 |
| 1 | 0 | 1 | 0.5 | 0.5 | 0.2 | 1 | 0 | 0.5 | 0.5 |
| 0 | 1 | 1 | 0.5 | 0.5 | 0.2 | 1 | 0 | 0.5 | 0.5 |
| 1 | 1 | 1 | 0.5 | 0.5 | 0.7 | 1 | 0 | 0.5 | 0.5 |

[60 Marks]

Q5 (b) Can you use the perceptron learning algorithm for learning the behaviour of an *XOR* gate? Give your reasons as to whether you can or cannot?

**ANSWER 5(b)**

Answer 5b. XOR gate cannot be modelled by simple perceptron since it doesn’t belong to the set of linearly separable problems. Single perceptron can represent a line in an input space where all inputs below this line have one output value and all inputs above this line have another value. Since input space of XOR problem cannot be divided by line to get all inputs related to 0 outputs on the one side and all inputs related to 1 outputs to the other side of the line, single perceptron can never model a XOR gate.

Q5 c. Describe the architecture of an adaptive neuro-fuzzy system that can learn the behaviour described in the above rule base governing the operation of an XOR gate.

**ANSWER 5(c)**

Answer 5c. In order to model XOR gate we need to design a neuro-fuzzy system with one hidden layer. In this system, first layer would comprise inputs x1 and x2 and a bias. Two input nodes together with bias would all be connected to one neuron in the hidden layer. Another neuron in the hidden layer would be the bias. All neurons from the hidden layer are connected to the one neuron in the output layer.