UNIVERSITY OF DUBLIN

TRINITY COLLEGE

School of Computer Science and Statistics
Department of Computer Science

BA Mod. (Computer Science) Trinity Term 2010
SS Examination

CS4001 Fuzzy Logic and Fuzzy Control Systems

Date Location Time: 3 Hours

Prof. Khurshid Ahmad

Instructions

(i) A total of THREE questions out of FIVE must be answered for full marks.

(ii) All questions carry equal marks.

(iii) Use of non-programmable calculators and log tables is permitted.
You must note the make and model of your calculator on your answer book.

Model Solutions to Question 1

Q1. This question is about fuzzy sets.

Bertie is an avid animal lover. His zoo contains a number of animals: duck-billed platypus, echidnas, horses, zebras, tigers, whales, dolphins, sparrows, robins, crows, penguins and kiwis.

Platypus and echidnas are monotermes- egg laying mammals: Bertie calls them 'half-mammals'. Horses, zebras and tigers – are the 'whole' mammals according to Bertie. He does not know how to classify whales and dolphins in his collection: these water borne mammals have lungs not gills, they have hair not scales, and they give live birth and don't lay eggs. Bertie calls them 10% mammals as whales and dolphins spend all their time in water. Then there is a collection of birds: flighted ones (sparrows, robins, crows) and non-flighted ones (penguins and kiwis); Bertie calls the non-flighted birds half birds.

Q1 a. Can you use a crisp set to describe the contents of Bertie's mammals? Give at least ONE reason to support your answer.

[10 Marks]

Answer 1a: Crisp sets are useful only when we have mutually exclusive sets where membership of objects under consideration is restricted to one single set. The categorization of monotermes and sea mammals, and that of non-flighted birds, will benefit when we have elastic membership – an object may weakly belong to one or more sets.

Q1 b. Using the convention of listing the members of a fuzzy sub-set, i.e the belongingness value and the name of the member of the sub-set, describe Bertie's mammals and birds as two sub-sets $A^{mammals}$ and A^{birds}.

Answer 1b.

$A^{mammals}$ ={0.5/platypus, 0.5/ echidnas, 1/horses, 1/zebras, 1/tigers, 0.1/whales, 0.1/dolphins, 0/sparrows, 0/robins, 0/crows, 0/penguins and 0/kiwis}

A^{birds} ={0/platypus, 0/ echidnas, 0/horses, 0/zebras, 0/tigers, 0/whales, 0/dolphins, 1/sparrows, 1/robins, 1/crows, 0.5/penguins and 0.5/kiwis}

[30 Marks]

Q1 c. Construct a subset $E^{mammals}$ which is a fuzzy intersection of $A^{mammals}$ and its complement $\tilde{A}^{mammals}$.

Answer 1c.

$$A^{mammals} \cap \widetilde{A}^{mammals} = E^{mammals}$$

$A'^{\,mammals}$: $\mu_{A'mammals}(x) = 1 - \mu_{A\,mammals}(x)$

$A'^{\,mammals}$ ={0.5/platypus, 0.5/ echidnas, 0/horses, 0/zebras, 0/tigers, 0.9/whales, 0.9/dolphins, 1/sparrows, 1/robins, 1/crows, 1/penguins and 1/kiwis}

$$A^{mammals} \cap \widetilde{A}^{mammals} : \mu_{A\,mammals \cap \widetilde{A}\,mammals}(x) = \min(\mu_{A\,mammals}, \mu_{\widetilde{A}\,mammals}).$$

$E^{mammals}$ ={0.5/platypus, 0.5/ echidnas, 0/horses, 0/zebras, 0/tigers, 0.1/whales, 0.1/dolphins, 0/sparrows, 0/robins, 0/crows, 0/penguins and 0/kiwis}

[30 Marks]

Q1 d. To what extent does the subset Amammals obeys the well known laws of crisp set theory: the Law of the Excluded Middle and the Law of Contradiction?

Answer 1d: Law of Contradiction states that

$$A \cap \widetilde{A} = \varnothing$$

But $E^{mammals} \neq \varnothing$ and thus $A^{mammals}$ violates the Law of Contradiction for crisp sets.

Law of excluded middle sates that

$$A \cup \widetilde{A} = X$$

The union

$$A^{mammals} \cup \widetilde{A}^{mammals} : \mu_{A\,mammals \cup \widetilde{A}\,mammals}(x) = \max(\mu_{A\,mammals}, \mu_{\widetilde{A}\,mammals}).$$

We have

$A \cup \widetilde{A} =$ {0.5/platypus, 0.5/ echidnas, 1/horses, 1/zebras, 1/tigers, 0.9/whales, 0.9/dolphins, 1/sparrows, 1/robins, 1/crows, 1/penguins and 1/kiwis}.

Thus in this case the Law of excluded middle is not satisfied.

[30 Marks]

Q2. **This question is about fuzzy sets and fuzzy knowledge representation**

Kruse, Gebhardt and Klawonn have discussed 'simple forms of representing fuzzy sets': 'Any fuzzy set μ of a universe of discourse X can be described by assigning a degree of membership function μ(x) to each element xεX. However, if the number of elements in X is very large or even countably infinite or if a continuum is used for X, then μ(x) is represented best with the help of an appropriate, possibly parametrized term, where the parameters can be adjusted according to the given problem' (*Foundations of Fuzzy Systems,* John Wiley &Sons, 1994, pp 13-14).

Q2 a. How will you represent the following every day notions using an appropriate fuzzy membership function: (i) the notion of being *tall*; (ii) that an apartment in the Dublin City Centre is about €250,000; and (iii) that a middle income earner in Ireland earns approximately between €80,000 and €200,000.

(Hint: Three commonly used membership functions are trapezoidal, Gaussian and piece-wise liner functions).

[15 Marks]

Answer 2 a.

(i) An interpretation of *tallness* is usually given by a non-decreasing monotonic function which is piece-wise linear:

$$\mu_{a,b}(x) = \begin{cases} 0 & if\ x \leq a \\ \dfrac{x-a}{b-a} & if\ a \leq x \leq b \\ 1 & if\ x \geq b \end{cases}$$

(ii) An interpretation of *aboutness* can be approximated by using the symmetrical triangular membership function:

$$\mu_{a,b}(x) = \begin{cases} 1 - \left|\dfrac{m-x}{d}\right| & if\ m-d \leq x \leq m+d \\ 0 \quad if\ x < m-d\ or\ x > m+d & \end{cases}$$

where d>0 and m ε R

(iii) The *in-betweeness* condition is approximated by using the so-called trapezoidal function:

$$\mu_{a,b,c,d} = \begin{cases} \left(\frac{x-a}{b-a}\right) & if\ a \leq x \leq b \\ 1 & if\ b \leq x \leq c \\ \left(\frac{x-d}{c-d}\right) & if\ a \leq x \leq b \\ 0 & if\ x < a\ OR\ x > d \end{cases}$$

Q2 b. Consider the following applicants for two advertisements, one for playing professional basketball and the other for being the coxswain of a rowing boat. The requirement for being a professional basketball player is that the candidate must be tall and that the candidate must be quick. For the coxswain the requirement is that the candidate should not be too tall.

Candidate's Name	**Height in meters**	**Speed in meters/second**
Gordon	0.97	65
Brian	1.65	30
Angela	1.75	27
Nicholas	1.78	32
Catherine	1.85	31
Shaquil	2.18	45
Tony	1.02	16

Usually, a person who is 1.5 meters or below in height is not considered *tall* and somebody who is over 2.13 meters in height is definitely considered *tall*. A *swift* person is definitely one who can move over 60 meters per second, but somebody who cannot move faster than 18 meters per second is not regarded as swift.

Q2 b.1 Derive the membership function for the fuzzy (sub-)sets *tall* and *swift* from the above descriptions.

Answer 2 b.1 The degree of membership for both fuzzy (sub-) sets can be given as a piece-wise linear function respectively:

$$\mu_{1.5,2.13}^{tall}(x) = \begin{cases} 0 & if\ x \leq 1.5 \\ \left(\frac{x-1.5}{0.63}\right), & if\ 1.5 \leq x \leq 2.13 \\ 1 & if\ x \geq 2.13 \end{cases}$$

$$\mu_{18,60}^{swift}(x) = \begin{cases} 0 & if\ x \leq 18 \\ \left(\frac{x-18}{42}\right), & if\ 18 \leq x \leq 60 \\ 1 & if\ x \geq 60 \end{cases}$$

[25 Marks]

Q2 b.3 Compute the complements, concentration, dilation, and support for the fuzzy (sub-) sets *tall* and *swift.*

Complements: $\overline{\mu_{1.5,2.13}^{tall}} = 1 - \mu_{1.5,2.13}^{tall}(x)$

Concentration: $\mu_{1.5,2.13}^{conc-\ tall}(x) = \left(\mu_{1.5,2.13}^{tall}(x)\right)^{n}, where\ n \geq 1$

Dilation: $\mu_{1.5,2.13}^{dil-\ tall}(x) = \left(\mu_{1.5,2.13}^{tall}(x)\right)^{\frac{1}{n}}, where\ n \geq 1$

Support: Assume A is a fuzzy subset of X; the <u>support</u> of A is the crisp subset of X whose elements all have non-zero membership grades in A:

$$supp(A) = \{x \mid \mu_A(x) > 0 \text{ and } x \in X\}$$

[25 Marks]

Q2 b.2. Using the membership function for *tall* and *swift* compute the possibility of each candidate either becoming a basket ball player or a coxswain

height (meters)	**Speed (m/sec)**	**X is TALL**	**X is SWIFT**	**Basket ball Player**	**Rowing boat coxswain**
0.97	65	0.00	1.00	0.00	1.00
1.65	30	0.21	0.29	0.21	0.79
1.75	27	0.38	0.21	0.21	0.62
1.78	32	0.42	0.33	0.33	0.58
1.85	31	0.54	0.31	0.31	0.46
2.18	45	1.00	0.64	0.64	0.00
1.02	16	0.00	0.00	0.00	1.00

[35 Marks]

Q3. **This question is about fuzzy rule based systems.**

Q3 a. What are the major tasks executed by a fuzzy rule-based systems? Describe these tasks in your own words.

Fuzzification involves the choice of variables, fuzzy input and output variables and defuzzified output variable(s), definition of membership functions for the input variables and the description of fuzzy rules. The membership functions defined on the input variables are applied to their actual values to determine the degree of truth for each rule premise.

Inference: The truth-value for the premise of each rule is computed and the conclusion applied to each part of the rule. This results in one fuzzy subset assigned to each output variable for each rule. MIN and PRODUCT are two inference methods. In MIN inferencing the output membership function is clipped off at a height corresponding to the computed degree of truth of a rule's premise. In PRODUCT inferencing the output membership function is scaled by the premise's computed degree of truth.

Composition: All the fuzzy subsets assigned to each output variable are combined together to form a single fuzzy subset for each output variable. In MAX composition, the combined fuzzy subset is constructed by taking the pointwise maximum over all the fuzzy subsets assigned to the output variable by the inference rule. The SUM composition is constructed by taking the pointwise sum over all the fuzzy subsets assigned to output variable by their inference rule.

Defuzzification: The fuzzy value produced by the composition stage needs to be converted to a single number or a crisp value. Two popular defuzzification techniques. (1) The CENTROID technique relies on using the centre of gravity of the membership function to calculate the crisp value of the output variable. (2) The MAXIMUM techniques use one of the variable values at which the fuzzy subset has its maximum truth value to compute the crisp value.

[15 marks]

Q3 b. The Liffey Mortgage Company, dealing in ordinary and sub-prime mortgages, was asked by the financial regulators about how the Company conducts its business. The Chief Executive Officer of Liffey Mortgage summarised the following rules the company uses in making lending decisions:

(i) **IF** Salary is Excellent **OR** Debts are Small
THEN Risk is Low

(ii) **IF** Salary is Good **AND** Debts are Large
THEN Risk is Normal

(iii) **IF** Salary is Poor
THEN Risk is High

(iv) **IF** Salary is less than 2.5*Mortgage
THEN Risk is Medium

The membership functions for the linguistic variables *Salary* and *Debts* are given in Euros (€) and the *Risk* is in percentage – 0% for no risk and 100% for very risky loans:

$$\mu_{Salary}^{Excellent}(x) = 0, \forall x \le 90; \quad \mu_{Salary}^{Excellent}(x) = 1, x \ge 120;$$

$$\mu_{Salary}^{Good}(x) = 0, \forall x \le 50 \ \& \ \forall x \ge 100; \quad \mu_{Salary}^{Good}(x) = 1, x = 75;$$

$$\mu_{Salary}^{Poor}(x) = 0, \forall x \ge 60; \quad \mu_{Salary}^{Poor}(x) = 1, x \le 10.$$

$$\mu_{Debt}^{Small}(x) = 0, \forall x \ge 50; \quad \mu_{Debt}^{Small}(x) = 1, x \le 10;$$

$$\mu_{Debt}^{L\arg e}(x) = 0, \forall x \le 15; \quad \mu_{Debt}^{L\arg e}(x) = 1, x \ge 60.$$

$$\mu_{Risk}^{Low}(x) = 0, \forall x \ge 40\%; \quad \mu_{Risk}^{Low}(x) = 1, x \le 20\%;$$

$$\mu_{Risk}^{Medium}(x) = 0, \forall x \le 20\% \ \& \ \forall x \ge 80\%; \quad \mu_{Risk}^{Medium}(x) = 1, 40\% \le x \le 60\%;$$

$$\mu_{Risk}^{High}(x) = 0, \forall x \le 60\%; \quad \mu_{Risk}^{High}(x) = 1, x \ge 80\%;$$

Q 3 b.1. Jim has applied for a € 200K mortgage for a penthouse apartment in Dublin: his salary is €140K and his debts amount to €60K. Use the rule base to calculate the risk associated with Jim. Your calculation should show the four tasks associated with the operation of a fuzzy rule-based system.

Answer 3 b.1 The salary is very large so there will be only one rule active; the large debt appears not to have any effect.

FUZZIFICATION:

Salary: 140 K Euros

$\mu_{Excellent}(140) = 1$

$\mu_{Good}(140) = 0$

$\mu_{Poor}(140) = 0$

Debts: 60k Euros

$\mu_{Small}(60K) = 0$

$\mu_{Large}(60K) = 1$

Question 3b.1 (continued)

INFERENCE:

Risk Evaluation

R1: $\mu_{\text{Salary}}^{\text{Excellent}}(x)$ **or** $\mu_{\text{Debts}}^{\text{Small}} = \max\left[\mu_{\text{Salary}}^{\text{Excellent}}, \mu_{\text{Debts}}^{\text{Small}}\right]$

$\mu_{\text{Risk}}^{\text{Low}} = \max[1, 0] = 1$

R2: $\mu_{\text{Salary}}^{\text{Good}}(x)$ **AND** $\mu_{\text{Debt}}^{\text{Large}}(x) = \min\left[\mu_{\text{Salary}}^{\text{Good}}, \mu_{\text{Debt}}^{\text{Large}}\right]$

$\mu_{\text{Risk}}^{\text{Normal}} = \min[0, 1] = 0$

R3: $\mu_{\text{Salary}}^{\text{Poor}} = 0$

ONLY ONE RULE (*R1*) WILL FIRE.

AGGREGATION OR COMPOSITION

$\mu_{Risk}^{Low}(x) = 0, \forall x \geq 40\%$; $\mu_{Risk}^{Low}(x) = 1, x \leq 20\%$;
$\mu_{Risk}^{Medium}(x) = 0, \forall x \leq 20\% \,\&\, \forall x \geq 80\%$; $\mu_{Risk}^{Medium}(x) = 1, 40\% \leq x \leq 60\%$;
$\mu_{Risk}^{High}(x) = 0, \forall x \leq 60\%$; $\mu_{Risk}^{High}(x) = 1, x \geq 80\%$;

[50 MARKS]

Answer 3 b.2

DEFUZZIFICATION:

In Mean of Maxima (MOM) Method the weighted sum and weighted membership are worked out, except that the membership function is given another alpha level cut corresponding to the maximum value of the output fuzzy set. The crisp value for MOM method is given as:

$$\eta = \frac{\sum\limits_{x \equiv Max(\mu Risk)} \mu_{Risk}(x) * x}{\sum\limits_{x \equiv Max(\mu Risk)} \mu_{Risk}(x)}$$

From Rules 1, 2 and 3 we have inferred the following values of risk:

Rule 1: μ=1

Rule 2: μ=0

Rule 3: μ=0

For the mean-of-maxima method we will use only those membership function values that are greater than or equal to 0.4:

$$\eta = \frac{(0+10+20)*1}{3 \times 1}\%$$

$$\eta = \frac{30}{3} = 10\%$$

The more widely used centre of area (or gravity) method requires a weighted average over all the values of the variables x, rather than just those where the membership function is the highest:

$$COG = \frac{\sum\limits_{x=0\%}^{100\%} \mu_{Risk}(x) x}{\sum\limits_{x=0\%}^{100\%} \mu_{Risk}}$$

$$= \frac{(0+10+20) \times 1 + 0.5 * 30}{(1+1+1+0.5)}\% = \frac{45}{3.5} = 12.857$$

The student is expected to comment that the COG method usually generates a higher value than the equivalent MoM method. The higher value, greater caution when risks are being calculated, is often traded for speed, as MOM is quicker to compute than COG. There is no clear motivation for selecting either methods.

[35 Marks]

[60 Marks]

Q 3 b.2. There are two methods which could be used in the *defuzzification* task: *centre-of-area* and *mean-of-maxima*. Justify the use of the method you have used in your calculation.

[25Marks]

Q4. **This question is about fuzzy control systems**

Noise pollution in major conurbations leads to a general degradation of quality of life. Fichera and colleagues have simulated noise pollution in a small town in Italy using fuzzy rule based systems. They have found that the noise level can be predicted using:

(i) The average number of vehicles per hour was computed as an aggregate of a variety of vehicles (cars, motor bikes, and heavy goods vehicles) per hour:

$$n_{eq} = n_{cars} + 3 * n_{motor\_bikes} + 6 * n_{heavy\_goods\_vehicles}$$

(ii) The so-called *equivalent continuous noise level (L_{AeqT})* is defined as a function of the number of vehicles per hour (n_{eq}), the average height $\hat{h}$, of the buildings along a road that has an average width $\hat{w}$:

$$L_{AeqT} = f(n_{eq}, \hat{h}, \hat{w}).$$

Fichera et al defined that the term set n_{eq} as comprising the linguistic variables *small* and *large*. They assume that the number of equivalent vehicles is definitely *small* if the vehicle count is less than or equal to 923 and the n_{eq} is definitely not *small* if the number is over 10489. Contrarily perhaps n_{eq} is definitely *large* if the number of equivalent vehicles is greater than or equal to 8944; if the vehicle count falls below 924 then the number of equivalent vehicles is definitely not *large*.

The height of the building term set *h* comprises *tall* and *low* buildings. A building that is below 12.7 metres is definitely a *low* building and it is definitely not *low* if the height is greater than 31.88 meters. Contrariwise, a *tall* building should measure more than 12.44 meters and a building with a height greater than or equal to 34.49 meters is definitely a *tall* building. The fuzzy rule set for computing the noise level is given as:

$$R_1 : IF\ n_{eq} is\ small\ \&\ h\ is\ low\ THEN\ L_{eq} = -148.6 - 0.087 n_{eq} + 24.38h + 0.24w$$
$$R_2 : IF\ n_{eq} is\ small\ \&\ h\ is\ tall\ THEN\ L_{eq} = -894.2 + 0.087 n_{eq} + 26.53h - 0.09w$$
$$R_3 : IF\ n_{eq} is\ l\arg e\ \&\ h\ is\ low\ THEN\ L_{eq} = 1180 - 0.071 n_{eq} - 26.99h - 0.61w$$
$$R_4 : IF\ n_{eq} is\ l\arg e\ \&\ h\ is\ tall\ THEN\ L_{eq} = 413.99 + 0.072 n_{eq} - 30.97h - 1.99w$$

Q4 a. Derive the membership functions for the linguistic variables *small, large, tall* and *low* from the description given above.

Answer 4a The membership functions are as follows:

small:

$$\mu(x) = 1\ \ if\ n_{eq} \le 923$$
$$\mu(x) = 0\ \ if\ n_{eq} \ge 10489$$
$$\mu(x) = \frac{10489 - x}{9566}\ \ if\ 923 \le n_{eq} \le 10489$$

large:

$$\mu(x) = 1\ \ if\ n_{eq} \ge 8944$$
$$\mu(x) = 0\ \ if\ n_{eq} \le 924$$
$$\mu(x) = \frac{x - 924}{8020}\ \ if\ 924 \le n_{eq} \le 8944$$

tall

$$\mu(x) = 1\ \ if\ h \ge 34.49$$
$$\mu(x) = 0\ \ if\ h \le 12.44$$
$$\mu(x) = \frac{34.49 - x}{22.05}\ \ if\ 12.44 \le h \le 34.49$$

low

$\mu(x) = 1 \;\; if\; h \leq 12.7$

$\mu(x) = 0 \;\; if\; h \geq 31.88$

$\mu(x) = \dfrac{x - 12.7}{19.18} \;\; if\; 12.7 \leq h \leq 31.88$

[20 Marks]

Q4 b. Consider a traffic observation study where it was found that on a road with a an average width of 30 metres, traffic varied during the day from a peak 7800 vehicles per hour at 0900 hours and 1700 hours to a low of 900 vehicles per hour at 0700 hours.

Time	0700	0900	1100	1300	1500	1700	1900
No. of Vehicles per hour	900	7800	5000	4700	5000	7800	3800

There were 10 buildings along the route with heights ranging from 10 meters to 18 meters:

{10, 18, 20, 12, 15, 14, 16, 12, 15.5, 18, 10}.

Compute the average equivalent noise level L_{AeqT} based on equivalent noise levels L_{eq} from the following observations using a fuzzy control system. Clearly show all the stages of computation: fuzzification, composition and inference for these observations.

The average number of vehicles observed in one hour was 5000 on a road with an average width of 30 metres and buildings along the way have an average height of 15 metres.

Answer 4b .Fuzzification: Neq =5000; h=15

Term	**Linguistic Variable**	**Membership Value**
Neq	**Small (Neq)**	**0.57**

	Large (Neq)	**0.51**
h	**Tall (h)**	**0.12**
	Low (h)	**0.88**

Inference + Composition

Neq=5000; h=15.0 M, w=30.0M

	Leq	**Union Neq & *h***	**Weight**	**Product**
Rule 1	**-210.7**	**Small +Low**	**0.57**	**-120.9**
Rule 2	**-63.95**	**Small+Tall**	**0.12**	**-7.669**
Rule 3	**401.85**	**Large+Low**	**0.51**	**204.23**
Rule 4	**249.74**	**Large+Tall**	**0.12**	**29.948**
SUM			**1.32**	**105.61**

OUTPUT:

LAeqt = 105.61/1.32= 79.9

Q4 c. Could you have carried out this task using Mamdani's formulation of a fuzzy control system?

This situation can be modelled by Mamdani model as well provided we could define the fuzzy sets for L_{eq} together with fuzzy sets for the width of the road (w). A new rule set has to be derived as well.

[80 marks]

Q5. **This question is about neuro-fuzzy systems**

The Speculative Triad Inc., employs three analysts, Gordon, B., Angela, M., and Brian, C., for recommending either to hold or sell shares. All three analyze share trading on the Irish Stock Exchange and relate their analysis to the price of copper, and then either give a hold signal or a sell signal. Now Grodon is an inveterate risk taker and Angela is his complete opposite, and Brian has his own unique way giving a buy or hold signal. An aggregate of their behaviour is shown in the table below:

Irish Stock Exchange Index	Price of a tonne of Copper	Brown Recommends	Angela Suggests	Brian Argues
Down	Down	Hold	Hold	Hold
Down	Up	Sell	Hold	Sell
Up	Down	Sell	Hold	Sell
Up	Up	Sell	Sell	Hold

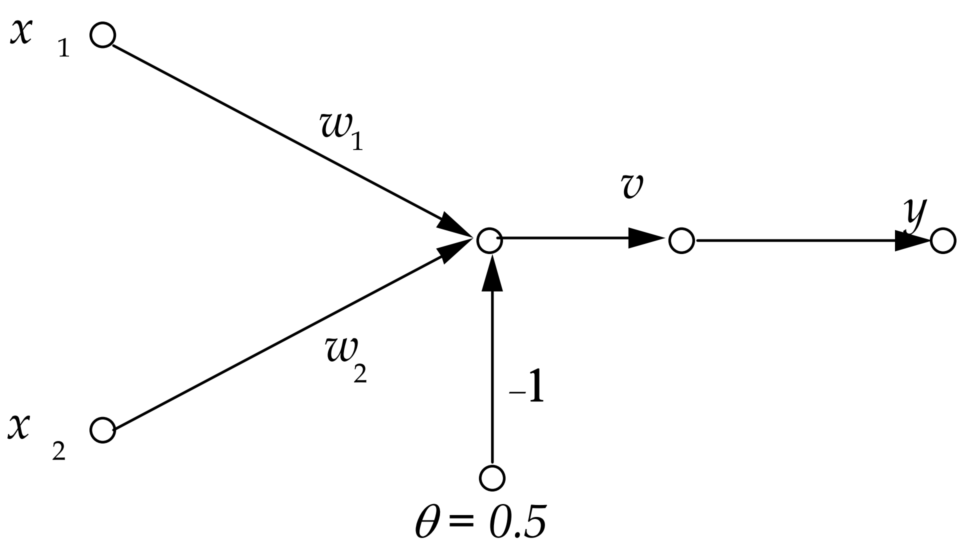
5(a) You have been asked to design a neural-network based system to learn the behaviour of the three analysts, Gordon, Angela and Brian. Can you recommend a single layer perceptron as a neural network system for learning the behaviour of all three analysts? Give at least two reasons to support your answer.

Answer 5 a. By assuming that UP=1 and DOWN=0, and HOLD=0 and SELL=1, the student should show the following decision table:

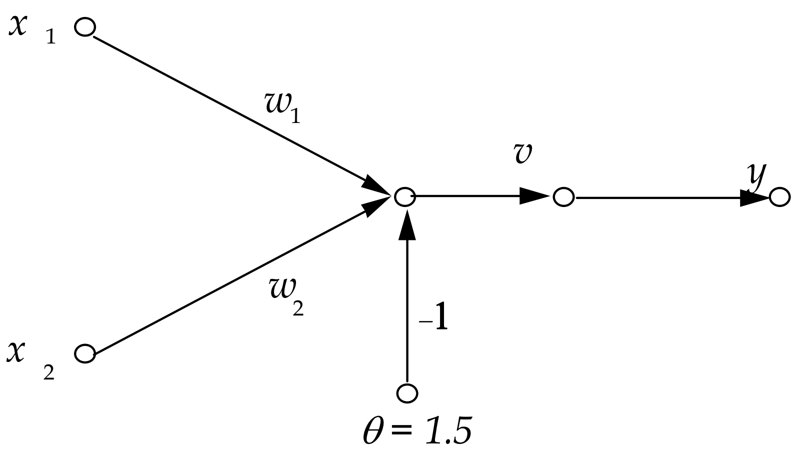
Irish Stock Exchange Index	Price of a tonne of Copper	Brown Recommends (OR)	Angela Suggests (AND)	Brian Argues (XOR)
0	0	0	0	0
0	1	1	0	1
1	0	1	0	1
1	1	1	1	0

And one can see that Brown's behaviour can be simulated using an AND network anbd Angela's by an OR network. Brian's behaviour follows that of an XOR network. Brown and Angel's behaviour can be simulated using a single layer network but Brian's behaviour requires a multi-layer perceptron:

Brown's Behaviour: $w_1= w_2=1$



Angela's Behaviour: $w_1= w_2=1$



[60 Marks]

5 (b) Consider the rule base for a fuzzy logic-circuit with two inputs (X_1 and X_2) and one output (Y):

	X1
X2	Y

The inputs can be about zero or nearly unity, and the output can be exactly zero or unity, according to the following rule base:

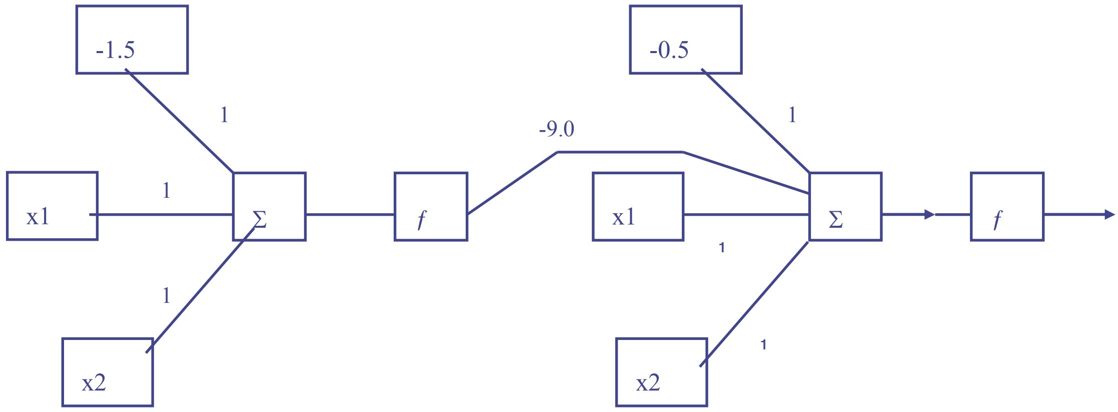
	I N P U T X_1		
		About Zero	Nearly Unity
	About Zero	0	0
INPUT X_2	Nearly Unity	0	1

Describe the architecture of an adaptive neuro-fuzzy system that can learn the behaviour described in the above rule base.

Illustrate your answer with neural network that can solve the above problem and clearly indicate the various inference processes the nodes of the adaptive will perform.

Answer 5b
XOR gate cannot be modelled by simple perceptron since it doesn't belong to the set of linearly separable problems. Single perceptron can represent a line in an input space where all inputs below this line have one output value and all inputs above this line have another value. Since input space of XOR problem cannot be divided by line to get all inputs related to 0 outputs on the one side and all inputs related to 1 outputs to the other side of the line, single perceptron can never model a XOR gate.

In order to model XOR gate we need to design a neuro-fuzzy system with one hidden layer. In this system, first layer would comprise inputs x1 and x2 and a bias. Two input nodes together with bias would all be connected to one neuron in the hidden layer. Another neurons in the hidden layer would be the bias and original inputs. All neurons from the hidden layer are connected to the one neuron in the output layer.



[40 Marks]