UNIVERSITY OF DUBLIN

TRINITY COLLEGE

School of Computer Science and Statistics
Department of Computer Science

BA Mod. (Computer Science) Trinity Term 2009
SS Examination

4BA13 Fuzzy Logic and Fuzzy Control Systems

Date Location Time: 3 Hours

Prof. Khurshid Ahmad

Instructions

(i) A total of THREE questions out of FIVE must be answered for full marks.

(ii) All questions carry equal marks.

(iii) Use of non-programmable calculators and log tables is permitted.
You must note the make and model of your calculator on your answer book.

Q1. **This question is about *fuzzy sets***.

P. J. Macvicar-Whelan reported in 1978 that he had conducted a "experimental and theoretical study of the categorization of human height". Macvicar-Whelan reported that: "Subjects of both sexes whose ages ranged from 6 to 72 were asked to class the height of both men and women using the labels VERY VERY SHORT [VVS], VERY SHORT [VS], SHORT [S], TALL [T], VERY TALL [VT], and VERY VERY TALL [VVT].

Here are two tables summarising the perceptions of Macvicar-Whelan's subjects:

Perceptions about men's height (numbers denote height in cms):

	Short			Tall		
	V V S	V S	S	T	V T	V V T
Definitely not	148	157	172	167	176	182
Possibly	137	143	157	179	189	198
Definitely	126	129	142	191	202	214

Perceptions about women's height (numbers denote height in cms):

	Short			Tall		
	V V S	V S	S	T	V T	V V T
Definitely not	153	155	161	160	170	174
Possibly	135	143	149	173	181	191
Definitely	117	131	137	186	192	208

Q1 a. Construct fuzzy membership functions (MF) for the fuzzy variable *height* including its terms *short* and *tall* for observation relating to both *men* and *women.* Hint: MFs are piece-wise linear.

[36 Marks]

Q1 b. Starting from the fuzzy membership of, say, *short* or *tall*, you can derive the membership functions for *very, very short* and *very short,* OR *very, very tall* and *very tall*. Perform this derivation for the men's domain and the women's domain.

[24 Marks]

Q1 c. Compare and contrast the membership functions you have obtained from your derivations for *short* or *tall*, with the membership function that can be computed from the perception tables above.

[20 Marks]

Q1 d. In what respect your results support or refute Macvicar-Whelan's observation that 'the hedge VERY causes a shift of the class frontier rather than a steepening of the membership function'.

[20 Marks]

Q2. **This question is about *aggregation operators.***

The general class of *AND* operators used in fuzzy modelling and control are the *t-norm* operators ***T***

$$T:[0,1]\mathbf{x}[0,1]\rightarrow [0,1]$$

The *t-norm* operators are *commutative, associative, non-decreasing* and have *neutral element* 1;

Q2 a. Assume truth values {0, ½, 1}. Tabulate the *t-norms* that can be identified using the definition of the operator *T* in the preamble above.

[30 Marks]

Q2 b. Let $\{T_i\}$ for i = 1,...,n be a set of continuous *t-norms*. An *ordinal sum* $T = (<T_i, a_i , e_i >)_i$ is a continuous *t-norm* which is on $[a_i,e_i]^2$ equal to the linear transformation of the *t-norm* T_i (defined on interval [0,1]) to interval $[a_i,e_i]$; and it is equal to the minimum *t-norm* outside of $\bigcup_{i=1}^{n}[a_i,e_i]^2$.

Derive the formula for *ordinal sum* $(<T_P,0, ¼>,< T_L,¾,1>)$.

[30 Marks]

Q2 c. *Cubic mean* is a *quasi-arithmetic mean* generated by $f(x) = x^3$.

2c (i) Describe a procedure that can be used to compute the *cubic mean* of 0.2458, 0.2458 and 0.2458 without using a calculator? [20 Marks]

2c (ii) From four numbers that should be aggregated by a *cubic mean* only one, 0.9, is known. The *cubic mean* of the other three is 0.5. Compute the *cubic mean* of all four numbers.

[20 Marks]

Q3. **This question is about fuzzy rule based systems.**

Gordon Motormouth has devised a new points based system for gauging the reaction of his customers who come to his restaurants. In this system, zero is the indication of the worst service and 10 of superb service. Customers have been asked to award a zero if they found food stale or rancid and 10 if they found food quality as grand. You will recall that Gordon's restaurant include 10% as the so-called 'service charges' or tips in the bill automatically.

Gordon has decided to build an expert system to automate the distribution of tips. He will use three rules of thumb to decide whether to give his workers a small share of the tip, ordinary share of the tip or a generous share of the tip:

IF the SERVICE is *POOR* AND FOOD is *RANCID* THEN TIP is SMALL

IF the SERVICE is *AVERAGE* AND FOOD is *GOOD* THEN TIP is ORDINARY

IF the SERVICE is *SUPERB* AND FOOD is GRAND THEN TIP is *GENEROUS*

Q3 a. If you believe that Gordon needs a fuzzy knowledge based system, then give at least two reasons that will support your decision. If you do not believe that Gordon needs a fuzzy system then give at least two reasons to support your answer.

[20 Marks]

Q3 b. Describe the linguistic variables and associated term sets required to build a fuzzy system that will help Gordon automate the distribution of tips.

[10 Marks]

Q3 c. Gordon is always in a hurry and we could discern few facts about what he means by *poor service*, *rancid food*, and *small tip.* He has left you with the job of finding the membership functions for all the terms that comprise the linguistic variables of Gordon's domain. Here are some hints for computing the membership functions:

The service is definitely poor, if the customer rating is less than or equal to 2, and it is definitely not poor if the customer rating is greater than or equal to 8. Similarly, the food is definitely rancid if the rating is 2 and definitely not rancid if the rating is 8 or more.

The tip has already been collected from the customer as service charge. Now, the worker will have a *small* tip if they receive a maximum of 1/3 of the service charge. If the worker receives more than 2/3 of the tip, then the tip is definitely *not small*.

Assume that *superb service*, *grand food* and *generous tip* can be modeled as the complement of *poor service*, *rancid food* and *small tip,* respectively.

The case of *average service* is a bit more complicated. We suggest that you assume the use of the special case of the bounded sum and use the following:

$$\mu_{average} = \min(1, 2 * \min(\mu_{poor}, \mu_{superb})),$$

where μ_{poor} is the membership function of the *poor-service* term, μ_{superb} is the membership function of the *superb-service* term. Use a similar strategy of using other two membership functions to define *good food* and *ordinary tip*.

[40 Marks]

Q3 d. Using the membership functions and the rule base in the preamble of Question 3 above, compute the tip to be given to a waiter when a customer gave 3 marks for service and 7 for the quality of food. Show the four stages of the computation clearly.

[30 Marks]

Q4. **This question is about fuzzy control systems**

'Feedback control' is defined as a mechanism for guiding or regulating the operation of a system or subsystems by returning to the input of the (sub)system a fraction of the output. The output of a control system *u(t)* can be related intuitively to the error signal e*(t)* by

$$u(t) = K_P\, e(t) + K_I \int_0^t e(\tau)\, d\tau + K_D \frac{de(t)}{dt}$$

where K_P, K_I *and* K_D are constants of proportionality related respectively to the current errors, sum of the recent errors, and the rate at which the error has been changing.

Q4 a. Describe in your words the difference between a classical operation of process control and a fuzzy control operation.

[20 Marks]

Q4 b. The design of a simple (fuzzy) air-conditioner can be effected through the operation of the following observations: Consider a rather vague customer, John Smith, for our fuzzy air-conditioner.

John thinks that it is definitely cold when the temperature falls below 10°C, the cold sets in at around 12.5°C, but sometimes he feels cold even at 15°C. He definitely feels hot when the temperature shoots above 27°C, but it can be mildly hot for him at temperatures up to 20°C. For John, the ambient temperature is just right when the temperature is 18°C; and the just right feeling starts to disappear when the temperature is within 4°C of the just right temperature.

When asked about the temperature ranges which may be considered cool and warm, as opposed to cold and hot respectively, the customer suggested that the feeling cool overlaps with the feeling cold and feeling just right and peaks at 15°C. John Smith thinks that it is positively warm at 25.5°C; he said that 20°C wasn't really warm neither was 29°C.

The engineer who designed a fuzzy air-conditioner related John's feelings of 'hot' and 'cold' to the speed of the air-conditioner's motor. The air-conditioner will *blast* cold air at 100 revolutions per minute (r.p.m.) when John feels hot, when it is cold then the system will operate at *minimal* (zero r.p.m.); for cool ambience the motor speed should be slow (30 r.p.m), for warm temperatures the speed will be *fast* (75 r.p.m). Finally, for *pleasant* ambient temperatures, the speed will be medium (50 r.p.m).

4b (i) Describe the fuzzy air-conditioner in terms of the linguistic variables, and the corresponding fuzzy sets for the variables.

[15 Marks]

(ii) Create a rule base from the description of John's feeling for *hot* and *cold* and the engineer's fuzzy control system above.

[20 Marks]

(iii) Would you be using a Takagi-Sugeno controller or a Mamdani controller for implementing your rule base? Give at least two reasons for your choice.

[15 Marks]

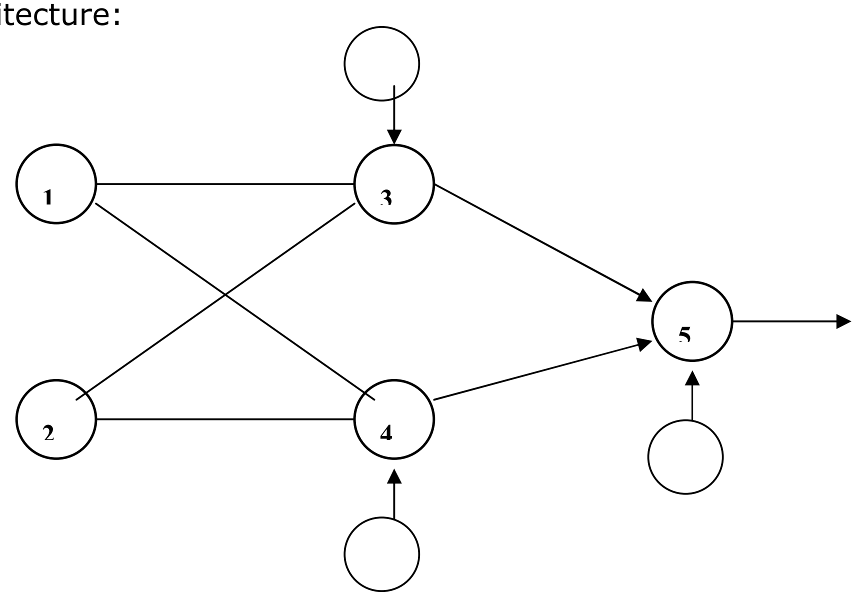
(iv). Compute the output of the air-conditioner on quite cold day with temperatures averaging at 12°C.

Describe the fuzzification, inference, composition and/or defuzzification stages of your computation.

[30 Marks]

Q5. **This question is about neuro-fuzzy systems**

Q5 a. Consider a multi-layer feed forward network with a 2-2-1 architecture:



We have $w_{03} = 0.1$; $w_{04} = 0.1$; and, $w_{05} = 0.2$, and the weight vectors are

$$\begin{bmatrix} w13 & w23 \\ w14 & w24 \end{bmatrix} = \begin{bmatrix} 0.2 & 0.1 \\ 0.1 & 0.3 \end{bmatrix};$$

and $[w_{35} \quad w_{45}] = [\,0.2 \quad 0.3]$

Train the network with a vector, $x^T = [0.1 \quad 0.9]$, with a desired output *d=0.9,* for one cycle. You may use the following formulas for computing the net input to a neuron (i), the sigmoid function (ii), the gradient for the output neuron (iii), and the gradients for the hidden neurons.

$$(i) \quad net_j = w_{0j} + \sum_{i=1}^{n} w_{ij} x_i$$

$$(ii) \quad o_j = \frac{1}{1 + e^{-net\,j}}$$

$$(iii) \; \delta_j = (d_j - o_j) o_j (1 - o_j)$$

$$(iv) \; \delta_j = (1 - o_j) o_j \sum_{k=1}^{m} \delta_k w_{kj}$$

[60 Marks]

Q5 b. Describe in your own words how a Takagi-Sugeno first order fuzzy control system can be described as a network that can be trained using the backpropagation algorithm. Illustrate your answer with a network diagram.

[25 Marks]

Q5 c. What are the key steps in training an adaptive neuro-fuzzy inference system in the forward pass and the backward pass.

[15 Marks]