BA Mod. (Computer Science) Trinity Term 2009
SS Examination

Solutions to
4BA13 Fuzzy Logic and Fuzzy Control Systems

Prof. Khurshid Ahmad

Q1. **This question is about *fuzzy sets*.**

1a: Fuzzy membership functions for fuzzy sets *short* and *tall* for men are given by

$$\mu_{A^{short}}(x) = \begin{cases} 1 & if \quad x \le 142 \\ \dfrac{172-x}{30} & if \quad 142 < x < 172 \\ 0 & if \quad 172 \le x \end{cases} \qquad \mu_{A^{tall}}(x) = \begin{cases} 0 & if \quad x \le 167 \\ \dfrac{x-167}{24} & if \quad 167 < x < 191 \\ 1 & if \quad 191 \le x \end{cases}$$

and fuzzy membership functions for fuzzy sets *short* and *tall* for women are given by

$$\mu_{B^{short}}(x) = \begin{cases} 1 & if \quad x \le 137 \\ \dfrac{161-x}{24} & if \quad 137 < x < 161 \\ 0 & if \quad 161 \le x \end{cases} \qquad \mu_{B^{tall}}(x) = \begin{cases} 0 & if \quad x \le 160 \\ \dfrac{x-160}{26} & if \quad 160 < x < 186 \\ 1 & if \quad 186 \le x \end{cases}$$

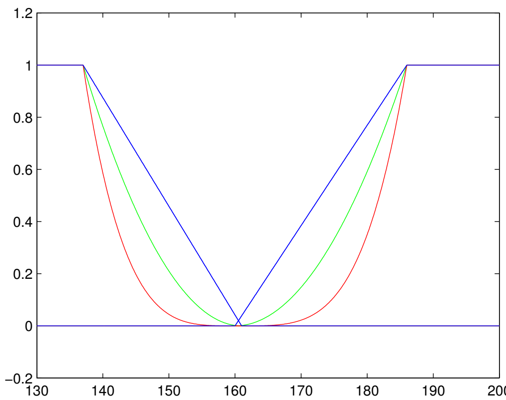
1b: Linguistic hedge *very* changes the membership function of a fuzzy set to its square. Since $0^2=0$ and $1^2=1$, the derived fuzzy sets have the following membership functions:

$$\mu_{A^{very\ short}}(x) = \begin{cases} 1 & if \quad x \le 142 \\ \left(\dfrac{172-x}{30}\right)^2 & if \quad 142 < x < 172 \\ 0 & if \quad 172 \le x \end{cases} \qquad \mu_{A^{very\ tall}}(x) = \begin{cases} 0 & if \quad x \le 167 \\ \left(\dfrac{x-167}{24}\right)^2 & if \quad 167 < x < 191 \\ 1 & if \quad 191 \le x \end{cases}$$

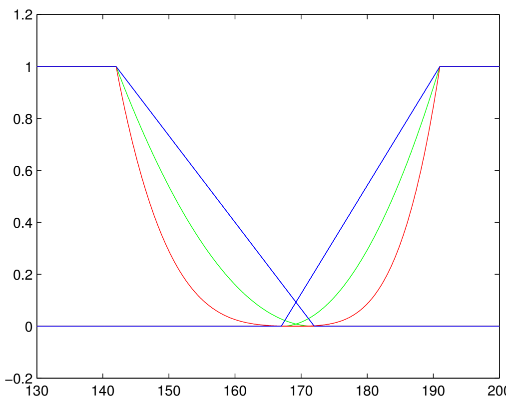
$$\mu_{A^{very\ very\ short}}(x) = \begin{cases} 1 & if \quad x \le 142 \\ \left(\dfrac{172-x}{30}\right)^4 & if \quad 142 < x < 172 \\ 0 & if \quad 172 \le x \end{cases} \qquad \mu_{A^{very\ very\ tall}}(x) = \begin{cases} 0 & if \quad x \le 167 \\ \left(\dfrac{x-167}{24}\right)^4 & if \quad 167 < x < 191 \\ 1 & if \quad 191 \le x \end{cases}$$

$$\mu_{B^{very\ short}}(x) = \begin{cases} 1 & if \quad x \le 137 \\ \left(\dfrac{161-x}{24}\right)^2 & if \quad 137 < x < 161 \\ 0 & if \quad 161 \le x \end{cases} \qquad \mu_{B^{very\ tall}}(x) = \begin{cases} 0 & if \quad x \le 160 \\ \left(\dfrac{x-160}{26}\right)^2 & if \quad 160 < x < 186 \\ 1 & if \quad 186 \le x \end{cases}$$

$$\mu_{B^{very\ very\ short}}(x) = \begin{cases} 1 & if \quad x \le 137 \\ \left(\dfrac{161-x}{24}\right)^4 & if \quad 137 < x < 161 \\ 0 & if \quad 161 \le x \end{cases} \qquad \mu_{B^{very\ very\ tall}}(x) = \begin{cases} 0 & if \quad x \le 160 \\ \left(\dfrac{x-160}{26}\right)^4 & if \quad 160 < x < 186 \\ 1 & if \quad 186 \le x \end{cases}$$



Women: blue – short(tall); green – very s(t); red – very very s(t)



Men: blue – short(tall); green – very s(t); red – very very s(t)

1c: Membership function for fuzzy sets *very short* (*tall*) and *very very short* (*tall*) based on the values given in above tables appear to be linear for both men's and women's domain. However, this is not the case for fuzzy sets obtained by means of linguistic hedge *very*. If we compare the values of these sets introduced in above tables with the derived function in the same points we get:

table	$\mu_{A_{very\ very\ short}}(126) = 1$	$\mu_{A_{very\ very\ short}}(137) = 0.5$	$\mu_{A_{very\ very\ short}}(148) = 0$
derived	$\mu_{A_{very\ very\ short}}(126) = 1$	$\mu_{A_{very\ very\ short}}(137) = 1$	$\mu_{A_{very\ very\ short}}(148) = 0.401$
table	$\mu_{A_{very\ very\ tallt}}(182) = 0$	$\mu_{A_{very\ very\ tallt}}(198) = 0.5$	$\mu_{A_{very\ very\ tallt}}(214) = 1$
derived	$\mu_{A_{very\ very\ tallt}}(182) = 0.1526$	$\mu_{A_{very\ very\ tallt}}(198) = 1$	$\mu_{A_{very\ very\ tallt}}(214) = 1$
table	$\mu_{A_{very\ short}}(129) = 1$	$\mu_{A_{very\ short}}(143) = 0.5$	$\mu_{A_{very\ short}}(157) = 0$
derived	$\mu_{A_{very\ short}}(129) = 1$	$\mu_{A_{very\ short}}(143) = 0.934$	$\mu_{A_{very\ short}}(157) = 0.25$
table	$\mu_{A_{very\ tallt}}(176) = 0$	$\mu_{A_{very\ tallt}}(189) = 0.5$	$\mu_{A_{very\ tallt}}(202) = 1$
derived	$\mu_{A_{very\ tallt}}(176) = 0.1406$	$\mu_{A_{very\ tallt}}(189) = 0.840$	$\mu_{A_{very\ tallt}}(202) = 1$

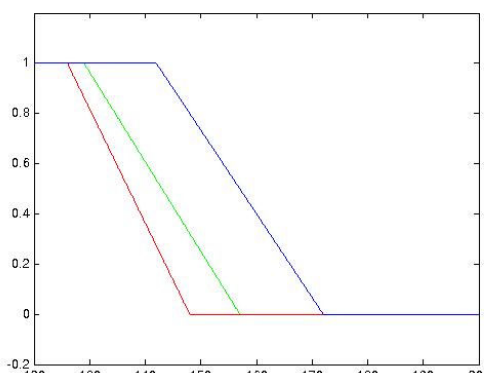
$$
\begin{array}{llll}
table & \mu_{B^{very\ very\ short}}(117)=1 & \mu_{B^{very\ very\ short}}(135)=0.5 & \mu_{B^{very\ very\ short}}(153)=0 \\
derived & \mu_{B^{very\ very\ short}}(117)=1 & \mu_{B^{very\ very\ short}}(135)=1 & \mu_{B^{very\ very\ short}}(153)=0.0123 \\
table & \mu_{B^{very\ very\ tallt}}(174)=0 & \mu_{B^{very\ very\ tallt}}(191)=0.5 & \mu_{B^{very\ very\ tallt}}(208)=1 \\
derived & \mu_{B^{very\ very\ tallt}}(174)=0.084 & \mu_{B^{very\ very\ tallt}}(191)=1 & \mu_{B^{very\ very\ tallt}}(208)=1
\end{array}
$$

$$
\begin{array}{llll}
table & \mu_{B^{very\ short}}(131)=1 & \mu_{B^{very\ short}}(143)=0.5 & \mu_{B^{very\ short}}(155)=0 \\
derived & \mu_{B^{very\ short}}(131)=1 & \mu_{B^{very\ short}}(143)=0.562 & \mu_{B^{very\ short}}(155)=0.062 \\
table & \mu_{B\ ^{very\ tallt}}(170)=0 & \mu_{B\ ^{very\ tallt}}(181)=0.5 & \mu_{B\ ^{very\ tallt}}(192)=1 \\
derived & \mu_{B\ ^{very\ tallt}}(170)=0.148 & \mu_{B\ ^{very\ tallt}}(181)=0.652 & \mu_{b\ ^{very\ tallt}}(192)=1
\end{array}
$$

From these values it can be seen that listed fuzzy sets have different shapes.

1d:

If the hedge *very* would steepen the membership functions the points with membership values 0 and 1 would not change the membership (since $0^p=0$ and $1^p=1$ for all p>0). However, by the comparison of values from 1c) we can see that this is not the case. The values indicate that in men's (women's) domain the fuzzy sets *short*, *very short* and *very very short* (*tall*, *very tall* and *very very tall*) can be modelled by the following piece-wise linear functions (see figure).



Men tabular: blue - short; green - very s.; red - very very s.

Q2 **This question is about aggregation operations on fuzzy sets**.

2a:
Properties of a t-norm imply the following equalities:
T(1,1) = 1, T(1, ½) = T(½, 1) = ½, T(1,0) = T(0,1) =0,
0 = T(1,0) ≥ T(½,0) ≥ T(0,0) ≥ 0
0 = T(0,1) ≥ T(0,½) ≥ T(0,0) ≥ 0. The only value which is not determined uniquely is T(½,½) . However, non-decreasingness of t-norms gives ½ = T(½, 1) ≥ T(½,½) ≥ T(½,0) =0. Therefore there are only two possibilities corresponding to two t-norms. The two t-norms corresponding to values T(½,½) = ½ and T(½,½) =0 are:

T_1

1	0	½	1
½	0	½	½
0	0	0	0
	0	½	1

T_2

1	0	½	1
½	0	0	½
0	0	0	0
	0	½	1

corresponding to minimum and Lukasiewicz t-norm, respectively.

2b:
The linear transformation of the product T_P (defined on [0,1]) to the interval [0, ¼] is given by

$$f^{-1}(T_p(f(x), f(y))) = \frac{4x \cdot 4y}{4} = 4 \cdot x \cdot y$$

where the corresponding linear transformation is f(x) = 4x.

The linear transformation of the Lukasiewicz t-norm T_L (defined on [0,1]) to the interval [¾,1] is given by

$$f^{-1}(T_L(f(x), f(y))) = \frac{\max(0, 4x - 3 + 4y - 3 - 1) + 3}{4} = \max(\frac{3}{4}, x + y - 1)$$

where the corresponding linear transformation is f(x) = 4x-3.

Therefore T= (<T_P,0, ¼>,< T_L,¾,1>) is given by

$$T(x,y) = \begin{cases} 4xy & if\,(x,y) \in [0,\frac{1}{4}]^2, \\ \max(\frac{3}{4}, x+y-1) & if\,(x,y) \in [\frac{3}{4},1]^2, \\ \min(x,y) & else. \end{cases}$$

2c:

(i) *Quasi-arithmetic* means are idempotent and therefore *cubic mean* of inputs with the same value (x,...,x) is x. Thus *cubic mean* of 0.2458, 0.2458 and 0.2458 is just 0.2458. Alternatively, the following computation can be done:

$$\sqrt[3]{\frac{0.2458^3 + 0.2458^3 + 0.2458^3}{3}} = \sqrt[3]{\frac{3 \cdot 0.2458^3}{3}} = \sqrt[3]{0.2458^3} = 0.2458.$$

(ii) Since *cubic mean* of the first three numbers is 0.5 we have $\sqrt[3]{\frac{x_1^{\,3} + x_2^{\,3} + x_3^{\,3}}{3}} = 0.5$ and $x_1^{\,3} + x_2^{\,3} + x_3^{\,3} = 3(0.5)^3 = 0.375$. Then the cubic mean of all four number is

$$\sqrt[3]{\frac{x_1^{\,3} + x_2^{\,3} + x_3^{\,3} + x_4^{\,3}}{4}} = \sqrt[3]{\frac{x_1^{\,3} + x_2^{\,3} + x_3^{\,3} + 0.9^3}{4}} = \sqrt[3]{\frac{0.375 + 0.9^3}{4}} = \sqrt[3]{\frac{0.375 + 0.729}{4}}$$

what is equal to 0.651.

Q3. **This question is about fuzzy rule based systems.**

3 a: Fuzzy logic enables us to define terms like poor, rancid, etc. as fuzzy concepts. It won't be easy define these terms by crisp sets. When fuzzy logic is employed, system exactly implements the intuitive rules given by Gordon. Therefore it is very good understandable although it is capable to model highly non-linear functions. On the other hand, rules given by Gordon indicate that the situation can be modelled also by a simple linear function, f.e. tip = a*food + b*service + c. However, in such a case the system works more like a black box.

3 b:
The term sets for service is {poor, average, superb} for food it is {rancid, good, grand} and for the output variable tip the term set is {small, ordinary, generous}.

3 c:

For input variable service we have

$$\mu_{A_{poor}}(x) = \begin{cases} 1 & if \quad x \le 2 \\ \frac{8-x}{6} & if \quad 2 < x < 8 \\ 0 & if \quad 8 \le x \end{cases}$$

$$\mu_{A_{superb}}(x) = 1 - \mu_{A_{poor}}(x) = \begin{cases} 0 & if \quad x \le 2 \\ \frac{x-2}{6} & if \quad 2 < x < 8 \\ 1 & if \quad 8 \le x \end{cases}$$

$$\min(\mu_{A_{poor}}(x), \mu_{A_{superb}}(x)) = \begin{cases} 0 & if \quad x \le 2 \\ \frac{x-2}{6} & if \quad 2 < x < 5 \\ \frac{8-x}{6} & if \quad 5 \le x < 8 \\ 0 & if \quad 8 \le x \end{cases}$$

$$\mu_{A_{average}}(x) = \min(1, 2 * \min(\mu_{A_{poor}}(x), \mu_{A_{superb}}(x))) =$$

$$= \begin{cases} 0 & if \quad x \le 2 \\ \frac{x-2}{3} & if \quad 2 < x < 5 \\ \frac{8-x}{3} & if \quad 5 \le x < 8 \\ 0 & if \quad 8 \le x \end{cases}$$

Similarly, for input variable food we have

$$\mu_{A_{rancid}}(x) = \begin{cases} 1 & if \quad x \le 2 \\ \dfrac{8-x}{6} & if \quad 2 < x < 8 \\ 0 & if \quad 8 \le x \end{cases}$$

$$\mu_{A_{grand}}(x) = 1 - \mu_{A_{rancid}}(x) = \begin{cases} 0 & if \quad x \le 2 \\ \dfrac{x-2}{6} & if \quad 2 < x < 8 \\ 1 & if \quad 8 \le x \end{cases}$$

$$\min(\mu_{A_{rancid}}(x), \mu_{A_{grand}}(x)) = \begin{cases} 0 & if \quad x \le 2 \\ \dfrac{x-2}{6} & if \quad 2 < x < 5 \\ \dfrac{8-x}{6} & if \quad 5 \le x < 8 \\ 0 & if \quad 8 \le x \end{cases}$$

$$\mu_{A_{good}}(x) = \min(1, 2 * \min(\mu_{A_{rancid}}(x), \mu_{A_{grand}}(x))) =$$

$$= \begin{cases} 0 & if \quad x \le 2 \\ \dfrac{x-2}{3} & if \quad 2 < x < 5 \\ \dfrac{8-x}{3} & if \quad 5 \le x < 8 \\ 0 & if \quad 8 \le x \end{cases}$$

For output variable tip we have

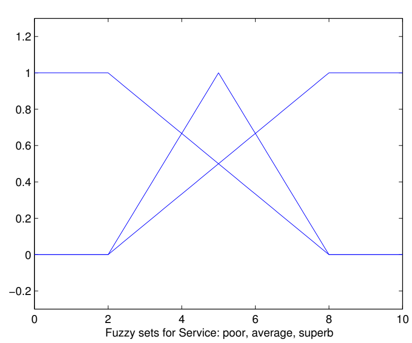
$$\mu_{A_{small}}(x) = \begin{cases} 1 & if \quad x \le \dfrac{1}{3} \\ 2 - 3x & if \quad \dfrac{1}{3} < x < \dfrac{2}{3} \\ 0 & if \quad \dfrac{2}{3} \le x \end{cases}$$

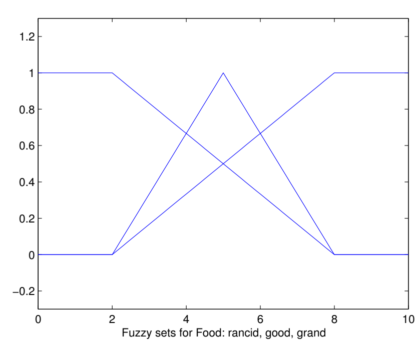
$$\mu_{A_{generous}}(x) = 1 - \mu_{A_{small}}(x) = \begin{cases} 0 & if \quad x \le \frac{1}{3} \\ 3x - 1 & if \quad \frac{1}{3} < x < \frac{2}{3} \\ 1 & if \quad \frac{2}{3} \le x \end{cases}$$

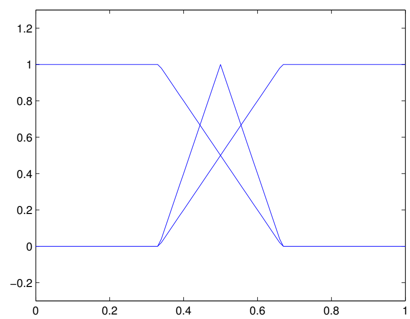
$$\min(\mu_{A_{small}}(x), \mu_{A_{generous}}(x)) = \begin{cases} 0 & if \quad x \le \frac{1}{3} \\ 3x - 1 & if \quad \frac{1}{3} < x < \frac{1}{2} \\ 2 - 3x & if \quad \frac{1}{2} \le x < \frac{2}{3} \\ 0 & if \quad \frac{2}{3} \le x \end{cases}$$

$$\mu_{A_{ordinary}}(x) = \min(1, 2 * \min(\mu_{A_{small}}(x), \mu_{A_{generous}}(x))) =$$

$$= \begin{cases} 0 & if \quad x \le \frac{1}{3} \\ 6x - 2 & if \quad \frac{1}{3} < x < \frac{1}{2} \\ 4 - 6x & if \quad \frac{1}{2} \le x < \frac{2}{3} \\ 0 & if \quad \frac{2}{3} \le x \end{cases}$$







Fuzzy sets for Tip: small, ordiner, generous

3 d:

1. **Fuzzification**:

for input value service = 3 we have the following membership degrees:

$$\mu_{A^{poor}}(3) = \frac{5}{6}, \quad \mu_{A^{average}}(3) = \frac{1}{3}, \quad \mu_{A^{\sup erb}}(3) = \frac{1}{6}$$

for input value food = 7 we have the following membership degrees:

$$\mu_{A^{rancid}}(7) = \frac{1}{6}, \quad \mu_{A^{god}}(7) = \frac{1}{3}, \quad \mu_{A^{grand}}(7) = \frac{5}{6}$$

2. **Rule firing**:

Rule 1 has firing degree FD1 = min(5/6,1/6) = 1/6
Rule 2 has firing degree FD2 = min(1/3,1/3) = 1/3.
Rule 3 has firing degree FD3 = min(1/6,5/6) = 1/6.

3. **Inference**:

In this phase we have to combine firing degrees with rule outputs. We get:

First output fuzzy set combined with firing degree

$$\mu_{O_1}(x) = \min(\frac{1}{6}, \mu_{A^{small}}(x)) = \begin{cases} \frac{1}{6} & if \quad x \le \frac{11}{18} \\ 2-3x & if \quad \frac{11}{18} < x < \frac{2}{3} \\ 0 & if \quad \frac{2}{3} \le x \end{cases}$$

Second output fuzzy set combined with firing degree

$$\mu_{O_2}(x) = \min(\frac{1}{3}, \mu_{A^{ordinary}}(x)) = \begin{cases} 0 & if \quad x \le \frac{1}{3} \\ 6x-2 & if \quad \frac{1}{3} < x < \frac{7}{18} \\ \frac{1}{3} & if \quad \frac{7}{18} \le x \le \frac{11}{18} \\ 4-6x & if \quad \frac{11}{18} < x < \frac{2}{3} \\ 0 & if \quad \frac{2}{3} \le x \end{cases}$$

Third output fuzzy set combined with firing degree

$$\mu_{O_3}(x) = \min(\frac{1}{6}, \mu_{A^{generous}}(x)) = \begin{cases} 0 & if \quad x \le \frac{1}{3} \\ 3x-1 & if \quad \frac{1}{3} < x < \frac{7}{18} \\ \frac{1}{6} & if \quad \frac{7}{18} \le x \end{cases}$$

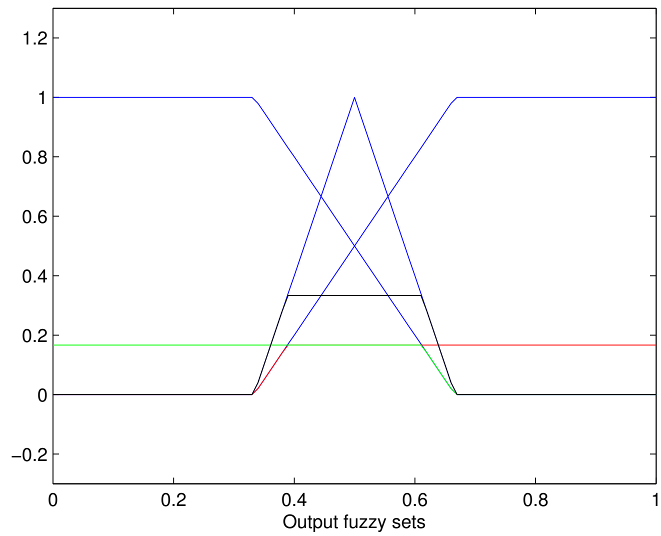
4. **Composition**

In this phase we need to combine all output fuzzy sets into just one fuzzy set

$$\mu_O(x) = \max(\mu_{O_1}(x), \mu_{O_2}(x), \mu_{O_3}(x)) = \begin{cases} \frac{1}{6} & if \quad x \le \frac{1}{3} \\ 6x-2 & if \quad \frac{1}{3} < x < \frac{7}{18} \\ \frac{1}{3} & if \quad \frac{7}{18} \le x \le \frac{11}{18} \\ 4-6x & if \quad \frac{11}{18} < x < \frac{2}{3} \\ \frac{1}{6} & if \quad \frac{2}{3} \le x \end{cases}$$

5. **Defuzzification**

In this phase the output fuzzy set needs to be defuzzified. We can use for example the center of gravity method. Since the output set is symmetric wrt. line x=1/2, the center of gravity lies on this line. Therefore the defuzzified value is 1/2.


1.2
1
0.8
0.6
0.4
0.2
0
−0.2
0
0.2
0.4
0.6
0.8
1
Output fuzzy sets

Q4. **This question is about Takagi-Sugeno-Kang controllers**

Q4 a. In the case of classical operations of process control one has to solve the non-linear function u. Furthermore, it is very important that one also finds the proportionality constants KI, KD, and KP.

In the case of fuzzy controller, the non-linear function is represented by a fuzzy mapping, typically acquired from human beings.

4b (i) Fuzzy air-conditioner system has the following linguistic variables with their respective term sets:

Temperature {cold, cool, pleasant, warm, hot}, Speed of the motor {minimal, slow, medium, fast, blast}. These fuzzy terms are modelled by fuzzy sets with following membership functions:

$$\mu_{A^{cold}}(x)=\begin{cases}1 & if \quad x\le 10\\ \dfrac{15-x}{5} & if \quad 10<x<15\\ 0 & if \quad 15\le x\end{cases} \qquad \mu_{A^{hot}}(x)=\begin{cases}0 & if \quad x\le 20\\ \dfrac{x-2}{7} & if \quad 20<x<27\\ 1 & if \quad 27\le x\end{cases}$$

$$\mu_{A^{cool}}(x)=\begin{cases}0 & if \quad x\le 10\vee x\ge 18\\ \dfrac{x-10}{5} & if \quad 10\le x\le 15\\ \dfrac{18-x}{3} & if \quad 15\le x\le 18\end{cases} \qquad \mu_{A^{warm}}(x)=\begin{cases}0 & if \quad x\le 20\vee x\ge 29\\ \dfrac{2x-40}{11} & if \quad 20\le x\le 25.5\\ \dfrac{58-2x}{7} & if \quad 25.5\le x\le 29\end{cases}$$

$$\mu_{A^{pleasant}}(x)=\begin{cases}0 & if \quad x\le 14\vee x\ge 22\\ \dfrac{x-14}{4} & if \quad 14\le x\le 18\\ \dfrac{22-x}{4} & if \quad 18\le x\le 22\end{cases}$$

$$\mu_{B^{minimalt}}(x)=0$$
$$\mu_{B^{slow}}(x)=30$$
$$\mu_{B^{medim}}(x)=50$$
$$\mu_{B^{fast}}(x)=75$$
$$\mu_{B^{blastt}}(x)=100$$

4b (ii) Rule base for air-conditioner fuzzy system is

If	temperature	is	cold	then	speed	is	minimal
If	temperature	is	cool	then	speed	is	slow
If	temperature	is	pleasant	then	speed	is	medium
If	temperature	is	warm	then	speed	is	fast

If temperature is hot then speed is blast

4b (iii) The system can be modelled by both mamdani and TSK fuzzy system, however the Takagi-Sugeno-Kang system is much easier and if we choose the zero-order TSK the consequents are described in the question. In all other cases the system needs to be more elaborated and either fuzzy sets or polynomial consequents need to be artificially added. TSK system needs no defuzzification and that also supports the choice of this system.

4b (iv) For a cold day, when the input temperature for the fuzzy system is 12^{o}C we have:

1. **Fuzzification**:

for input variable temperature we have

$$\mu_{A^{cold}}(12) = 0.6, \quad \mu_{A^{cool}}(12) = 0.4, \quad \mu_{A^{pleasant}}(12) = 0, \quad \mu_{A^{warm}}(12) = 0, \quad \mu_{A^{hot}}(12) = 0.$$

2. **Rule firing:**

Rule 1 has firing degree FD1 = 0.6
Rule 2 has firing degree FD2 = 0.4
Rule 3 has firing degree FD3 = 0
Rule 4 has firing degree FD4 = 0
Rule 5 has firing degree FD5 = 0

3. **Inference**:

In this phase we have to combine firing degrees with rule outputs. We get:

$\mu_{O_1}(x) = 0.6 \cdot 0 = 0$

$\mu_{O_2}(x) = 0.4 \cdot 30 = 12$

$\mu_{O_3}(x) = 0 \cdot 50 = 0$

$\mu_{O_4}(x) = 0 \cdot 75 = 0$

$\mu_{O_5}(x) = 0 \cdot 100 = 0$

4. **Composition**:

In zero-order TSK fuzzy system there is no defuzzification, therefore we have just to sum up the normalized outputs of the rules. Since sum of the firing degrees is 1 the outputs are already normalized. We get

$\mu_{O_1}(x) = 0 + 12 + 0 + 0 + 0 = 12$

Thus the output of the system is 12 r.p.m.

Q5. **This question is about neuro-fuzzy systems**

5a: We have $w_{03} = 0.1$; $w_{04} = 0.1$; and, $w_{05} = 0.2$, and the weight vectors are

$$\begin{bmatrix} w13 & w23 \\ w14 & w24 \end{bmatrix} = \begin{bmatrix} 0.2 & 0.1 \\ 0.1 & 0.3 \end{bmatrix};$$

and

$$x^T = [0.1 \quad 0.9];$$

and

$$[w_{35} \quad w_{45}] = [0.2 \quad 0.3].$$

We assume learning rate n = 0.1.

FORWARD PASS:

$$\begin{aligned} net_3 &= w_{03} + \sum x_i w_{i3} \\ &= w_{03} + x_1 w_{13} + x_2 w_{23} \\ &= 0.1 + 0.1 * 0.2 + 0.9 * 0.1 \\ &= 0.21 \end{aligned}$$

$$\begin{aligned} o_3 &= \frac{1}{1 + \exp(-0.21)} \\ &= 0.5523 \end{aligned}$$

and,

$$\begin{aligned} net_4 &= w_{04} + \sum x_i w_{i4} \\ &= w_{04} + x_1 w_{14} + x_2 w_{24} \\ &= 0.1 + 0.1 * 0.1 + 0.9 * 0.3 \\ &= 0.38 \end{aligned}$$

$$\begin{aligned} o_4 &= \frac{1}{1 + \exp(-0.38)} \\ &= 0.5939 \end{aligned}$$

OUTPUT LAYER:

$$\begin{aligned} net_5 &= w_{05} + \sum o_i w_{i5}, \ where\ i = 3,4 \\ &= w_{05} + o_3 w_{35} + o_4 w_{45} \\ &= 0.2 + 0.5523 * 0.2 + 0.5939 * 0.3 \\ &= 0.4886 \end{aligned}$$

$$\begin{aligned} o_5 &= \frac{1}{1 + \exp(-0.4886)} \\ &= 0.6198 \end{aligned}$$

BACKWARD PASS:

ERRORS – OUTPUT NODES:

$$\delta_5 = (d_5 - o_5)o_5(1 - o_5)$$
$$= (0.9 - 0.6198) * 0.6198 * (1 - 0.6198)$$
$$= 0.0660$$

ERRORS – HIDDEN NODES:

$$\delta_4 = (1 - o_4)o_4\delta_5 w_{45}$$
$$= (1 - 0.5939) * 0.5939 * 0.066 * 0.3$$
$$= 0.0048$$
$$\delta_3 = (1 - o_3)o_3\delta_5 w_{35}$$
$$= (1 - 0.5523) * 0.5523 * 0.066 * 0.2$$
$$= 0.0033$$

Weight update:

$$w_{35}^{new} = w_{35} + \eta * \delta_5 * o_3$$
$$= 0.2 + 0.1 * 0.066 * 0.5523$$
$$= 0.203645$$

$$w_{45}^{new} = w_{45} + \eta * \delta_5 * o_4$$
$$= 0.3 + 0.1 * 0.066 * 0.5939$$
$$= 0.30392$$

$$w_{13}^{new} = w_{13} + \eta * \delta_3 * x_1$$
$$= 0.2 + 0.1 * 0.033 * 0.1$$
$$= 0.20033$$

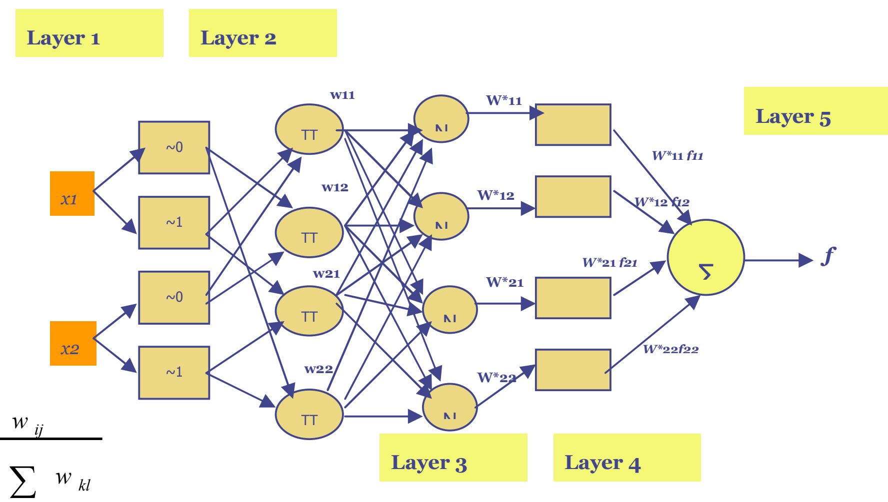
$$w_{23}^{new} = w_{23} + \eta * \delta_3 * x_2$$
$$= 0.1 + 0.1 * 0.033 * 0.9$$
$$= 0.10297$$

$$w_{14}^{new} = w_{14} + \eta * \delta_4 * x_1$$
$$= 0.1 + 0.1 * 0.048 * 0.1$$
$$= 0.10048$$

$$w_{23}^{new} = w_{23} + \eta * \delta_3 * x_2$$
$$= 0.3 + 0.1 * 0.048 * 0.9$$
$$= 0.30432$$

5b: The equivalent Takagi Sugeno Network that can be trained using backpropagtion algorithm

The solution is a zero-order Takagi-Sugeno model with two inputs (x1 and x2) and one output *f*.



$$w^*_{ij} = \frac{w_{ij}}{\sum_k \sum_l w_{kl}}$$

5c:

The training pattern to be adapted for training an ANFIS system:

	Forward Pass	Backward Pass
Parameters: Premise	No change	Gradient Descent
Parameters: Consequent	Least Square Estimator	No change
Compute	Node Outputs	Error Signals