A sequential circuit is specified by a time sequence of inputs, outputs, and internal states.

Synchronous clocked sequential circuit

(a) Block diagram

(b) Timing diagram of clock pulses
A finite-state machine (FSM) or finite-state automaton (FSA, plural: automata), finite automaton, or simply a state machine, is a mathematical model of computation. It is an abstract machine that can be in exactly one of a finite number of states at any given time.

Examples are vending machines, which dispense products when the proper combination of coins is deposited, elevators, whose sequence of stops is determined by the floors requested by riders, traffic lights, which change sequence when cars are waiting, and combination locks, which require the input of combination numbers in the proper order.

A state is a description of the status of a system that is waiting to execute a transition. A transition is a set of actions to be executed when a condition is fulfilled or when an event is received.

Real computers are finite state machines: they have finite memory so there's a finite number of states the machine can be in.
Common Examples of Synchronous FSM

- Up and Down Binary Counters
- Shift Registers
- Sequence Detectors
- Controllers
State Equations

A(t + 1) = A(t)x(t) + B(t)x(t)
B(t + 1) = A'(t)x(t)

A(t + 1) = Ax + Bx
B(t + 1) = A'x

A state equation (also called a transition equation) specifies the next state as a function of the present state and inputs.

y = (A + B)x'
The time sequence of inputs, outputs, and flip-flop states can be enumerated in a state table (sometimes called a transition table).

The derivation of a state table requires listing all possible binary combinations of present states and inputs. In this case, we have eight binary combinations from 000 to 111. The next-state values are then determined from the logic diagram or from the state equations.

In general, a sequential circuit with m flip-flops and n inputs needs $2^{m+n}$ rows in the state table. The binary numbers from 0 through $2^{m+n} - 1$ are listed under the present-state and input columns. The next-state section has m columns, one for each flip-flop. The binary values for the next state are derived directly from the state equations. The output section has as many columns as there are output variables. Its binary value is derived from the circuit or from the Boolean function in the same manner as in a truth table.
The binary number inside each circle identifies the state of the flip-flops. The directed lines are labelled with two binary numbers separated by a slash.

The input value during the present state is labelled first, and the number after the slash gives the output during the present state with the given input.

<table>
<thead>
<tr>
<th>Present State</th>
<th>Next State</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$x = 0$</td>
<td>$x = 0$</td>
</tr>
<tr>
<td>$A$ $B$</td>
<td>$A$ $B$</td>
<td>$A$ $B$</td>
</tr>
<tr>
<td>0 0</td>
<td>0 0 0 1</td>
<td>0 0 1  1</td>
</tr>
<tr>
<td>0 1</td>
<td>0 0 1 1</td>
<td>1 0 1  0</td>
</tr>
<tr>
<td>1 0</td>
<td>0 0 1 0</td>
<td>1 0 1  0</td>
</tr>
<tr>
<td>1 1</td>
<td>0 0 1 0</td>
<td>1 0 1  0</td>
</tr>
</tbody>
</table>
The state diagram gives a pictorial view of state transitions and is the form more suitable for human interpretation of the circuit's operation.

It is important to remember that the bit value listed for the output along the directed line occurs during the present state and with the indicated input, and has nothing to do with the transition to the next state.

For example, the directed line from state 00 to 01 is labelled 1/0, meaning that if the sequential circuit is in the present state 00 and the input is 1, the output is 0. After the next clock edge, the circuit goes to the next state, 01. If the input changed to 0, then the output becomes 1, but if the input remained at 1, the output stays at 0. This information is obtained from the state diagram along the two directed lines emanating from the circle with state 01. A directed line connecting a circle with itself indicates that no change of state occurs.
The state diagram gives a pictorial view of state transitions and is the form more suitable for human interpretation of the circuit's operation.

The state diagram clearly shows that, starting from state 00, the output is 0 as long as the input stays at 1.

The first 0 input after a string of 1’s gives an output of 1 and transfers the circuit back to the initial state, 00.

The machine represented by this state diagram acts to detect a zero in the bit stream of data.

00 zero was detected in last cycle, or reset, wait for a 1
  if x=0 or 1 output is 0, if x stays at zero stay in this state
  if x goes to 1 next state 01
01 last cycle was a 1 and last output a zero
  if x is 0, output a 1, new zero detected, next 00
  if x is 1, output a zero next state 11
11 two 1’s were found
  if x is 0 output a 1 next state 00
  if x is 1 output 0 next 10
10 three or more ones were found
  if x is 1 output a zero and stay here
  if x is zero, output a 1 next state 00
D-Type Flip-Flop with Set/Reset

Set/reset events are asynchronous to the clock edge.

The edge-triggered D flip-flop uses three SR latches. Two latches respond to the external D (data) and Clk (clock) inputs. The third latch provides the outputs for the flip-flop. The S and R inputs of the output latch are maintained at the logic-1 level when Clk = 0.
If $D = 0$ when $\text{Clk}$ becomes 1, $R$ changes to 0. This causes the flip-flop to go to the reset state, making $Q = 0$. If there is a change in the D input while $\text{Clk} = 1$, terminal R remains at 0 because $Q$ is 0. Thus, the flip-flop is locked out and is unresponsive to further changes in the input.
At every rising edge of clock, if reset is not asserted, the state of the machine is updated by the first always block;

when state is updated by the first always block, the change in state is detected by the sensitivity list mechanism of the second always block; then the second always block updates the value of next_state (it will be used by the first always block at the next tick of the clock); the third always block also detects the change in state and updates the value of the output. In addition, the second and third always blocks detect changes in x_in and update next_state and y_out accordingly.
Signal transitions that are caused by input signals that change on the active edge of the clock race with the clock itself to reach the affected flip-flops, and the outcome is indeterminate (unpredictable).

Conversely, changes caused by inputs that are synchronized to the inactive edge of the clock reach stability before the active edge, with predictable outputs of the flip-flops that are affected by the inputs.
Stream of 1s

valid Mealy output

Mealy glitch
initial #200 $finish;
initial begin t_clock = 0; forever #5 t_clock = ~t_clock; end

initial fork
// Statements with the fork . . . join block execute in parallel.
// so the time delays are relative to a common reference of t = 0
  t_reset = 0;
  t_x_in = 0;
  #2  t_reset = 1;
  #87 t_reset = 0;
  #89 t_reset = 1;
  #10 t_x_in = 1;
  #30 t_x_in = 0;
  #40 t_x_in = 1;
  #50 t_x_in = 0;
  #52 t_x_in = 1;
  #54 t_x_in = 0;
  #70 t_x_in = 1;
  #100 t_x_in = 0;
  #120 t_x_in = 1;
  #160 t_x_in = 0;
  #170 t_x_in = 1;
join
In the theory of computation, a Mealy machine is a finite-state machine whose output values are determined both by its current state and the current inputs. This is in contrast to a Moore machine, whose output values are determined solely by its current state.
We will adopt the convention of using the flip-flop input symbol to denote the input equation variable and a subscript to designate the name of the flip-flop output. For example, $D_Q = x + y$ specifies an OR gate with inputs $x$ and $y$ connected to the D input of a flip-flop whose output is labelled with the symbol $Q$.

\[
D_A = A \oplus x \oplus y
\]

\[
A(t + 1) = A \oplus x \oplus y
\]

The expression specifies an odd function and is equal to 1 when only one variable is 1 or when all three variables are 1.
State Table for Sequential Circuit with JK Flip-Flops

<table>
<thead>
<tr>
<th>Present State</th>
<th>Input</th>
<th>Next State</th>
<th>Flip-Flop Inputs</th>
</tr>
</thead>
<tbody>
<tr>
<td>A B x</td>
<td></td>
<td>A B</td>
<td>( I_A ) ( K_A ) ( I_B ) ( K_B )</td>
</tr>
<tr>
<td>0 0 0</td>
<td></td>
<td>0 1</td>
<td>0 0 1 0</td>
</tr>
<tr>
<td>0 0 1</td>
<td></td>
<td>0 0</td>
<td>0 0 0 1</td>
</tr>
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<td></td>
<td>1 1</td>
<td>1 1 1 0</td>
</tr>
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<td></td>
<td>1 0</td>
<td>1 0 0 1</td>
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</tr>
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</table>

\[
\begin{align*}
I_A &= B \\
K_A &= Bx' \\
I_B &= x' \\
K_B &= A'x + Ax' = A \oplus x
\end{align*}

S1 01 0 1 1 1 1 0
S0 00 0 0 0 0 0 0
S2 10 1 1 0 0 1 1
S3 11 1 1 0 0 0 0
reg [1: 0] state;
parameter S0 = 2'b00, S1 = 2'b01, S2 = 2'b10, S3 = 2'b11;
always @ (posedge clock, negedge reset)
  if (reset == 0) state <= S0; // Initialize to state S0
  else case (state)
    S0: if (~x_in) state <= S1; else state <= S0;
    S1: if (x_in) state <= S2; else state <= S3;
    S2: if (~x_in) state <= S3; else state <= S2;
    S3: if (~x_in) state <= S0; else state <= S3;
  endcase
assign y_out = state;
endmodule
(1) the output depends on only the state,
(2) reset "on-the-fly" forces the state of the machine back to S0 (00)