Prove the following theorems algebraically:
(a) $X(X' + Y) = XY$
(b) $X + XY = X$
(c) $XY + XY' = X$
(d) $(A + B)(A + B') = A$

Simplify each of the following expressions by applying one of the theorems. State the theorem used.
(a) $X'Y'Z + (X'Y'Z)'$
(b) $(AB' + CD)(B'E + CD)$
(c) $ACF + AC'F$
(d) $A(C + D'B) + A'$
(e) $(A'B + C + D)(A'B + D)$
(f) $(A + BC) + (DE + F)(A + BC)'$

For each of the following circuits, find the output and design a simpler circuit having the same output. (Hint: Find the circuit output by first finding the output of each gate, going from left to right, and simplifying as you go.)
Multiply out and simplify to obtain a sum of products:
(a) \((A + B)(C + B)(D' + B)(ACD' + E)\)
(b) \((A' + B + C')(A' + C' + D)(B' + D')\)

Factor each of the following expressions to obtain a product of sums:
(a) \(AB + C'D'\)
(b) \(WX + WY'X + ZYX\)
(c) \(A'B'C + EF + DEF'\)
(d) \(XYZ + W'Z + XQ'Z\)
(e) \(ACD' + C'D' + A'\ C\)
(f) \(A + BC + DE\)
(The answer to (f) should be the product of four terms, each a sum of three variables.)

Draw a circuit that uses only one AND gate and one OR gate to realize each of the following functions:
(a) \((A + B + C + D)(A + B + C + E)(A + B + C + F)\)
(b) \(WXYZ + VXYZ + UXZY\)

Simplify the following expressions to a minimum sum of products.
(a) \([(AB)' + C'D]'\)
(b) \([A + B(C' + D)]'\)
(c) \([(A + B')C'(A + B)(C + A)']\)

Find \(F\) and \(G\) and simplify:

(a)

(b)