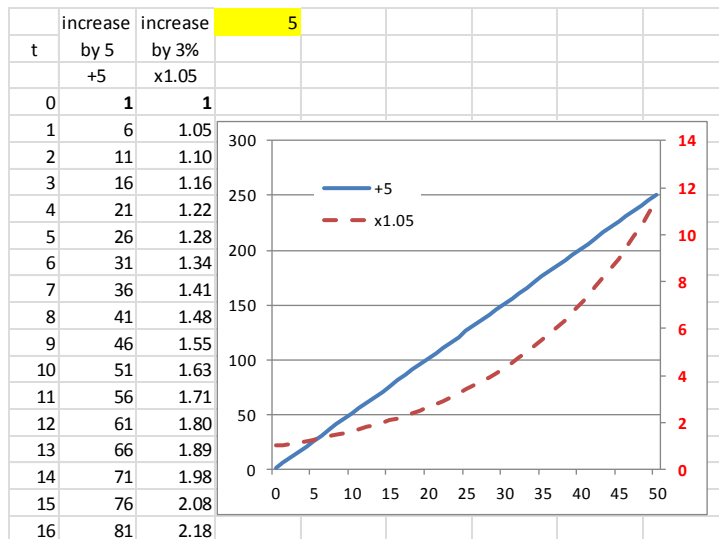


Additive and Multiplicative Change

- What's the difference?
- How do I recognise which to use?
- Are there cases where it doesn't matter?



Additive:

Multiplicative

equiv

Prev value +5

Prev value + 5%

Prev value \times **1.05**

See [Add.vs.Mult Excel file](#)

Experiment by

Changing factor from 5

Changing initial values from 1

Hint

:try large/small/negative

The Log Scale

In many cases the “log scale” is in fact natural

⇒ Conclusions, predictions, insights simple

Closely related

The use of “%age change” is natural

⇒ Change naturally thought of as multiplicative

Mathematic Concepts

- *%age* *mathematical concept*
invented to deal with multiplicative change.
 - commonly used, but some people get confused.
 - Eg, increasing the price by 50%, and then decreasing it by 33.33%, leads to the original price.
 - Decreasing by 50% doesn't get back to the same place!
- *logs* *mathematical concept*
invented to deal with multiplicative change.
 - not commonly used: lots of people get confused.
 - tricks MINITAB into working with multiplicative change

Logs

Formally

$$y = 10^{\log_{10} y}$$

Examples

$$(y = 100) \Leftrightarrow (y = 10^2) \Leftrightarrow (\log_{10} y = 2) \Leftrightarrow (\log_{10}(100) = 2)$$

$$(y = \sqrt{10}) \Leftrightarrow (y = 10^{1/2}) \Leftrightarrow (\log_{10} y = 1/2) \Leftrightarrow (\log_{10}(\sqrt{10}) = 1/2)$$

$$(y = 4) \Leftrightarrow (y = 10^{0.60206}) \Leftrightarrow (\log_{10} y = 0.60206) \Leftrightarrow (\log_{10} 4 = 0.60206)$$

Key advantage: Multiplication \rightarrow Addition

$$x \times y = 10^{\log_{10} x} \times 10^{\log_{10} y} = 10^{(\log_{10} x) + (\log_{10} y)}$$

Thus

$$\log_{10}(x \times y) = \log_{10}(x) + \log_{10}(y)$$

If z is obtained by **multiplying** x by y

then $\log_{10}(z)$ is obtained by **adding** $\log_{10}(x)$ to $\log_{10}(y)$

Logs

Example

$z = 4000$ obtained by multiplying 4 and 10 and 10 and 10

$\log_{10} 4000$ obtained by adding $\log_{10} 4$ and $\log_{10} 10$ and $\log_{10} 10$ and $\log_{10} 10$

Thus $\log_{10} 4000 = 0.60206 + 1 + 1 + 1 = 3.60206$

$z = 0.04$ obtained by multiplying 4 and 10^{-1} and 10^{-1}

$\log_{10} 0.04$ obtained by adding $\log_{10} 4$ and $\log_{10} 10^{-1}$ and $\log_{10} 10^{-1}$

Thus $\log_{10} 0.04 = 0.60206 + (-1) + (-1) = -1.39794$

Logs

Since (increasing y by 10%) \Leftrightarrow (multiplying y by 1.1)

$\Rightarrow (z = y \times 1.1) \Leftrightarrow (\log_{10} z = \log_{10} y + 0.04139)$

Also (increasing y by 10% twice) \Leftrightarrow (multiplying y by 1.1^2)

\Leftrightarrow (multiplying y by 1.21)

\Leftrightarrow (increasing y by 21%)

$\Rightarrow (z = y \times 1.21) \Leftrightarrow (\log_{10} z = \log_{10} y + 2 \times 0.04139)$

Thus

(increasing y by 10% forty times) \Leftrightarrow (multiplying y by $1.1^{40} = 45.26$)

\Leftrightarrow (increasing $\log_{10} y$ by adding 40×0.04139)

Task: rework for increase of 5% forty times

Recognising Additive/ Multiplicative

Negative Values?

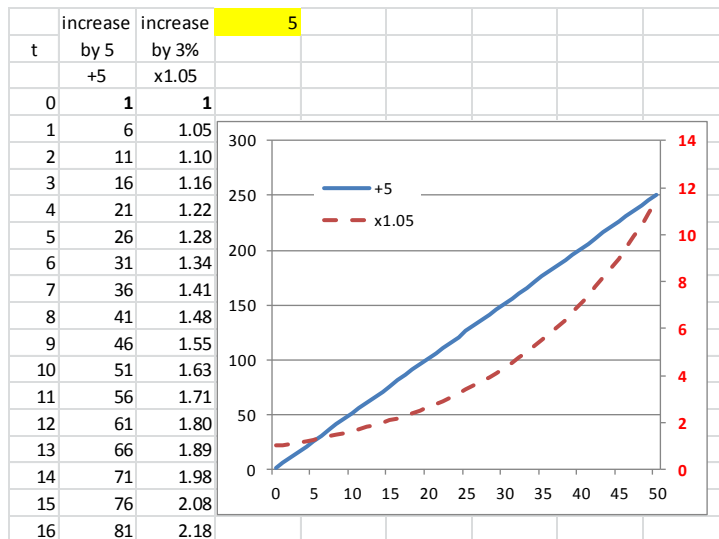
- Constant additive changes can lead to negative values
If OK with negatives \Rightarrow Additive Model may be OK
- Constant %age changes can never lead to negatives
Even if the changes are negative %
Logs can be negative, but 'antilog' are ALWAYS positive.
If Not OK with negatives \Rightarrow Additive Model may give silly ans.
- Negative temperatures OK. Additive Change *may be OK*
- Negative prices, weights - ?? Multiplicative Change *may be better*

Additive and Multiplicative Change

Experiment by:

- Changing factor from 5
- Changing initial values from 1
- Hint: try large/small/negative
- Find values where Mult gives (nearly) straight lines
(so change could be regarded as either additive or multiplicative)

See [Add.vs.Mult Excel file](#)



+ 5% is a mathematical concept/notation
invented to discuss mult change

Additive and Multiplicative Change

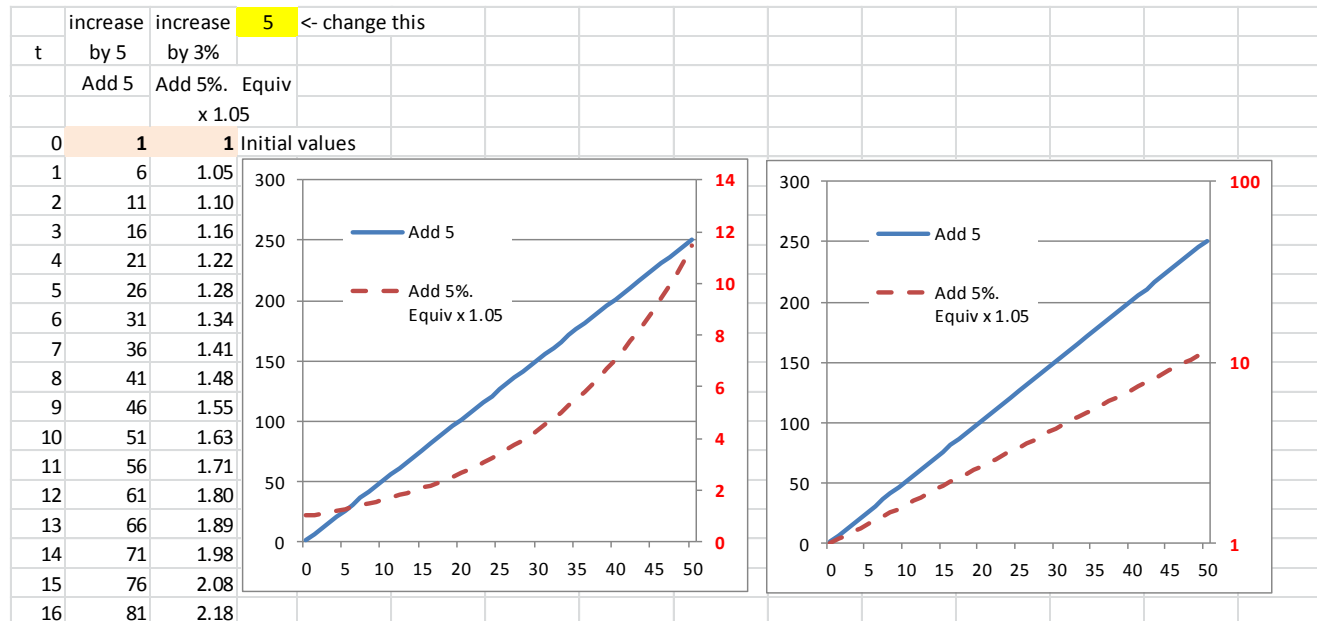
Investigate Log scales

(format axis in Excel)

Try increase of 50%, 100%

See [Add.vs.Mult Excel file](#)

The “log scale” is a mathematical device to present multiplicative growth.



Additive and Multiplicative Change

Mathematical notation

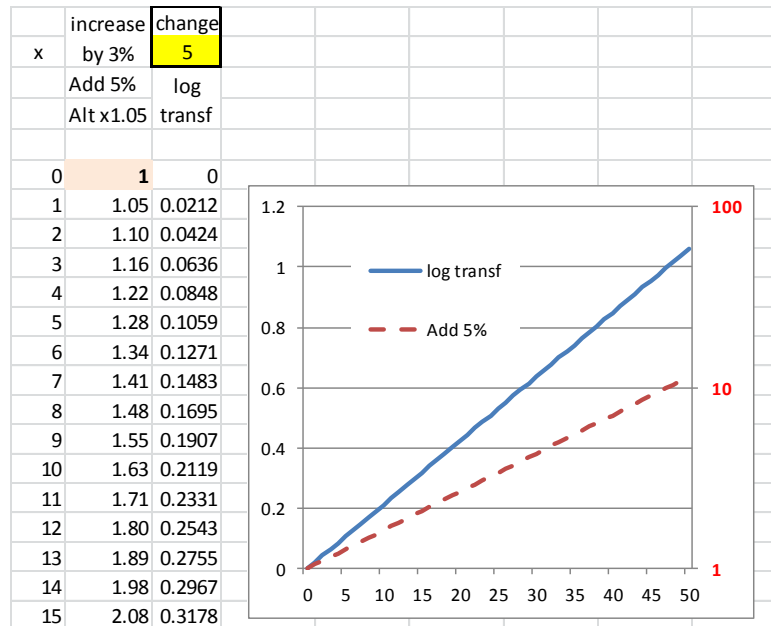
Add: $y = \alpha + \beta x$ Mult: $y = \alpha \beta^x$

Eg $y = 2 + 3x$ $y = 2(1.03)^x$

$$\log(y) = \log(\alpha) + x \log(\beta)$$

The “log transformation” is a mathematical device to mathematically manipulate multiplicative growth.

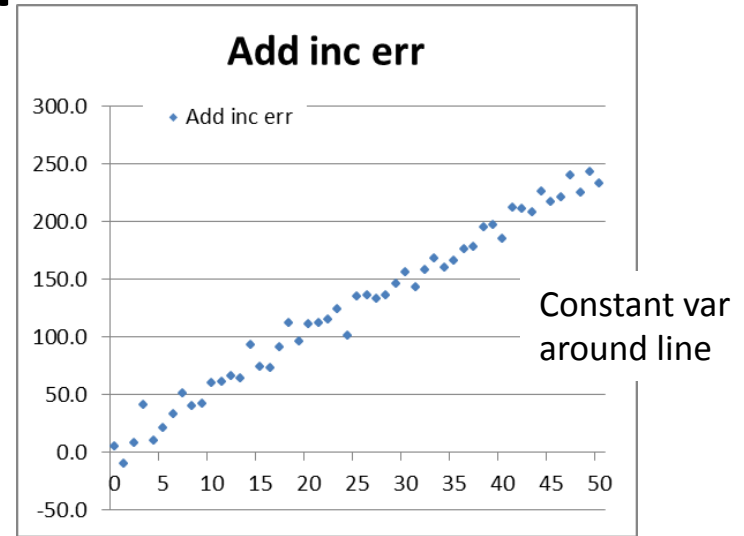
See [Add.vs.Mult Excel file](#)



Additive and Multiplicative Error

Add: $y = \alpha + \beta x \pm \text{rand error}$

Eg $y = 2 + 3x + \varepsilon; \quad \varepsilon \sim N(0, (10)^2)$



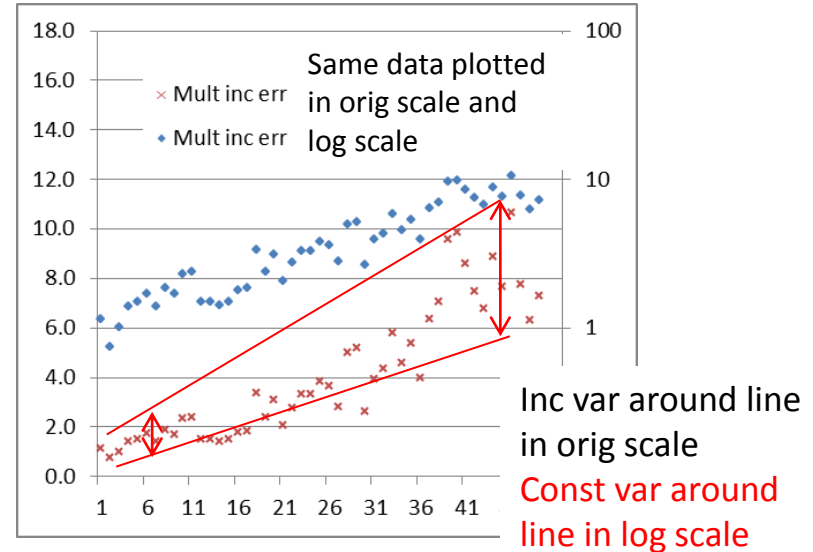
Mult: $y = \alpha\beta^x (1 \pm \text{rand error})^x$

Eg $y = 2(1.03)^x (1 + \varepsilon);$

$\log(y) = \log(\alpha) + x \log(\beta) + \log(1 + \varepsilon)$

$\log(y) = \log(\alpha) + x \log(\beta) + u; u \sim N(0, (0.1)^2)$

$1 + \varepsilon = \text{antilog}(u) = 10^u$



See [Add.vs.Mult Excel file](#)

Symptoms of Additive/ Multiplicative

Random variation *constant*

⇒ Additive Model *may be OK*

Dist of random variation *symmetric*

⇒ Additive Model *may be OK*

Random variation increasing

⇒ Multiplicative Model *may be better*

Dist of random variation *skew* (mostly small, sometimes very large)

⇒ Multiplicative Model *may be better*

Recognising Additive/ Multiplicative

Constant additive changes can lead to negative values

If OK with negatives \Rightarrow Additive Model may be OK

Constant %age changes can never lead to negatives

Even if the changes are negative percents

Logs can be negative, but 'antilog' are ALWAYS positive.

Negative temperatures don't confuse anyone.

Negative prices, volumes, weights can be confusing.

Think twice about a method that might take you there, such as regression applied to prices.

Regression applied to log prices can never lead to negative PRICES.

When log transform?

Log transf of Y and/or some X values indicated when:

- Y values must be positive
 - esp when Y values clearly constrained below in data
 - esp when variability in Y increasing with \hat{Y} (hetero-skedastic)
 - often $Y=0$ not meaningful
- Y and/or X values
 - positive and range over orders of magnitudes
 - often evidenced by very skew dist of Y and/or X
- “Small change” in Y and/or X values
 - Naturally expressed in %age terms
 - Change $1 \rightarrow 2 \gg \gg$ Change $1,000,001 \rightarrow 1,000,002$
- **Underlying relationships multiplicative**

Why log transform?

Multiple Linear Regression / MINITAB

- Can only handle cases where
 - change 1 \rightarrow 2 equiv to Change 1,000,001 \rightarrow 1,000,002
 - Y unconstrained (-ve values OK)
- Model details sensitive to
 - Outliers in Y – large residual
 - Outliers in X – influential
- Makes prediction intervals
 - Using (symmetric) Normal dist
- **Can lead to better insight**

Insight

\Leftrightarrow See things simply

\Leftrightarrow Naturally (Add/ Mult)

\Leftrightarrow Distinguish

Big picture/details

(details: eg lack of normality ,
some big residuals,
increasing variance,
.....)

Other approaches

- Use more flexible techniques than
Multiple Linear Regression
- Eg Generalised Linear Models (glm)
- Log scale & Log transform
not the only alternatives

Generalised Linear Models glm

- Special case: Mult Lin Reg with Normal errors
 - For given X , Avg $Y = \text{lin comb of } X$
 - For given X , Random Var in Y approx Normal, SD constant
- Other cases:
 - For given X , Avg Y *computed via* lin comb of X
 - For given X , SD of Random Var in Y *depends on* Avg Y
 - Eg *Logistic regression* Binary Y ; $Y = \text{proportion}$
- No transformations used:
 - But not least squares algorithm
- Can sometimes approximate by Transform and MLR
- **glm better general platform**

Technicalities with log transform

- Cannot be calculated if any $Y = 0$ $X=0$
 - Ask: how come value is 0?
 - Can use variation eg $\log(Y + 1)$
 $\log(Y + 0.5 \times \text{smallest } Y)$
- Not *necessary* to take logs of *all* variables
- Back transform
 - Fitted $\log Y \pm 2s \Rightarrow 10^{\text{Fit}-2s}, 10^{\text{Fit}}, 10^{\text{Fit}+2s}$
 - Note back transform necessarily >0
- Interpret coeffs
 - Increase $\log(X)$ by 1 \Leftrightarrow increase x factor of 10
 - Increase $\log(Y)$ by β \Leftrightarrow increase y by factor of 10^β

MLR with other transforms

- When Log is natural
- When Y is a proportion, bounded by 0 and 1

$$-\log\left(\frac{Y}{1-Y}\right) = \log(Y) - \log(1-Y)$$

- When Y is a count

$$-\sqrt{Y}$$

- glm better