The central role of the Propensity Score in Observational Studies for Causal Effects

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Motivation

- World War II: want to minimize bomber losses to enemy fire.
- What areas should receive more protection?
Motivation

- Assume damage is uniformly distributed
- Returning aircraft hit in less vulnerable parts
- “[Abraham] Wald proposed that the Navy instead reinforce the areas where the returning aircraft were unscathed”

https://en.wikipedia.org/wiki/Abraham_Wald
**Motivation**

Causal effects

- Observe responses: \( r_1, \ldots, r_n \).

- Treatment assignment \( z_i = \begin{cases} 1 & \text{assigned experimental treatment} \\ 0 & \text{otherwise} \end{cases} \)

- We either observe \( r_i \mid z_i = 1 \) or \( r_i \mid z_i = 0 \).

- We want to estimate average treatment effect (ATE):

\[
ATE = \mathbb{E}[r_1] - \mathbb{E}[r_0].
\]
Causal effects

- Covariates $x_1, \ldots, x_n$. These can influence results (confounders/lurking variables).

- For valid inference, we require that $(r_1, r_0) \perp z \mid x$.

- Best solution: randomise treatment, e.g., RCTs.

- But this isn’t always possible.
A balancing score $b(x)$, is a function of $x$ such that

$$x \perp z \mid b(x).$$

Trivial choice is $b(x) = x$.

Better choices are many-to-one functions of $x$.

Some choices coarser (finer) than others:

$$b_1(x) \subseteq b_2(x).$$
Propensity score

\[ e(x) = \mathbb{P}(z = 1|x). \]

- This is called the propensity score – propensity for exposure to treatment.

- Assume

\[ p(z|x) = \prod_{i=1}^{n} e(x_i)^{z_i} \{1 - e(x_i)\}^{1-z_i}. \]

- N.B., \( \mathbb{E}[z] = e(x) \).
Example 2

Figure 3.4: Distributions of covariate values for the treated and non-treated group

(a) X values

(b) J values

(a) Estimated ATE and true ATE
Rosenbaum & Rubin show that:

- \( x \perp z \mid e(x) \) i.e., \( e(x) \) is a balancing score
- \( e(x) \subseteq b(x) \subseteq x \) i.e., \( e(x) \) is the coarsest balancing score
- \( (r_1, r_0) \perp z \mid x \) and \( 0 < \mathbb{P}(z = 1 \mid x) < 1 \)
  \( \Rightarrow (r_1, r_0) \perp z \mid b(x) \) and \( 0 < \mathbb{P}(z = 1 \mid b(x)) < 1 \).
  i.e., if treatment assignment is strongly ignorable given \( x \), then it is strongly ignorable given any balancing score.
How to use the propensity score

- We can estimate $e(x)$ using, e.g., logistic regression. Once estimated, can use propensity scores by:
  - Matched sampling
  - Subclassification (stratification)
  - Covariance adjustment
Examples

- We can estimate $e(x)$ using, e.g., logistic regression. Once estimated, can use propensity scores by:
  - Matched sampling
  - Subclassification (stratification)
  - Covariance adjustment
Stratification not always possible.