Question 1. A box contains 5 red and 5 blue marbles. Two marbles are withdrawn randomly. If they are the same color, then you win $1.10; if they are different colors, then you lose $1.00. Calculate:
(a) The expected value of the amount you win
(b) The variance of the amount you win.

Question 2. Suppose you carry out a poll following an election. You do this by selecting \( n \) people uniformly at random and asking whether they voted or not, letting \( X_i \) = 1 if person \( i \) voted and \( X_i \) = 0 otherwise. Suppose the probability that a person voted is 0.6.
(a) Calculate \( E[X_i] \) and \( Var(X_i) \).
Let \( Y = \sum_{i=1}^{n} X_i \).
(c) What is \( E[Y] \)? Is it the same as \( E[X] \) or different, and why?
(d) What is \( E[\frac{1}{n} Y] \)?
(e) What is the variance of \( \frac{1}{n} Y \) (express in terms of \( Var(X) \))?
Hints: use linearity of the expectation and the fact that people are sampled independently.

Question 3. Suppose that 2 balls are chosen without replacement from an urn consisting of 5 white and 8 red balls. Let \( X_i \) equal 1 if the \( i \)’th ball selected is white, and let it equal 0 otherwise.
(a) Give the joint probability mass function of \( X_1 \) and \( X_2 \)
(b) Are \( X_1 \) and \( X_2 \) independent? (Use the formal definition of independence to determine this)
(c) Calculate \( E[X_2] \)
(d) Calculate \( E[X_2|X_1 = 1] \)