ST3009 Mid-Term Test 2019

Attempt **all** questions. Time: 1 hour 30 mins.

1. A (large) bag contains 4 white and 3 black sheep

a) Suppose four sheep are drawn from the bag without replacement. What is the probability that they are alternately of different colours? [5 marks]

Suppose now that each sheep is put back into the bag after being drawn.

b) What is the probability that the four sheep are alternately of different colours now? [5 marks]

c) Let X be the number of sheep drawn from the bag that are white. Calculate:
   (i) E[X] [5 marks]
   (ii) Var(X) [5 marks]

d) State the definition of statistical independence for random variables. [5 marks]

e) Let Y be the number of sheep drawn from the bag that are black. Are random variables X and Y independent? Explain your answer using the definition of statistical independence. [5 marks]

2.

a) Define what is meant by the terms "sample space", "random event" and "random variable". Give examples of each. [5 marks]

b) Consider two random variables X and Y that take values in set {0,1}. Starting from the definition of the expectation and variance of a random variable show that

(i) $E[aX]=aE[X]$ [5 marks]

(ii) $Var(aX)=a^2Var(X)$ [5 marks]

c) Let X be a random variable which takes value 1 when a user clicks on a displayed advert, and 0 otherwise.

(i) How can we interpret E[X] and why might its value be of interest to an advertiser? When might the value of E[X] be less useful (hint: think about the variance)? [5 marks]

ii) Let Y be a random variable which takes value 1 when a user has visited web site https://Y.com. Suppose $Prob(X=1)=0.001$, $Prob(Y=1|X=1)=0.1$ and $Prob(Y=1|X=0)=0.01$. Using Bayes Rule calculate $Prob(X=1|Y=1)$. Given an advertiser has knowledge of the value of $Prob(X=1|Y=1)$ vs $Prob(X=1)$ how might a user visiting https://Y.com affect whether the advertiser displays an advert to that user or not ? [5 marks]

3. Five people play the game of "odd-man-out" to determine who pays for a meal. In this game, each person flips a coin. If one person's coin comes up different to all others (i.e. there is one H and four T's or there is one T and four H's), then that

person pays. Otherwise, everyone flips again. They go on doing this until someone is chosen.

a) What is the probability that one person's coin comes up differently from the others at a given round? [5 marks]

b) What is the expected number of times they must flip before they know who should pay? Hint $\sum_{i=1}^{\infty} ix^{i-1} = \frac{1}{(1-x)^2}$ [5 marks]

c) Explain how to generate a Bernoulli random variable (similar to a coin toss) using then rand() function in matlab. [5 marks]

d) Write pseudo-code for a short simulation of this game that outputs the number of flips made. [5 marks]