NOTE: There are more example questions at the end of Chapter 3 of the course textbook “A First Course in Probability” by Sheldon Ross.

**Question 1.** Independent flips of a coin that lands on heads with probability \( p \) are made. What is the probability that the first four outcomes are
(a) H,H,H,H?
(b) T,H,H,H?

**Question 2.** Eight bits are transmitted across a lossy wireless link. Each bit is corrupted independently with probability \( p \).
(a) What is the probability that no bits are corrupted?
(b) What is the probability that 8 bits are corrupted?
(c) What is the probability that 3 bits are corrupted?

**Question 3.** A bag contains 5 red balls and 10 black balls. Three balls are drawn out independently with replacement (the balls are put back into the bag after being drawn).
(a) What is the probability that all three balls are red?
(b) What is the probability that one red and two black balls are drawn?
(c) Suppose now that each ball is not put back in the bag after being drawn i.e. the balls are drawn without replacement. Are the first and second balls drawn still independent? Explain using the formal definition of independence.

**Question 4.** Suppose two 6-sided dice are rolled. Let \( A \) be the event that the first die comes up 3 and \( B \) that the sum of the dice is 2. Are these events independent? Explain using the formal definition of independence.

**Question 5.** In a class, there are 4 freshman men, 6 freshman women, and 6 sophister men. How many sophister women must be present if gender and year are to be independent when a student is selected uniformly at random?

**Question 6.** Suppose events \( E \) and \( F \) are independent, show that \( E \) and \( F^c \) are therefore also independent. Hint: start from \( P(E) = P(E \cap F) + P(E \cap F^c) \).

**Question 7.** A simplified model for the movement of the price of a stock supposes that on each day the stocks price either moves up 1 unit with probability \( p \) or moves down 1 unit with probability \( 1-p \). The changes on different days are assumed to be independent.
(a) What is the probability that after 2 days the stock will be at its original price?
(b) What is the probability that after 3 days the stocks price will have increased by 1 unit?
(c) Given that after 3 days the stocks price has increased by 1 unit, what is the probability that it went up on the first day?

**Question 8.** A true/false question is to be posed to a husband-and-wife team on a quiz show. Both the husband and the wife will independently give the correct answer with probability \( p \). Which of the following is a better strategy for the couple?
(a) Choose one of them and let that person answer the question.
(b) Have them both consider the question, and then either give the common answer if they agree or, if they disagree, flip a coin to determine which answer to give.