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School of Computer Science and Statistics

Extra Questions ST3009: Statistical Methods for Computer Science

NOTE: There are many more example questions in the course textbook "A First Course in Probability" by Sheldon Ross.

Question 1. (a) What is the number of ways that we can select 3 consonants from 7 ?

(b) What is the number of ways we can select 2 vowels from 4 ?

(c) Out of 7 consonants and 4 vowels, how many ways can 3 consonants and 2 vowels be selected ?

(d) How many ways can 3 consonants and 2 vowels (5 letters) be rearranged ?

Solutions

- Number of ways of selecting 3 consonants from 7 is $\binom{7}{3}$.
- Number of ways of selecting 2 vowels from 4 is $\binom{4}{2}$.
- So number of ways of selecting 3 consonants from 7 and 2 vowels from 4 is $\binom{4}{2}\binom{7}{3} = 210$
- Number of ways of arranging 5 letters among themselves is $5! = 120$

Question 2. In a group of 6 boys and 4 girls, four children are to be selected. In how many different ways can they be selected such that at least one boy should be there?

Solutions We have 4 options. 1. We can select 4 boys, which can be done in $\binom{6}{4}$ ways. 2. We can select 3 boys and 1 girl, which can be done in $\binom{6}{3}\binom{4}{1}$ ways. 3. We can select 2 boys and 2 girls, which can be done in $\binom{6}{2}\binom{4}{2}$ ways. 4. We can select 1 boy and 3 girls, which can be done in $\binom{6}{1}\binom{4}{3}$ ways. So total number of ways is $\binom{6}{4} + \binom{6}{3}\binom{4}{1} + \binom{6}{2}\binom{4}{2} + \binom{6}{1}\binom{4}{3} = 209$.

Question 3. A bag contains 1 red ball and 4 black balls.

(a) In how many different ways can I draw 2 balls from the bag, i.e. one after another without replacement, where when counting all balls of the same colour are treated as being the same.

(b) In how many different ways can I draw 2 balls from the bag when after drawing each ball I put it back into the bag.

Solutions

- The first ball can be red or black. If the first ball is red, the second ball must be black. If the first ball is black, the second ball can be red or black. So there are $1 + 2$ different arrangements of coloured balls.
- The first ball can be red or black. The second ball can also be red or black, regardless of the choice of the first ball. So there are now 2×2 different arrangements.

Question 4. I toss a coin and if it comes up heads I roll two 4-sided die and otherwise two 6-sided die.

(a) How many arrangements of heads/tails and die outcomes are possible (counting the 4 and 6 sided dice separately).

(b) Out of these arrangements, in how many do the dice rolls sum to 3 ?

(c) In how many do the dice rolls sum to 10 ?

Solutions

- $1 \times 6 \times 6 + 1 \times 4 \times 4 = 52$.
- Each of the two dice can sum to 3 by rolling (1, 2), (2, 1). So in total there are 4 ways that the dice rolls sum to 3.
- The two 4-sided dice can never sum to 10. The two 6-sided dice can sum to 10 as (4, 6), (5, 5), (6, 4) i.e. in a total 3 ways.

Question 5. From a group of 5 women and 7 men:
(a) A committee of 3 is to be formed. How many different committees are possible?
(b) How many different committees consisting of 2 women and 3 men can be formed?
(c) What if 2 of the men are feuding and refuse to serve on the committee together?
Solutions

- There are $\binom{12}{3}$ possible committees
- There are $\binom{5}{2}$ possible groups of 2 women and $\binom{7}{3}$ groups of 3 men, so $\binom{5}{2}\binom{7}{3}$ possible committees consisting of 2 women and 3 men.
- There are $\binom{5}{3}$ ways to select 3 men excluding those feuding, when we select one of the feuding men then there are $\binom{5}{2}$ ways to select the other two men i.e. excluding the other feuding man. So there are $\binom{5}{3} + 2\binom{5}{2} = 30$ ways to select 3 men and $\binom{5}{2}$ possible groups of 2 women. So there are $30\binom{5}{2}$ possible committees.

Question 6. (a) How many different 7-place license plates are possible if the first 2 places are for letters and the other 5 for numbers?
(b) And how many when no letter or number can be repeated in a single license plate.
Solutions

- There are 26^2 ways to choose 2 letters and 10^5 ways to choose 5 numbers, so $26^2 \times 10^5$ possible number plates
- There are 26×25 ways to choose 2 letters without duplicates and $10 \times 9 \times 8 \times 7 \times 6$ ways to select 5 numbers without duplicates.

Question 7. A deck of playing cards contains 52 cards consisting of four suits (hearts, spades, clubs or diamonds) and 13 ranks or values (1,2, . . . ,10,jack, queen, king).
(a) You draw one card from the deck and then draw another (without replacement). How many ways could you draw a pair (two cards of the same rank) e.g. 4 of hearts and 4 of spades.
(b) Suppose you now draw three cards. In how many ways can you draw a pair now (two cards of the same rank, the third of a different rank)
Solutions

- We select 1 from the 13 possible ranks, which can be done in 13 ways. For the pair we select two cards from the four suits which can be done in $\binom{4}{2}$ ways. So there are $13\binom{4}{2}$ ways to draw a pair.
- There are $13\binom{4}{2}$ pairs amongst two cards. For the third card we select from one of the remaining 12 ranks, and then from one of the four suits. So there are $12\binom{4}{1}$ ways to select the third card and so $13\binom{4}{2}12\binom{4}{1}$ ways to draw a pair from three cards.

Question 8. I am stocking a fish tank. The fish shop has four types of sea creature: piranha, crocodile, tuna, catfish. Piranha's and crocodile's can share the tank with each other but not with tuna or catfish as they'll just eat these.

(a) How many combinations of two sea creatures can I place in my tank ?

(b) How many combinations of three sea creatures ?

Solutions

- Two sea creatures. From the piranha and crocs I can select 2×2 different combinations (I can have two piranha, two crocs or a croc and a piranha). Otherwise I can select two tuna, two catfish or a tuna plus catfish. So there are 6 possible combinations of sea creatures.

- Three sea creatures. From the piranha and crocs I can select 3 piranha, 3 crocs, 2 piranha and 1 croc or 1 piranha and 2 crocs. Otherwise I can select 3 tuna, 2 tuna and 1 catfish, 1 tuna and 2 catfish or 3 catfish. So 8 possible combinations.

Question 9. John, Jim, Jay, and Jack have formed a band consisting of 4 instruments.

(a) If each of the boys can play all 4 instruments, how many different arrangements are possible?

(b) What if John and Jim can play all 4 instruments, but Jay and Jack can each play only piano and drums (you can assume all four player have an instrument)?

Solutions

- 4! ways for four people to select four instruments

- Jay and Jack can select two instruments in 2 different ways. Of the remaining two instruments John and Jim can select these in 2 different ways. So there are $2 \times 2 = 4$ different arrangements.

Question 10. In how many ways can 3 novels, 2 mathematics books, and 1 chemistry book be arranged on a bookshelf if

(a) the books can be arranged in any order?

(b) the mathematics books must be together and the novels must be together?

(c) the novels must be together, but the other books can be arranged in any order?

Solutions

- 6!=720

- There are 2! ways to arrange the maths books and 3! ways to arrange the novels when they must be kept together. The novels, maths books and chemistry books can be arranged in 3! ways. So in total there are 3!2!3! = 72 arrangements.

- There are 3! ways to arrange the novels and 3! ways to arrange the rest. The novels and other books can be arranged in 2! ways (i.e. novels first then the others, the others then the novels) so there are 3!3!2! = 72 arrangements.