1) The four conditions of Arrow’s theorem are (U) unrestricted domain, (D) no dictatorship, (P) Pareto’s condition and (I) independence of irrelevant alternatives. Show, by example, that we can find a Social Welfare Function which satisfies any three of the four conditions. In each case provide a counterexample of the fourth condition.

2) Suppose that there are two individuals A and B and they are both facing a decision under uncertainty. There are two decisions \(d_1\) and \(d_2\) and two possible states \(\theta_1\) and \(\theta_2\). The reward if \(\theta_i\) is true and \(d_j\) is chosen is \(r_{ij}\). A and B each specify a utility \(u_A\) and \(u_B\) over the rewards and a probability \(p_A\) and \(p_B\) for \(\theta_1\). Each can hence maximise expected utility to decide which of the decisions they prefer. However, a choice must be made jointly.

Suppose they agree to average their utilities and probabilities and find their group decision by maximising expected utility for this preference and belief specification. Clearly this method for ranking decisions allows no dictators and has unrestricted domain. If we only have two decisions then independence of irrelevant alternatives is not an issue. However, does the method in this special case always obey the Pareto principle?

Either prove or give a counterexample to the proposition that is A and B each individually prefer decision \(d_1\) to \(d_2\), then the group decision method will also do so.