1)

a) State the minimax theorem for two-person zero-sum games. [5 Marks]

b) Odysseus and his men find themselves trapped inside the cave of the cyclops Polyphemus, who barbarously feasts on them for breakfast and supper. Odysseus considers fighting Polyphemus. Odysseus can either (1) throw his sword at Polyphemus' eye, or (2) take a defensive position enabling him to dodge Polyphemus' boulder and await his chance for the next play. Polyphemus can either (1) throw a boulder at Odysseus, or (2) walk up to Odysseus and attempt to smash his brain. Odysseus' utility is minus Polyphemus' utility. Odysseus' utility for each possible outcome of the game is:

\[
\begin{array}{c|cc}
P: & P1 & P2 \\
O: & O1 & O2 \\
& -2 & 1 \\
& 0 & -1
\end{array}
\]

Give one reason why it is advantageous for both Odysseus and Polyphemus to play a randomized strategy. [3 Marks]

c) Which randomized strategy should Odysseus play, and what is his expected utility for this strategy? [15 Marks]

d) Explain in one sentence how your solution of the game indicates as to whether Odysseus should fight Polyphemus, or not. [2 Marks]

2) The government is planning its budget for renewable energy for the next 10 years. They can either invest in infrastructure for existing technologies, or invest in research into new technologies. If they cannot decide then the budget will remain unspent. The economic status after 5 and 10 years for the two alternatives are (expressed in utility):

<table>
<thead>
<tr>
<th></th>
<th>No investment</th>
<th>Invest in Infrastructure</th>
<th>Invest in Research</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 years</td>
<td>0</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>10 years</td>
<td>-5</td>
<td>3</td>
<td>10</td>
</tr>
</tbody>
</table>

a) Explain why this problem can be considered to fit in the framework of bargaining theory. [2 Marks]
b) Find the Nash point and the equitable distribution point for this problem and explain the corresponding strategies for the government.  

[15 Marks]

c) Which of the Nash point axioms does the equitable distribution point fail to guarantee, and briefly discuss the interpretation of this axiom. 

[4 Marks]

d) Explain whether or not the Nash point found in part b) can be used to suggest a division of the total budget over infrastructure and research, if such a division would be possible. 

[4 Marks]

3) Placebo Pharmaceuticals is eager to expand its product portfolio with a drug against cardiovascular diseases. Three alternative strategies have been identified. The first is to hire a research team of 200 people to develop the new drug. However, to develop a new drug is expensive (50, using ‘million dollars’ as monetary units throughout) and it is far from certain that the team will manage to successfully develop a drug that meets regulatory requirements enforced by the Food and Drug Administration (FDA). The probability of such success is estimated to be 0.1.

The second alternative is to acquire a small company, Cholesterol Business Inc., that has already developed a drug that is currently undergoing clinical trials. If the trials are successful the FDA will license the product, leading to probability 0.8 of successful development of the drug. The costs to take over Cholesterol Business Inc. are 120.

The third alternative is to buy a license for 170 from a rival company to produce and market an already existing drug. This option has no uncertainties involved.

It is also possible (at cost 0) to decide not to expand the portfolio with such a drug.

The utility of a profit \( L \geq 0 \) is \( u(L) = L \), so monetary value, but for a loss (‘negative profit \( L \leq 0 \)) the utility is \( u(L) = 2L \). Revenue will depend on future demand for the new drug, which will be high with probability 0.3, leading to revenue 200, or low otherwise, leading to revenue 100.

a) Draw a decision tree to represent this decision problem and derive the optimal decision. Advise the company what to do. 

[20 Marks]

b) Suppose it is possible to perform extensive market research in order to learn, with certainty, the future demand for the new drug. What is the maximum amount you would recommend the company pay for this market research? 

[5 Marks]