Section 2: Eigendecomposition.

Ensure you have completed the previous worksheet before advancing to this one.

1 Selecting Elements of Matrices


Read this data frame into R, giving it the name music in the process (see previous worksheet for details).

The file music will have been read into R as a ‘data frame’, but we generally will wish to work with matrices. To convert the data frame into a matrix enter the following:

> music=as.matrix(music)

Now to select any element of the matrix music we enter into R a command of the form music[i,j], where i is the desired row number and j is the desired column number. For example, enter the following:

> music[1,1]

This finds the element in the first row and column of music and you should have been returned with something of the form [1] "Dancing Queen".

To select an entire row we enter a command of the type music[i,] (with i the desired row), and similarly, to select an entire column we enter a command of the type music[,j] (with j the desired column). For example, try entering the following commands:

> music[1,]

> music[,2]

2 Matrix Eigendecomposition

Eigendecomposition of matrices in R is simple when using the inbuilt eigen function. Have a look at the help file for this command.
**Task:** Create a matrix $A$ such that:

$$ A = \begin{pmatrix} 1 & 2 \\ 2 & 5 \end{pmatrix} $$

> 

Now enter the following command:

> res=eigen(A)

> res

The above assigns the result of the eigendecomposition of $A$ to the name `res`. Typing `res` then calls the result up. Note that it would have been fine to simply enter `eigen(A)` to see the result of the function, but assigning it with a name makes it easier to use in the future. If the code has been correctly entered you should have been returned with the following:

$\text{values}$

[1] 5.8284271 0.1715729

$\text{vectors}$

[,1]     [,2]
[1,] 0.3826834 -0.9238795
[2,] 0.9238795  0.3826834

The numbers following the term `values` are the eigenvalues of the matrix $A$ (given in decreasing order), whilst the columns of the matrix returned after the term `vectors` are the corresponding orthonormal eigenvectors. That is to say the output informs us that the matrix $A$ has one eigenvalue of 5.83 that corresponds to orthonormal eigenvector $(0.38, 0.92)$, and another smaller eigenvalue of 0.17 that corresponds to orthonormal eigenvector $(-0.92, 0.38)$.

To request the eigenvalues and eigenvectors of the matrix $A$ directly, now enter the following:

> res$values

> res$vectors

**Task:** Write an R code that directly returns only the first eigenvalue of $A$:

> 

**Task:** Write an R code that directly returns only the second eigenvector of $A$:

>
3 Matrix Multiplication

In order to double check that the results are the appropriate eigenvalues and eigenvectors of the matrix $A$ we need to know how matrix multiplication works in $\mathbb{R}$. Remembering that the $\times$ character is used for ordinary multiplication in $\mathbb{R}$, the sequence $\%\times\%$ replaces this for when multiplying matrices.

Enter the following:

```r
> A%*%res$vector[,1]
```

Now check the results agree with the following:

```r
> res$value[1]*res$vector[,1]
```

**Task:** Check that the second eigenvalue and eigenvector are indeed correct:

```r
>
> 

Sometimes when multiplying a vector by a matrix we need to take the transpose of one or more of the terms in order to ensure that the matrix or vector dimensions permit multiplication. In $\mathbb{R}$ this is performed with the `t` function. Enter the following:

```r
> a=c(1,2)
> a
```

This creates a generic vector in $\mathbb{R}$ that will automatically be considered as either a row vector or column vector as appropriate:

```r
> a%*%A
> A%*%a
```

Now enter the following:

```r
> a=matrix(c(1,2),nrow=1,ncol=2)
> a
```

This has told $\mathbb{R}$ to specifically treat the vector $a$ as a row vector, hence only $a%*%A$ will now work. Now try:

```r
> t(a)
> A%*%t(a)
```
You will see the transpose function $t$ changes the row vector $a$ into the column vector $t(a)$.

**Task:** The cranial length and cranial breadth of 35 female frogs have expected value $\mu = (23, 24)^T$ and covariance matrix:

\[
\begin{pmatrix}
17.7 & 20.3 \\
20.3 & 24.4
\end{pmatrix}
\]

What are the eigenvalues and eigenvectors of the covariance matrix?

> 

> 

> 

> 

4 **Plots**

As a final task today we will examine how to create some simple plots in R. Try and understand how the following code works (remember to use help files if you are uncertain):

> x=sample(c(-1000:1000), size=200, replace=FALSE)

> x=sort(x)

> y=x^2+3*x+1

> plot(x,y, type="l", col="red")

**Task:** Use R to produce a plot of the curve $y = x^3 + x^2 - 2x + 4$.

> 

> 

> 

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