



## **Prospect Theory**

Khurshid Ahmad,  
Chair of Computer Science  
Trinity College, Dublin, IRELAND  
13-15<sup>th</sup> Feb 2013

# Utility Theory and After

To a person considered by himself, the value of a pleasure or pain considered by itself, will be greater or less, according to the four following circumstances: 1. Its *intensity*, 2. Its *duration*; 3. Its certainty or uncertainty; 4. Its propinquity or remoteness. (IV, 4, italics added).



**Jeremy Bentham**  
(1748-1832)



**Stuart Mill**  
(1806-1876)



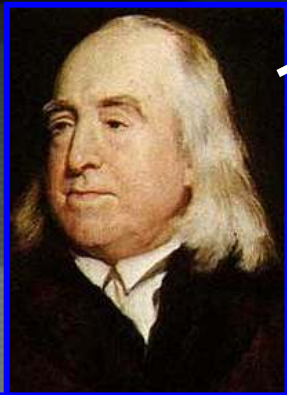
**Maynard Keynes**  
(1883-1946)



**Daniel Kahneman**  
(1934-)

# Utility Theory and After

To a person considered by himself, the value of a pleasure or pain considered by itself, will be greater or less, according to the four following circumstances: 1. Its *intensity*, 2. Its *duration*; 3. Its certainty or uncertainty; 4. Its propinquity or remoteness. (IV, 4, italics added).



**Jeremy Bentham**  
(1748-1832)



**Stuart Mill**  
(1806-1876)



**Maynard Keynes**  
(1883-1946)



**Daniel Kahneman**  
(1934-)

# Utility Theory and After

There are three main puzzles associated with aggregate stock market behavior:

- (i) the equity premium puzzle;
- (ii) the volatility puzzle; and
- (iii) the predictability puzzle.

**Barberis, Nicholas., Huang Ming., and Santos, Tano. (2001). Prospect Theory and Asset Prices. *The Quarterly Journal of Economics*. Vol CXVI (Iss. 1) (available at <http://web.cenet.org.cn/upfile/881.pdf>)**



# Prospect Theory

In 1979, Kahnemann and Tversky presented their 'critique of *expected utility theory* as a descriptive model of decision making under risk' and put forward their own model - *prospect theory*.

It appears that when faced with risky prospects, people typically made choices that are not consistent with the expected utility theory:

1. People underweight outcomes that are merely probable in comparison with outcomes that are obtained with certainty. This tendency, called the certainty effect, contributes to risk aversion in choices involving sure gains and to risk seeking in choices involving sure losses.

2. People generally discard components that are shared by all prospects under consideration. This tendency, called the isolation effect, leads to inconsistent preferences when the same choice is presented in different forms.

**Daniel Kahneman & Amos Tversky. (1979) Prospect Theory: An Analysis of Decision under Risk *Econometrica*, Vol. 47 (No. 2) (Mar., 1979), pp. 263-292**

# Prospect Theory

**In Kahnemann and Tversky *prospect theory*, 'value is assigned to gains and losses rather than to final assets and in which probabilities are replaced by decision weights'.**

**The value function is normally concave for gains, commonly convex for losses, and is generally steeper for losses than for gains.**

**Decision weights are generally lower than the corresponding probabilities, except in the range of low probabilities.**

**Overweighting of low probabilities may contribute to the attractiveness of both insurance and gambling.**

**Daniel Kahneman & Amos Tversky. (1979) Prospect Theory: An Analysis of Decision under Risk  
*Econometrica*, Vol. 47 (No. 2) (Mar., 1979), pp. 263-292**

# Prospect Theory

Kahnemann:

Our normative treatment of the utility of temporally extended outcomes adopts a hedonic interpretation of utility, but no endorsement of Bentham's view of pleasure and pain as sovereign masters of human action is intended. Our analysis applies to situations in which a separate value judgment designates experienced utility a criterion for evaluating outcomes.'

# Notes on Prospect Theory: Two Systems View

	PERCEPTION	INTUITION SYSTEM 1	REASONING SYSTEM 2
PROCESS	Fast Parallel Automatic Effortless Associative Slow-learning		Slow Serial Controlled Effortful Rule-governed Flexible
CONTENT	Percepts Current stimulation Stimulus-bound	Conceptual representations Past, Present and Future Can be evoked by language	



# Notes on Prospect Theory: Framing

## Framing of Outcomes:

‘Risky prospects are characterized by their possible outcomes and by the probabilities of these outcomes. The same option can be framed in different ways. For example, the possible outcome of a gamble can be framed either as gains or losses relative to the status quo or as asset positions that incorporate initial wealth or as asset positions that incorporate initial wealth. **Invariance requires that such changes in the description outcomes should not alter the preference order**’ (Kahneman and Tversky 2000:4) (Emphasis added)

# Notes on Prospect Theory: Framing

## Framing of Outcomes:

Imagine that the U.S. is preparing for the outbreak of an unusual Asian disease, which is expected to kill 600 people. Two alternative programs to combat the disease have been proposed. Assume that the exact scientific estimates of the consequences are as follows:

**If Program A is adopted, 200 people will be saved.**

**If Program B is adopted, there is 1/3 probability that 600 people will be saved and 2/3 probability that no people will be saved.**

**Which of the two programs would you favor?**

# Notes on Prospect Theory: Framing

## Framing of Outcomes:

Imagine that the U.S. is preparing for the outbreak of an unusual Asian disease, which is expected to kill 600 people. Two alternative programs to combat the disease have been proposed. Assume that the exact scientific estimates of the consequences are as follows:

**In Kahneman and Tversky (2000:5) the results were**

Program	Your response
A: 200 people will be saved	
B: 1/3 chance 600 be saved; 2/3 chance all die	

# Notes on Prospect Theory: Framing

## Framing of Outcomes:

Imagine that the U.S. is preparing for the outbreak of an unusual Asian disease, which is expected to kill 600 people. Two alternative programs to combat the disease have been proposed. Assume that the exact scientific estimates of the consequences are as follows:

**In Kahneman and Tversky (2000:5) the results were**

Program	Poll Results
A: 200 people will be saved	72%
B: 1/3 chance 600 be saved; 2/3 chance all die	28%

# Notes on Prospect Theory: Framing

## Framing of Outcomes:

Imagine that the U.S. is preparing for the outbreak of an unusual Asian disease, which is expected to kill 600 people. Two alternative programs to combat the disease have been proposed. Assume that the exact scientific estimates of the consequences are as follows:

**In Kahneman and Tversky (2000:5) the results were**

Program	Your response
C: 400 people will die	
D: 1/3 chance that nobody will die; 2/3 chance all 600 will die	



# Notes on Prospect Theory: Framing

## Framing of Outcomes:

Imagine that the U.S. is preparing for the outbreak of an unusual Asian disease, which is expected to kill 600 people. Two alternative programs to combat the disease have been proposed. Assume that the exact scientific estimates of the consequences are as follows:

**In Kahneman and Tversky (2000:5) the results were**

Program	Poll Results
C: 400 people will die	22%
D: 1/3 chance that nobody will die; 2/3 chance all 600 will die	78%

# Notes on Prospect Theory: Framing

## Framing of Outcomes:

Failure of invariance against changes in description. Spot the difference between A & C and B & D. The failures are common to expert and naïve; failures persist when A,B, C and D are asked within minutes of each other

Program	Poll Results
A: 200 people will be saved	72%
C: 400 people will die	22%
B: 1/3 chance 600 be saved; 2/3 chance all die	28%
D: 1/3 chance that nobody will die; 2/3 chance all 600 will die	78%

# Notes on Prospect Theory: Framing

## Framing of Outcomes:

Failure of invariance against changes in description.

<b>A: 200 people will be saved</b>	<b>Risk Averse</b>	<b>72%</b>
<b>C: 400 people will die</b>		<b>22%</b>
<b>B: 1/3 chance 600 be saved; 2/3 chance all die</b>		<b>28%</b>
<b>D: 1/3 chance that nobody will die; 2/3 chance all 600 will die</b>	<b>Risk seeking</b>	<b>78%</b>

# Notes on Prospect Theory: Framing

## Framing of Outcomes:

**Framing effects resemble perceptual illusions more than computational errors. Violation of the dominance requirement of rational choice**

<b>Gamble</b>	<b>Your response</b>
<b>A: 25% chance to win \$240; 75% chance to lose \$760</b>	
<b>B: 25% chance to win \$250; 75% chance to lose \$750</b>	

# Notes on Prospect Theory: Framing

## Framing of Outcomes:

**Framing effects resemble perceptual illusions more than computational errors. Violation of the dominance requirement of rational choice**

<b>Gambling Decision</b>	<b>Poll Results</b>
<b>E: 25% chance to win \$240; 75% chance to lose \$760</b>	<b>0%</b>
<b>F: 25% chance to win \$250; 75% chance to lose \$750</b>	<b>100%</b>



# Notes on Prospect Theory: Framing

## Framing of Outcomes:

Framing effects resemble perceptual illusions more than computational errors. The so-called dominance requirement of rational choice is obeyed in certain cases. For example:

Gamble	Poll Results
A: 25% chance to win \$240; 75% chance to lose \$760	0%
B: 25% chance to win \$250; 75% chance to lose \$750	100%

# Notes on Prospect Theory: Framing

## Framing of Outcomes:

But there are many instance where dominance choice principle is violated

Gambling Decisions	Your response
<b>Decision 1: Choose between</b>	
A: 100% chance or a sure gain of \$240	
B: 25% chance of gaining \$1000; 75% chance to gain nothing	
<b>Decision 2: Choose between</b>	
C: 100% chance or a sure loss of \$750	
D: 75% chance of losing \$1000; 25% chance of losing nothing	

# Notes on Prospect Theory: Framing

## Framing of Outcomes:

But there are many instance where dominance choice principle is violated

Gambling Decisions	Poll Results
<b>Decision 1: Choose between</b>	
A: A sure gain of \$240	84%
B: 25% chance of gaining \$1000; 75% chance to gain nothing	16%
<b>Decision 2: Choose between</b>	
C: A surety of a \$750 loss	13%
D: 75% chance of losing \$1000; 25% chance of losing nothing	87%

# Notes on Prospect Theory: Framing

## Framing of Outcomes:

But there are many instance where dominance choice principle is violated

Gambling Decisions	Poll Results
<b>RISK AVERSE Decision 1: Choose between</b>	
A: A sure gain of \$240	84%
B: 25% chance of gaining \$1000; 75% chance to gain nothing	16%
<b>RISK SEEKING Decision 2: Choose between</b>	
C: A surety of a \$750 loss	13%
D: 75% chance of losing \$1000; 25% chance of losing nothing	87%

# Notes on Prospect Theory: Framing

## Framing of Outcomes:

But there are many instance where dominance choice principle is violated

		s h a r e s		s t o c k s			
		A	B	C	D	E	F
Disease	You						
	K&T	72%	28%	22%	72%		
Gambling 1	You						
	K&T					0%	100%
Gambling 2	You						
	K&T	84%	16%	13%	87%		



# Notes on Prospect Theory: Framing

## Framing of Outcomes:

But there are many instance where dominance choice principle is violated. Decisions A&D dominate over B&C. Now, if you take options A&D together means a 75% chance of a loss of \$760 (\$1000-\$240) and 25% (A: 100%- D 75%) chance of wining \$240. This combination (A&D) was approved by 84% of respondents but rejected by all when the same decision was framed in Decision E!!!!

Gambling Decisions	Poll Results
<b>RISK AVERSE Decision 1: Choose between</b>	
A: A sure gain of \$240	84%
B: 25% chance of gaining \$1000; 75% chance to gain nothing	16%
<b>RISK SEEKING Decision 2: Choose between</b>	
C: A surety of a \$750 loss	13%
D: 75% chance of losing \$1000; 25% chance of losing nothing	87%

Gambling Decision	Poll Results
E: 25% chance to win \$240; 75% chance to lose \$760	0%
F: 25% chance to win \$250; 75% chance to lose \$750	100%

# Notes on Prospect Theory: Framing

## Framing of Outcomes:

But there are many instance where dominance choice principle is violated. Decisions A&D dominate over B&C. Now, if you take options B&C together suggests that a 25% chance to win \$250 and a 75% chance to lose \$750. The combination B&C has an approval of 13%. BUT, but, a differently framed Decision (F) had an approval rating of 100% !!!!

Gambling Decisions	Poll Results
<b>RISK AVERSE Decision 1: Choose between</b>	
A: A sure gain of \$240	84%
B: 25% chance of gaining \$1000; 75% chance to gain nothing	16%
<b>RISK SEEKING Decision 2: Choose between</b>	
C: A surety of a \$750 loss	13%
D: 75% chance of losing \$1000; 25% chance of losing nothing	87%

Gambling Decision	Poll Results
E: 25% chance to win \$240; 75% chance to lose \$760	0%
F: 25% chance to win \$250; 75% chance to lose \$750	100%

# Notes on Prospect Theory: Framing

## Framing of Outcomes:

One might argue that frame invariance should be preserved as invariance is 'normatively essential, intuitively compelling, and psychologically unfeasible' (Kahneman and Tversky 2000:6). One way of doing it will be to look at each decision in terms of total assets rather than in terms of gains or losses – but this is usually not possible except in ruinous circumstances.

Gambling Decisions	Poll Results
<b>RISK AVERSE Decision 1: Choose between</b>	
A: A sure gain of \$240	84%
B: 25% chance of gaining \$1000; 75% chance to gain nothing	16%
<b>RISK SEEKING Decision 2: Choose between</b>	
C: A surety of a \$750 loss	13%
D: 75% chance of losing \$1000; 25% chance of losing nothing	87%

Gambling Decision	Poll Results
E: 25% chance to win \$240; 75% chance to lose \$760	0%
F: 25% chance to win \$250; 75% chance to lose \$750	100%

# Notes on Prospect Theory: Framing

## Framing of Outcomes:

Equally, it is not possible to compound all the outcomes of concurrent decisions (A&B and C&D) because this will be beyond the limits of intuitive computation. And the case of pandemics and other major catastrophies, one is being asked to 'aggregate' overall mortality, mortality due to diseases, or number of fatalities associated with the particular disease in consideration.

Program	Poll Results
A: 200 people will be saved	72%
C: 400 people will die	22%
B: 1/3 chance 600 be saved; 2/3 chance all die	28%
D: 1/3 chance that nobody will die; 2/3 chance all 600 will die	78%

# The Value Function

Kahneman and Tversky's *prospect theory* helps in computing utility over gains and losses – *returns*- rather than levels of wealth. The value function in prospect theory is defined as:

$$v(x) = \begin{cases} x^\alpha & \text{if } x \geq 0 \\ -\lambda(-x)^\beta & \text{if } x < 0 \end{cases}$$

Kahneman and Tversky have estimated  $\alpha$ ,  $\beta$  and  $\lambda$ , the loss aversion coefficient, as

$$\begin{aligned} \alpha &\approx 0.88; \\ \beta &\approx 0.88; \\ \lambda &\approx 2.25 \end{aligned}$$

Benartzi, Shlomo & Thaler, Richard H. (2000). Myopic Loss Aversion and Equity Premium Puzzle. In (Eds.) Daniel Kahneman & Amos Tversky. *Choices, Values and Frames*. New York: Cambridge Univ. Press. pp 301-316

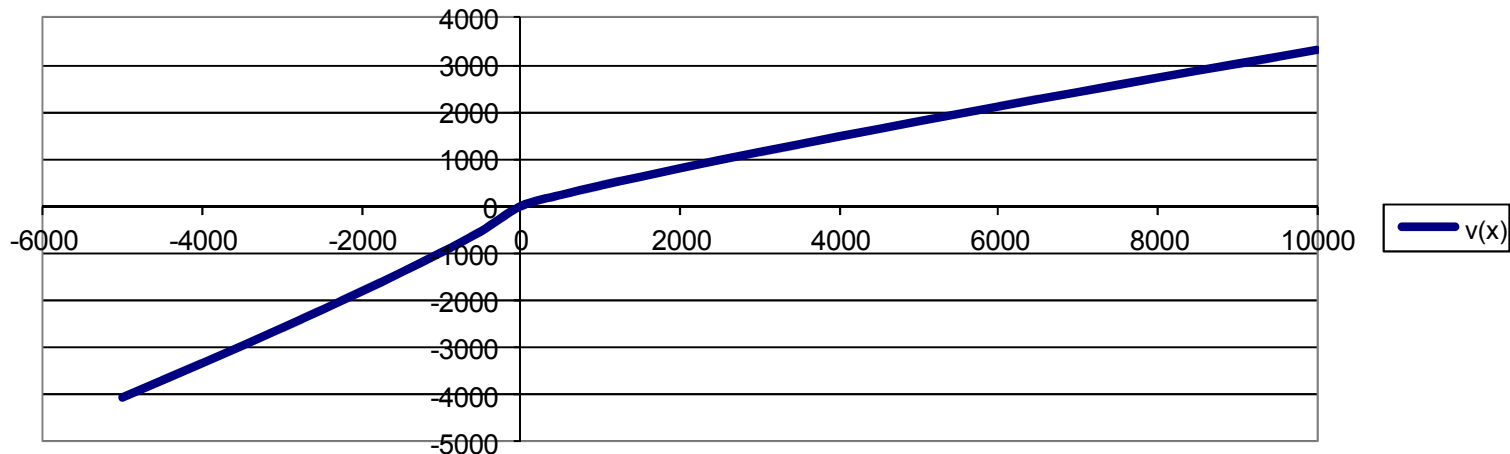


# The Value Function

FINANCIALS

**A Value Function**

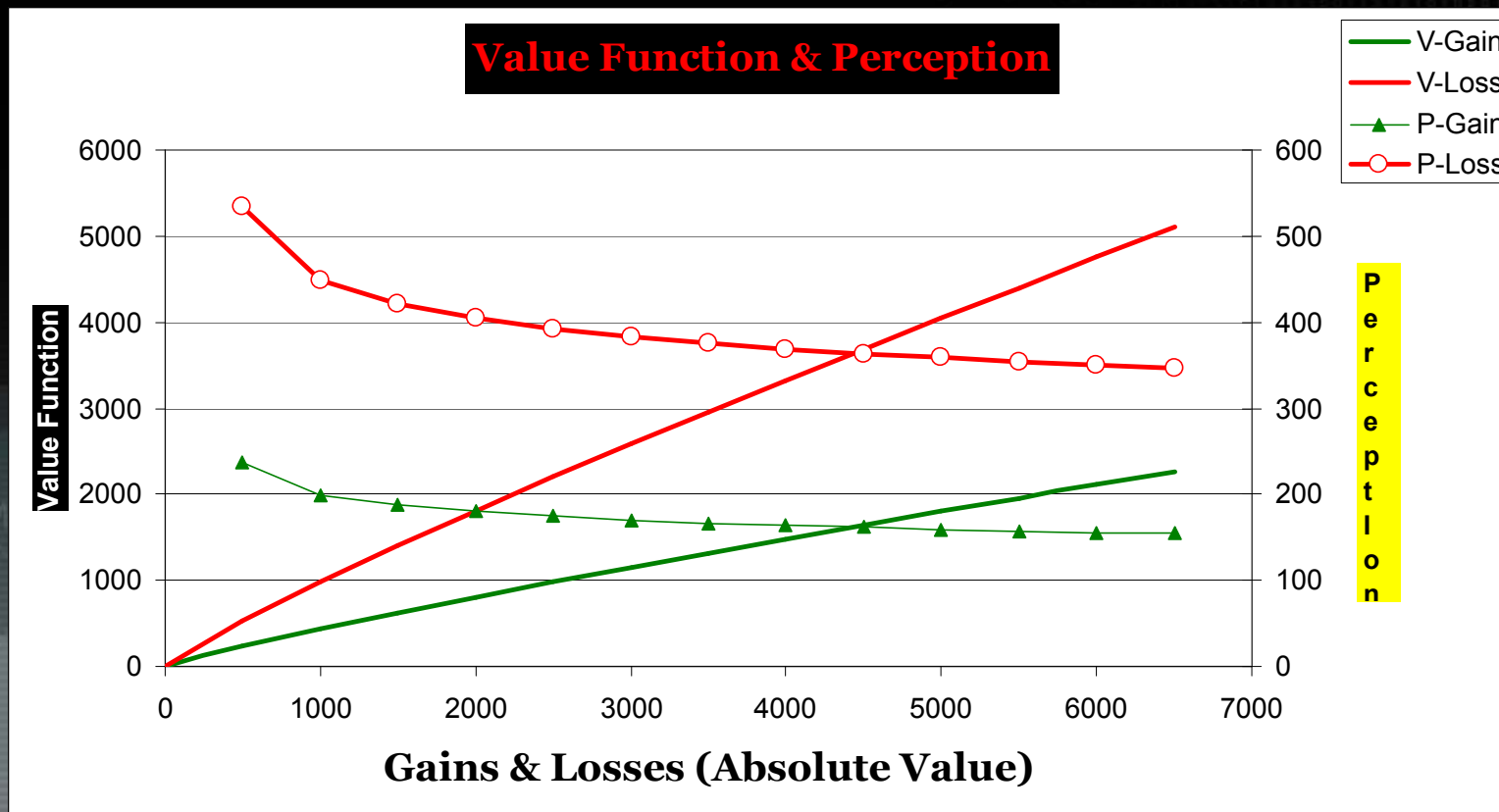
**Risky Value Choice**



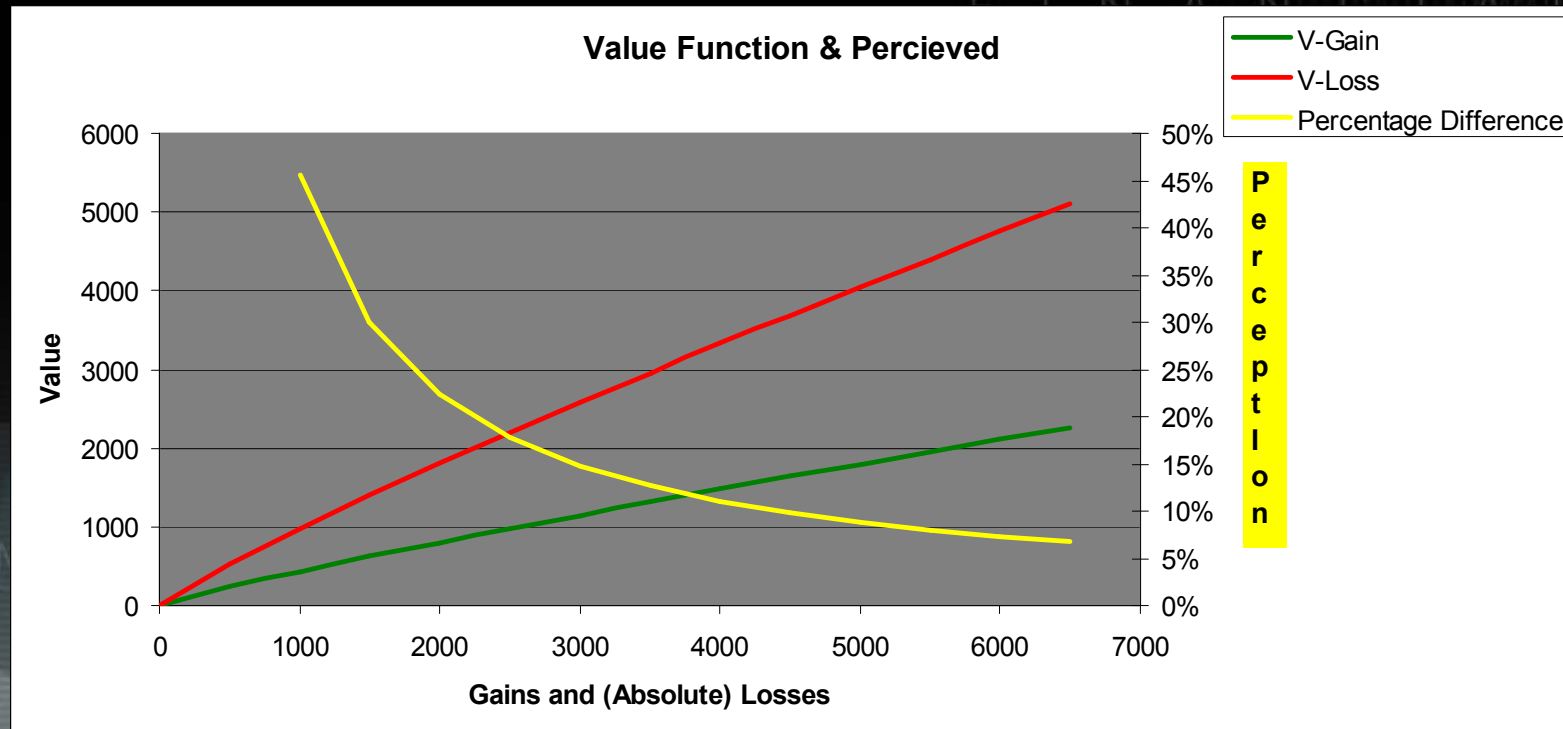
**Gains(+)** & **Losses (-)**

# The Value Function

FINANCIALS



# The Value Function



# Value Function and Mental Accounting

Principles of Hedonic Framing: alternative frames of mental accounts lead to different prospective value.

<b>Observation</b>	<b>Action</b>
Gain function is concave	<b>Segregate gains</b>
Loss Function is convex	<b>Integrate Losses</b>
Offset Loss Aversion	<b>Integrate smaller losses with larger gains</b>
Gain function is steepest at the origin	<b>Segregate small gains → silver linings</b>

Thaler, Richard (2000). 'Mental Accounting Matters. In (eds.) Daniel Kahneman and Amos Trevisky., pp 241-268

# Prospect Theory: Mental Accounting

## What will you do to save €5?

Your local 'superstore', Loc-Money sells calculators and DVD players. A well known brand of calculator will cost €15 and an equally well known DVD player is selling for €125. There is a chatty salesperson who just lets you know that there are two new specialist stores opened – Zippity specializing in calculators (and computers) and Seeall specialising in DVDs. Both are discounting heavily to get more custom and are giving €5 euros less than Loc-Money, but are 20 minutes walk from Loc-Money.

**Calculator: Will you go to Zippity to save €5 on the purchase of the calculator?**

**DVD: Will you go to Seeall to save €5 on the purchase of the DVD?**

# Prospect Theory: Mental Accounting

What will you do to save €5?

How can we explain the illusions we experience in mental accounting?

**Calculator:** Will you go to Zippity to save €5 on the purchase of the calculator?

**DVD:** Will you go to Seeall to save €5 on the purchase of the DVD?



# Prospect Theory: Mental Accounting

FINANCIALS

		Stay at Loc-Money	Go to Zippitty
Calculator	YOU!!		
	K&T	32%	68.00%
		Stay at Loc-Money	Go to Seeall
DVD	YOU!!		
	K&T	71%	29%

# Prospect Theory: Mental Accounting

		Stay at Loc-Money	Go to Zippitty
<b>Calculator</b>	<b>YOU!!</b>		
	<b>K&amp;T</b>	<b>32%</b>	<b>68.00%</b>
		Stay at Loc-Money	Go to Seeall
<b>DVD</b>	<b>YOU!!</b>		
	<b>K&amp;T</b>	<b>71%</b>	<b>29%</b>

The DVD and calculator example suggests that mental accounting is 'piece meal and topical': 'the utility of the saving must be associated with the differences in values rather than the value of the difference' (Thaler 2000:245).

# **Prospect Theory: Mental Accounting**

**Shafir, Diamond and Tversky (2000)**  
discuss the tendency to think in terms  
of nominal rather than real  
monetary values. “A nickel aint worth a  
dime anymore.”

# Prospect Theory: Mental Accounting

Suppose you bought a case of good 1982 Bordeaux in the futures market for \$20 a bottle. The wine now sells at auction for about \$75 a bottle. You decided to drink a bottle of this wine with your dinner. Which of the following best captures your feeling of the cost to you of drinking this bottle? (Thaler 2000:252)

Proposition	Your Choice
\$0	25%
\$20	18%
\$20 plus interest	6%
\$75	12.5%
-\$55	37.5
<b>Total</b>	

Thaler, Richard (2000). 'Mental Accounting Matters. In (eds.) Daniel Kahneman and Amos Trevisky., pp 241-268

# Prospect Theory: Mental Accounting

Suppose you bought a case of good 1982 Bordeaux in the futures market for \$20 a bottle. The wine now sells at auction for about \$75 a bottle. You decided to drink a bottle of this wine dinner. Which of the following best captures your feeling of the cost to you of drinking this bottle?

Proposition	Poll Result
\$0	30
\$20	18
\$20 plus interest	7
\$75	20
-\$55	25
Total	100

# Prospect Theory: Mental Accounting

Suppose you bought a case of good 1982 Bordeaux in the futures market for \$20 a bottle. The wine now sells at auction for about \$75 a bottle. You decided to drink a bottle of this wine **over** dinner. Which of the following best captures your feeling of the cost to you of drinking this bottle?

1. No loss or gain (\$0)
2. I lost \$20
3. I lost \$20 plus interest
4. I lost \$75
5. I gained \$55 (\$75-\$20)



# Prospect Theory: Mental Accounting

Two competing bookstores have in stock an identical leather bound edition of a leather bound of Oscar Wilde's collected writing. Store A bought for \$ 20 each. Tom of Store **A** sold 100 copies for \$44 a copy. Store **B** bought the same book a year later, and with 10% inflation paid \$22 per book and sold a 100 copies for \$45. Which store made a better deal selling the books?

1. Store A	
2. Store B	

# Prospect Theory: Mental Accounting

Two competing bookstores have in stock an identical leather bound edition of a leather bound of Oscar Wilde's collected writing. Store A bought for \$ 20 each. Tom of Store **A** sold 100 copies for \$44 a copy. Store **B** bought the same book a year later, and with 10% inflation paid \$22 per book and sold a 100 copies for \$45. Which store made a better deal selling the books?

1. Store A	87%
2. Store B	13%

# **Prospect Theory: Mental Accounting**

**Accounting: The action or process of reckoning, counting, or computing; numeration, computation. Now esp. the management of financial affairs, e.g. those of a business enterprise (OED).**

**Accounting is governed by the laws of the Land, conventions, and precedence.**

# Prospect Theory: Mental Accounting

Accounting: System of recording and summarizing business and financial transactions in books, and analyzing, verifying, and reporting results.

Mental Accounting: The set of cognitive operations used by individuals and households to organize, evaluate, and keep track of financial activities → psychology of choice.

Thaler, Richard (2000). 'Mental Accounting Matters. In (eds.) Daniel Kahneman and Amos Trevisky., pp 241-268

# Prospect Theory: Mental Accounting

**Mental Accounting:** The set of cognitive operations used by individuals and households to organize, evaluate, and keep track of financial activities.

Three 'components' of mental accounting:

1. How outcomes are perceived and experienced & how decisions are made and subsequently evaluated;
2. Assignment of activities to specific accounts;
3. Frequency by which accounts are evaluated and read.

**Thaler, Richard (2000). 'Mental Accounting Matters. In (eds.) Daniel Kahneman and Amos Tversky., pp 241-268**

# Prospect Theory: Mental Accounting

**Mental Accounting:** The set of cognitive operations used by individuals and households to organize, evaluate, and keep track of financial activities.

Accounting decisions as to which category to assign a purchase, whether to combine with others in that category, and how often to 'balance' the [mental account] 'books' can affect the perceived attractiveness of choices.

Thaler, Richard (2000). 'Mental Accounting Matters. In (eds.) Daniel Kahneman and Amos Trevisky., pp 241-268



# Prospect Theory: Mental Accounting

**OED: When a thing which is the subject of an obligation..must be delivered in specie [In kind; in respect of kind; specifically], the thing is not fungible, i.e. that very thing, and not another thing of the same or another class in lieu of it must be delivered. Where the subject of the obligation is a thing of a given class, the thing is said to be fungible, i.e. the delivery of any object which answers to the generic description will satisfy the terms of the obligation.**

Thaler: Mental accounting violates the economic notion of fungibility. Money in one mental account is not a perfect substitute for money in another account. Because of violations of fungibility, mental accounting matters.

Thaler, Richard (2000). 'Mental Accounting Matters. In (eds.) Daniel Kahneman and Amos Trevisky., pp 241-268

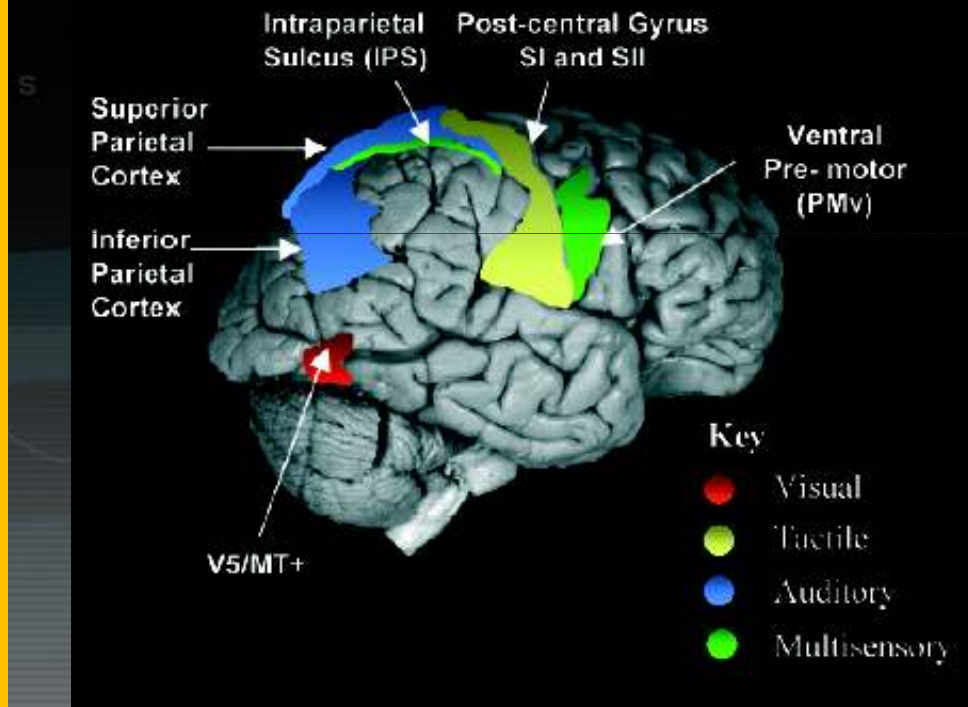
## Numerosity: *distance* and *magnitude* effect phenomena → Psychophysical 'Laws'

- The greater occurrence of errors found when comparing numbers that are close together in magnitude as opposed to further apart is known as the distance effect.
- The magnitude effect is the drop in performance observed when comparing numbers that are equal in distance, but have larger magnitudes.
- Fechner's law states that the perceived intensity of a number stimulus is proportional to the logarithm of the actual intensity → hence the internal representation of number is *compressed* at higher magnitudes.

# Neural Correlates of Behaviour: Modality and Neuronal Correlation

## Neural underpinnings of Multisensory **Motion Integration**:

‘In addition to [...] modality-specific motion-processing areas, there are a number of brain areas that appear to be responsive to motion signals in more than one sensory modality [....] the **IPS**, [...] **precentral gyrus** can be activated by auditory, visual or tactile motion signals’



Soto-Faraco, S. *et al* (2004). 'Moving Multisensory Research Along: Motion Perception Across Sensory Modalities'. *Current Directions in Psy. Sci.* Vol 13(1), pp 29-32

## **Learning to Compute: Cross-Modal Interaction and Spatial Attention**

**The key to spatial attention is that different stimuli, visual and auditory, help to identify the spatial location of the object generating the stimuli.**

**One argument is that there may be a neuronal correlate of such crossmodal interaction between two stimuli.**

**Information related to the location of the stimulus (*where*) and identifying the stimulus (*what*) appears to have correlates at the neuronal level in the so-called *dorsal* and *ventral* streams in the brain.**

# Learning to Compute: Cross Modal Interaction and 'Numerical' Neurons

*Numerosity, numerons, single neuron arithmetic and number sense in humans and some primates.*

- Observations on enumeration without having been taught a number system, *subitisation* or visual enumeration, or approximate calculation without rigorously carrying out arithmetic procedures, lead to the speculation that there may be areas in the brain where the visuo-spatial information about the objects, for instance, is processed such that the number information is preserved

## **Learning to Compute: Numerosity, Number Sense and 'Numerons'**

- Number sense has played a major role in psychology where many earlier studies were dedicated to 'the mathematical description of how a continuum of sensation, such as loudness or duration' is represented in the brain/mind.**
- The 19<sup>th</sup> century psychophysicist, Gustav Fechner, had observed that 'the intensity of subjective sensation increases as the logarithm of the stimulus intensity'.**
- One of the 21<sup>st</sup> century rendition of this 'law' is that the 'external stimulus is scaled into a logarithmic internal representation of sensation'**

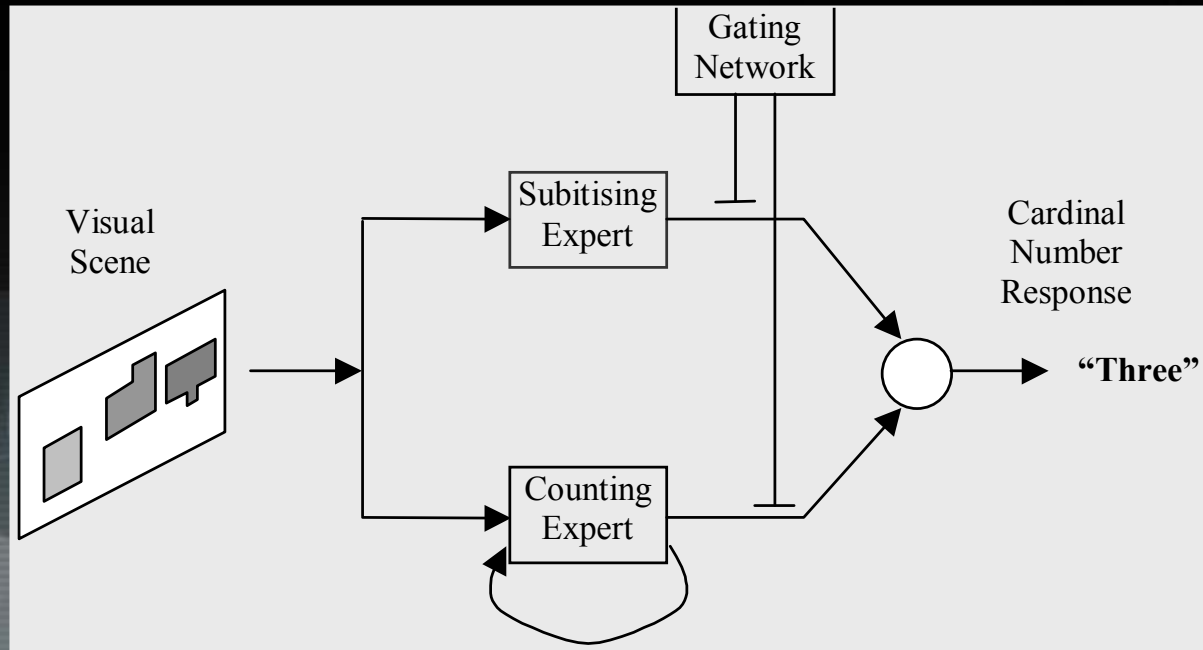


## **Learning to Compute: Numerosity, Number Sense and 'Numerons'**

- Number related behaviours 'depend on the capacity to abstract information from sensory inputs and to retain it in memory' and that in monkeys this capacity is in the 'prefrontal cortex' and there are reports of activation in humans in proximate regions of the brain.**
- As predicted by Fechner, there is a compressed scaling of numerical information, and this information is stored in the prefrontal cortex of the monkey and the parietal cortex of the human.**

# Learning to Compute: Numerosity, Number Sense and 'Numerons'

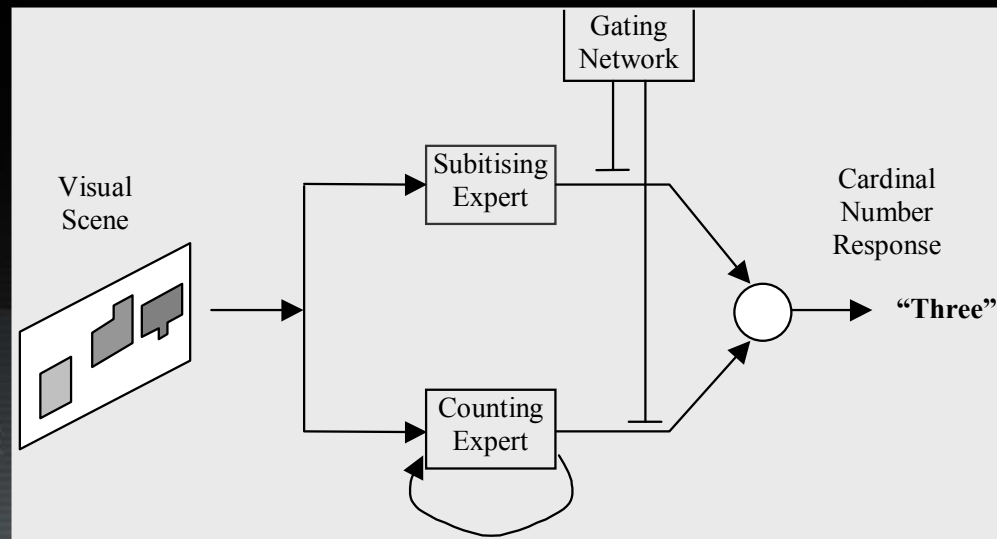
**A model of networks and routes to counting and subitisation in our brain**



**The gating network is a time-keeper: if one is in a hurry, then the subitisation network is opened but if we have time then the gating network activates the counting network.**

# Learning to Compute: Numerosity, Number Sense and 'Numerons'

**A model of networks and routes to counting and subitisation in our brain**



**Brain imaging studies tell us that when people subitise the brain images show a high activation in the brain areas believed to be involved in visual processing and spatial awareness areas of the brain. Imaging studies and simulations of counting show that verbal processing and temporal sequencing areas play a major role here.**

## **Learning to Compute: Numerosity, Number Sense and 'Numerons'**

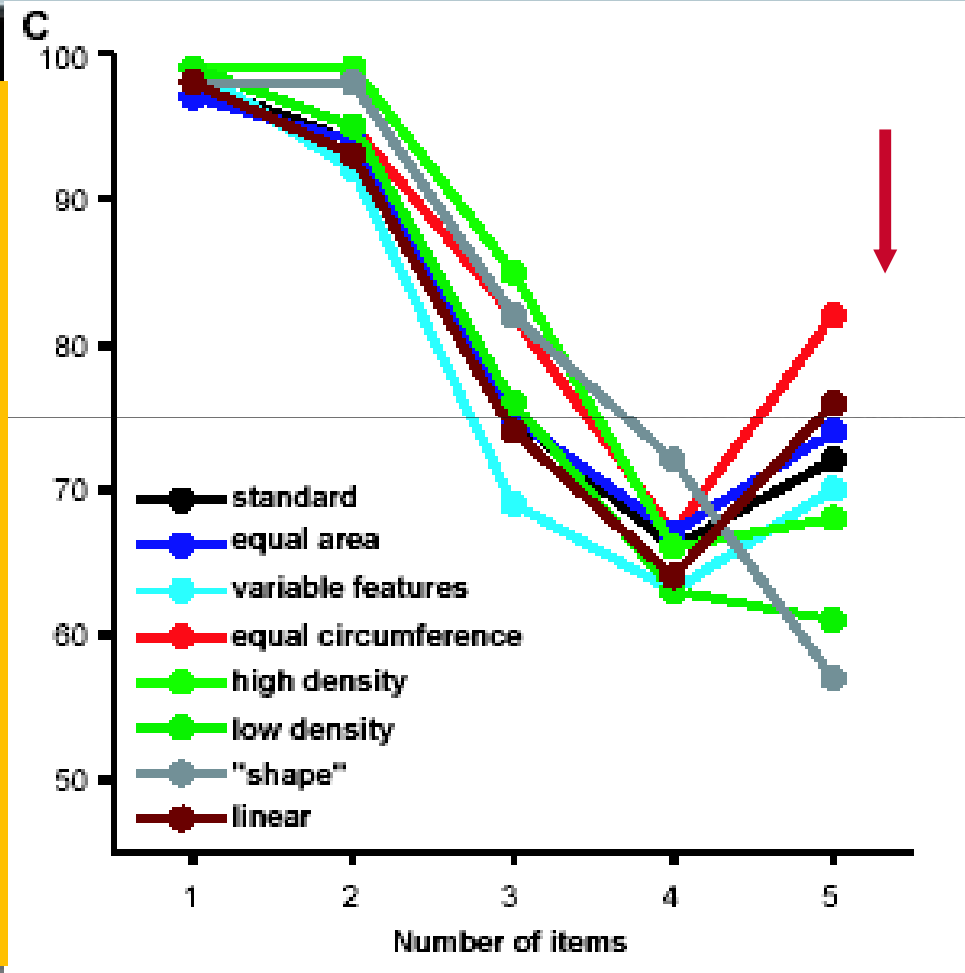
**Nieder et al report over a third of the 352 randomly selected neurons from the lateral prefrontal cortex of two monkeys 'showed activity that varied significantly with the number of items in the sample display': this suggests that certain neurons specialise as 'number detectors' – the illusive numerons perhaps have been found.**

**The two key areas involved are the principal sulcus and arcuate sulcus<sup>1</sup>.**

<sup>1</sup>Nieder, A., & Miller, E. K. (2003). 'Coding of Cognitive Magnitude: Compressed Scaling of Numerical Information in Primate Prefrontal Cortex'. *Neuron* Vol 37, pp 149-157.

# Learning to Compute: Numerosity, Number Sense and 'Numerons'

'Monkeys watched two displays (first sample, then test) separated by a 1-s delay. [the displays varied in shape, size, texture and so on.] They were trained to release a lever if the displays contained the same number of items. Average performance of both monkeys was significantly better than chance for all tested quantities, with a decline when tested for higher quantities similar to that seen in humans performing comparable tasks.



Andreas Nieder, David J. Freedman, Earl K. Miller (2002). 'Representation of the Quantity of Visual Items in the Primate Prefrontal Cortex'. *Science* Vol. 297, pp 1709-11.

## Learning to Compute: Numerosity, Number Sense and 'Numerons'

- The compressed number line theory can be used to explain the observation that neonates and monkeys, and adults in a hurry, can accurately enumerate quantities less than 5 without recourse to overt counting.
- Higher numbers cannot be enumerated with any accuracy through visual enumeration or *subitisation* and that within the numbers 1-5, there is a diminution in accuracy as we approach the higher number.
- **Subitisation is sometimes related to the existence of 'preverbal numerical abilities'**

# Learning to Compute: Numerosity, Number Sense and 'Numerons'

- **Vernon Smith (2008)** has looked at the phenomenon of the wisdom of the crowds both in terms of the success stories associated with this phenomenon and with the disasters as well.
- He cites the success stories where the crowds can correctly tell the weight of cattle in a fairground, or groups of individual making a correct guess of the number of small objects in a bottle. But he notes that while 'it is rare for any individual's estimate of a number to lie between the mean and the true number' and as the size increases the estimation error increases as well (2008:183-184). The errors of individuals are distributed independently. This leads Smith to argue that 'sample mean will approximate the true value as the sample size increases'
- **Subitisation is sometimes related to the existence of 'preverbal numerical abilities'**

**Smith, Vernon. (2008). *Rationality in Economics*. Cambridge: Cambridge University Press**



## Learning to Compute: Numerosity, Number Sense and 'Numerons'

- **Vernon Smith (2008) has looked at the phenomenon of the wisdom of the crowds both in terms of the success stories associated with this phenomenon and with the disasters as well.**
- **He cites the disaster stories as well: 'In asset markets, however, traders are observed to bubble and crash on the long path to a rational expectation outcome' (2008:182)**

• **Subitisation is sometimes related to the existence of 'preverbal numerical abilities'**

**Smith, Vernon. (2008). *Rationality in Economics*. Cambridge: Cambridge University Press**

# Processes in Prospect Theory

<http://io.uwinnipeg.ca/~epritch1/jdm99b.htm>

<b><i>Mental operation</i></b>	<b><i>That facilitates the Decision Makers to</i></b>
1. <i>coding</i>	(1.1) define an (arbitrary?) reference point (1.2) cast the options in terms of gains and losses
2. <i>combination</i>	aggregate the likelihood of events that present identical outcomes
3. <i>segregation</i>	(3.1) focus on the aspects they find most relevant to the problem, (3.2) omit others aspects
4. <i>cancellation</i>	ignore the dimensions in the evaluation of two alternatives that are identical
5. <i>simplification and dominance</i>	(5.1) round up probabilities (5.2) discard small probabilities and treat highly likely outcomes as certainties

# Processes in Prospect Theory

Prospect theory assumes the following:

**Discontinuity of weighting function**

**Subadditivity**

**Subcertainty**

**Regressiveness**

**Reference Point**

**Slope of value function** - Prospect theory assumes that losses have greater weight than gains, which explains why people tend to be risk seeking for losses but not for gains.

# Processes in Prospect Theory

**Translation** - People take the outcome of the available alternatives and translate them into subjective values, and similarly they translate the probabilities associated with those outcomes into decision weight ( $\tilde{\pi}$ )

**Combination** - After the translation, the values & weights are combined into prospect values.

**Decision** - The prospect values are then applied to make the appropriate decision - whether that be an evaluation or a choice.

<http://io.uwinnipeg.ca/~epritch1/jdm99b.htm>

# **What is new about Prospect Theory?**

**Psychologists look at individuals and work in behavioural economics focuses on ‘the contradictions between observed behaviour and the predictions of classical models of choice, bargaining and competition’; the behavioural psychologists ‘have moved the argument from an account of theoretical possibilities and anomalies to deeper empirical investigations’ (Smith 2008: 152).**

# What is new about Prospect Theory?

According to Vernon Smith, Kahnemann and Tversky's contribution is in their 'modifications in both utility and probability weighting functions of standard expected utility theory' (ibid; emphasis added). Their empirical tests verified (i) that it is changes in wealth relative to individual's current asset state is critical to our understanding; and (ii) people choose as if they are risk seeking in losses and risk averse in gains. These two propositions, according to Smith, can be traced back to Adam Smith and does not find Kahnemann and Tversky to be averse to the 'axioms of standard expected utility theory'.

# After Prospect Theory?

**You have just been through a lecture where we have looked at two different theories of decision making in economics and finance.**

**Almost all theories provide a partial explanation, description and sometimes prediction of a complex phenomenon. The explanation, description and prediction of human behaviour is not an easy task. So utility theory provides an understanding some economic behaviour and prospect theory of other economic behaviour.**

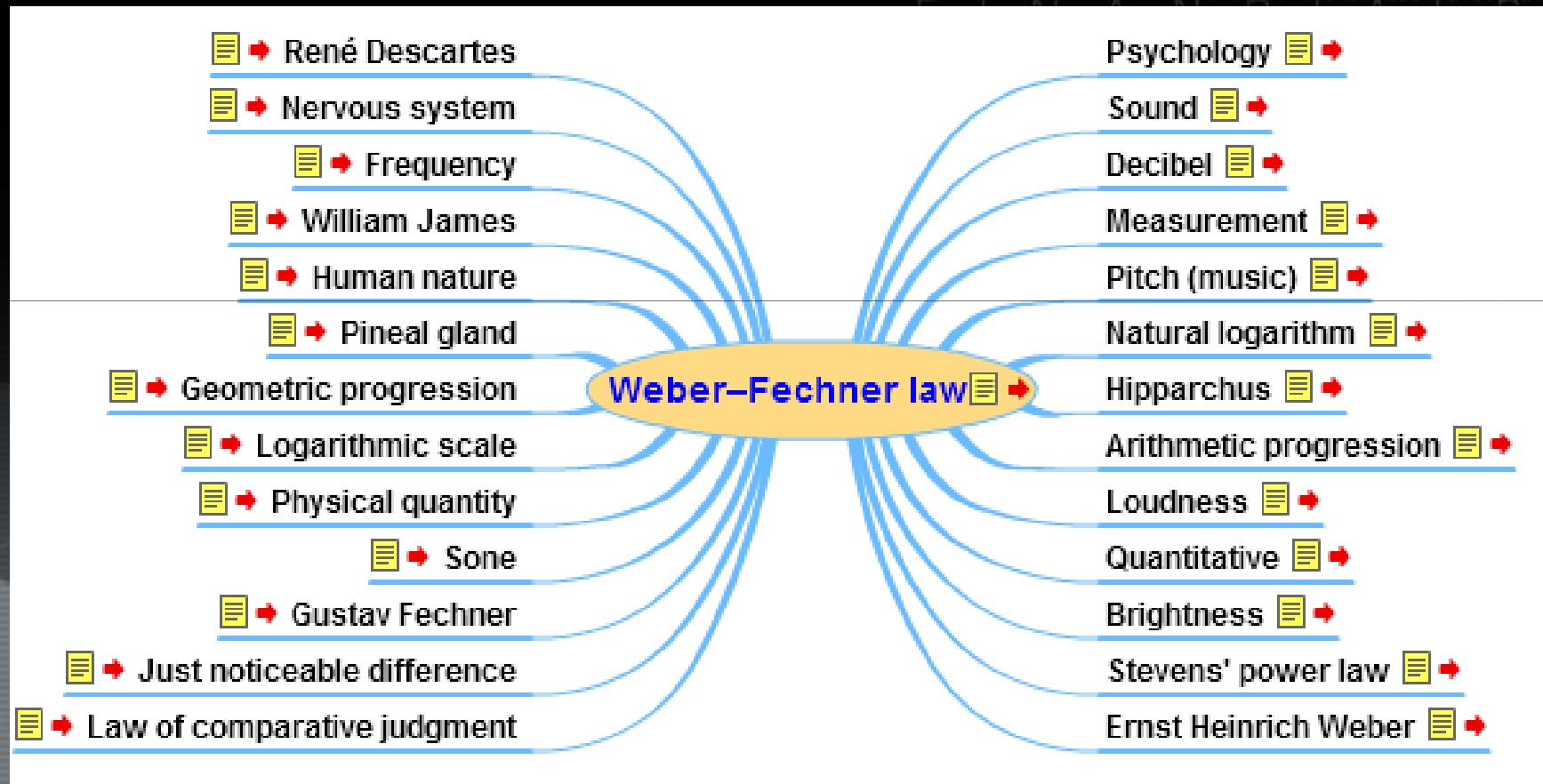
**Theories are constantly modified, updated, refined and occasionally discarded. Experimentation and theorising have a symbiotic relationship – one reinforces and eggs the other one on. Here is an example from the growth of nuclear physics**



# Processes in Prospect Theory

The behavioural outcome of an estimation task may simply be the result of multiple, diverse scaling schemes at different processing stages. In other words, at the behaviour level, it may look like magnitude estimations are following Weber-Fechner laws, but the underlying neural code could actually look quite different.

# Processes in Prospect Theory



# Neuroscience and Economics

**It can be hypothesized that different criteria are applied to select one or more features of each of the interacting modalities – sometimes the features can be aggregated to achieve super-addition, such that the whole is greater than the sum of the individual features, and at other times some features can be relegated in importance such that the whole is less than the sum leading to sub-addition. Yet, sometimes a simple addition of the modalities suffices. The well-known cocktail party effect relies on the super-addition of low-level linguistic information with the visual information of facial changes that enables listeners to ‘listen’ in noisy environments. The collapse of enterprises and markets on rumours, despite encouraging quantitative information about the performance of their assets, is the sub-addition of linguistic information with numerical..**

# Processes in Prospect Theory

**Multi-criteria decision making has a long history in social sciences and recently have been used in environmental sciences, images classification and financial forecasting. The different ways in which features are aggregated depends on context, data density and uncertainty and it appears that the importance of criteria is measured by means of a capacity. In effect, it has been found out the criteria can be aggregated by means of the so-called fuzzy integrals – for cardinal evaluations it is the Choquet integral appears to be the key and for ordinal evaluations it is the Sugeno integral.**