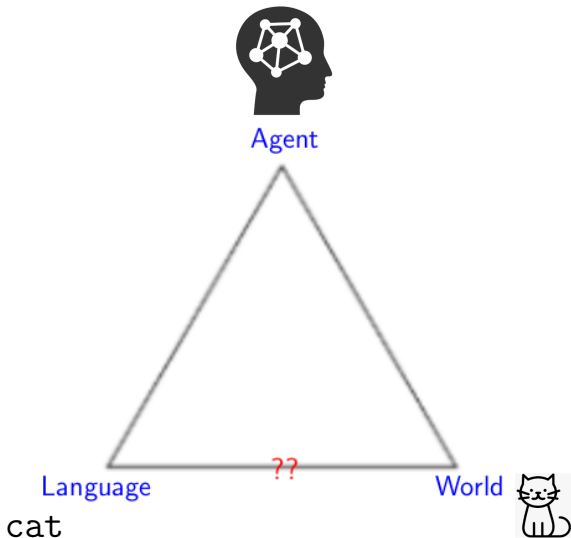


# Triadic Temporal Representations & Deformations

[Tim.Fernando@tcd.ie](mailto:Tim.Fernando@tcd.ie)

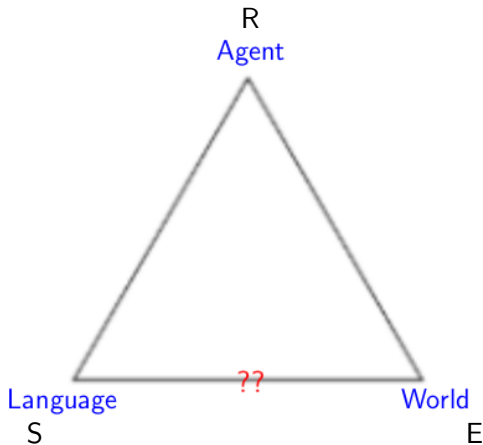
Nancy, 20 June 2023

# Triadic Temporal Representations & Deformations



# Triadic Temporal Representations & Deformations

REICHENBACH: tense (S-R) & aspect (R-E)



# Triadic Temporal Representations & Deformations

representations  $\rightsquigarrow$  patterns

*Pattern Theory*, formulated by Ulf Grenander, is a mathematical formalism to describe **knowledge of the world** as patterns.

- Wikipedia

# Triadic Temporal Representations & Deformations

representations  $\rightsquigarrow$  patterns

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*the pattern should not merely describe the 'pure' situation that underlies reality but the 'deformed' situation that is actually observed in which the pure pattern may be hard to recognize. This generalizes, for example, Chomsky's idea of the deep structure of an utterance vs. its surface structure, where **deep**  $\sim$  **pure** and **surface**  $\sim$  **deformed**.*

- Mumford 2019

# Triadic Temporal Representations & Deformations

**David Bryant Mumford** (born 11 June 1937) is an American mathematician known for his work in algebraic geometry and then for research into vision and pattern theory. He won the Fields Medal and was a MacArthur Fellow. In 2010 he was awarded the National Medal of Science.

**David Mumford**



David Mumford in 2010

pattern  $\approx$  pure situation + deformations  
e.g., output  $\approx$  input + noise (Shannon noisy channel)

(1) Facebook bought Instagram.

(2) Facebook owns Instagram.

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(5)  $\boxed{\neg \text{own}(x, y)} \xrightarrow{\text{buy}(x, y)} \boxed{\text{own}(x, y)}$  transition

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**Proposal:** extract **finite automata** from knowledge graphs

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**Proposal:** extract **finite automata** from knowledge graphs,  
allowing for **refinements** and **alternatives**

Deformations: **institution** as triad (Goguen)

# TALK OUTLINE

§1 Transitions from finite automata

§2 Strings as compressed models

§3 Granularity: sigs & reducts

§4 Deformations: institution as triad (Goguen)

# TALK OUTLINE

§1 Transitions from finite automata

§2 Strings as compressed models

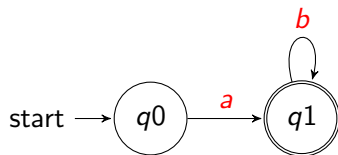
§3 Granularity: sigs & reducts

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# Labelled transition $q \xrightarrow{a} q'$

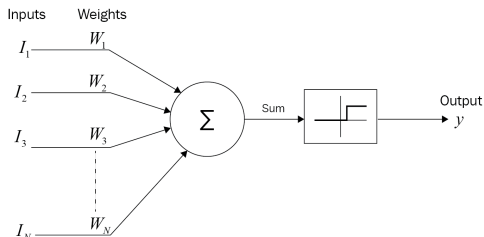
	automata	
$q$	state	
$a$	symbol	



$q_0 \xrightarrow{a} q_1 \in F$   
 $q_0 \xrightarrow{a} q_1 \xrightarrow{b} q_1 \in F$   
 $\vdots$   
regular expression  $ab^*$

# Labelled transition $q \xrightarrow{a} q'$

	automata	Kleene 1956 (nerve nets)
$q$	state	$(v_1, \dots, v_m)$
$a$	symbol	{active input cells}



$$I_1, I_2 \dots I_N = q, a$$

$(v_1 \dots v_m)$  records the values  $v_i$  of  $m$  inner cells

$$\text{output } q' = (y_1 \dots y_m)$$

## Labelled transition $q \xrightarrow{a} q'$

	automata	Kleene 1956 (nerve nets)	action languages
$q$	state	$(v_1, \dots, v_m)$	fluent, value
$a$	symbol	{active input cells}	(elementary) action

Gelfond & Lifschitz 1998 ... symbolic AI (J. McCarthy)

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Gelfond & Lifschitz 1998 ... symbolic AI (J. McCarthy)

- *fluent* (Newton), inertia (frame problem)

- *action signature* (**V**, **F**, **A**)

names **A** for actions + state information **V**, **F**

$$\boxed{\neg \text{own}(x, y)} \xrightarrow{\text{buy}(x, y)} \boxed{\text{own}(x, y)}$$

$$\boxed{(\text{own}(x, y), 0), \text{buy}(x, y)} \mid \boxed{(\text{own}(x, y), 1)}$$

# TALK OUTLINE

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## Strings in Reichenbach

Simple Past:  $E \approx R$     $R < S$

$$\boxed{E, R} \ \& \ \boxed{R | S} = \boxed{E, R | S}$$

## String sets in Reichenbach

Simple Past:  $E \approx R \quad R < S$

$$\boxed{E, R} \ \& \ \boxed{R | S} = \boxed{E, R | S}$$

Posterior Past:  $R < E \quad R < S$

$$\boxed{R | E} \ \& \ \boxed{R | S} = \boxed{R} \underbrace{(\boxed{E, S} + \boxed{E | S} + \boxed{S | E})}_{\text{trichotomy}}$$

## String sets in Reichenbach and Allen

Simple Past:  $E \approx R \quad R < S$

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$$\boxed{l | r} \ \& \ \boxed{l' | r'} = 13 \text{ Allen relations}$$



## String sets in Reichenbach and Allen

<i>X meets Y</i>	m	mi	XXXYYY
<i>X overlaps Y</i>	o	oi	XXX YYY
<i>X during Y</i>	d	di	XXX YYYYYY
<i>X starts Y</i>	s	si	XXX YYYYY

$\boxed{l} \boxed{r} \ \& \ \boxed{l'} \boxed{r'}$  = 13 Allen relations

$m(x, y) \ \& \ d(y, z) \rightsquigarrow \{s, o, d\}(x, z)$

$\boxed{x} \boxed{y} \ \& \ \boxed{z} \boxed{y, z} \boxed{z} = (\epsilon + \boxed{x} + \boxed{z}) \boxed{x, z} \boxed{y, z} \boxed{z}$

## X meets Y: compression

Stative      delete stutters      XXXYYY  $\rightsquigarrow$ 

x	y
---	---

X *meets* Y

## X meets Y: compression

Stative      delete stutters      XXXYYY  $\rightsquigarrow$ 

x	y
---	---

X meets Y



Transition      

l	r
---	---

## X meets Y: compression

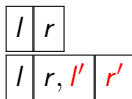
Stative      delete stutters      XXXYYY  $\rightsquigarrow$ 

x	y
---	---

X meets Y



Transition



## X meets Y: compression two ways

Stative      delete **stutters**      XXXYYY  $\rightsquigarrow$ 

x	y
---	---

X meets Y



Transition



delete   (S-words, Durand & Schwer 2008)

No change:       $q \xrightarrow{\square} q$

*no time without change* (Aristotle)

## String as model: *no time without change* (Aristotle)

$$\boxed{l \mid r, l' \mid r'} \text{ as } \langle \{1, 2, 3\}, \llbracket S \rrbracket, \llbracket P_l \rrbracket, \llbracket P_r \rrbracket, \llbracket P_{l'} \rrbracket, \llbracket P_{r'} \rrbracket \rangle$$
$$\llbracket S \rrbracket := \{(1, 2), (2, 3)\}$$

$$\llbracket P_l \rrbracket := \{1\}, \quad \llbracket P_r \rrbracket := \{2\}, \quad \llbracket P_{l'} \rrbracket := \{2\}, \quad \llbracket P_{r'} \rrbracket := \{3\}$$

$$ntwoc_{A,V} := \forall i \left( \bigvee_{a \in A} P_a(i) \vee \bigvee_{u \in \Sigma V} \delta_u(i) \right)$$

$$\delta_u(i) := P_u(i) \wedge \neg \exists j (iSj \wedge P_u(j))$$

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# J.A. Wheeler

**John Archibald Wheeler** (July 9, 1911 – April 13, 2008) was an American **theoretical physicist**. He was largely responsible for reviving interest in **general relativity** in the United States after **World War II**. Wheeler also worked with **Niels Bohr** in explaining the basic principles behind **nuclear fission**. Together with **Gregory Breit**, Wheeler developed the concept of the **Breit–Wheeler process**. He is best known for popularizing the term "**black hole**,"<sup>[1]</sup> as to objects with gravitational collapse already predicted during the early 20th century, for inventing the terms "**quantum foam**", "**neutron moderator**", "**wormhole**" and "**it from bit**", and for hypothesizing the "**one-electron universe**". **Stephen Hawking** referred to him as the "hero of the black hole story".<sup>[2]</sup>

**John Archibald Wheeler**



Wheeler before the Hermann Weyl-Conference 1985 in Kiel, Germany

## J.A. Wheeler: *it from bit*

*every **it** — every particle, every field of force, even the spacetime continuum itself — derives its function, its meaning, its very existence entirely — even if in some contexts indirectly — from the apparatus-elicited answers to yes-or-no questions, binary choices, **bits**.*

- Information, physics, quantum: the search for links, 1990

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## A-compression for $ntwoc_{A,V}$

**Theorem.** For all  $s \in \mathcal{B}_{A,V}^*$ ,

$$s \models ntwoc_{A,V} \iff s = \kappa_A(s)$$

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where

$$\kappa_A(s) := \begin{cases} \epsilon & \text{if } s = \epsilon \text{ or } s = \square \\ s & \text{else if } \text{length}(s) = 1 \end{cases}$$
$$\kappa_A(\alpha \alpha' s) := \begin{cases} \kappa_A(\alpha' s) & \text{if } \alpha = \square \text{ or } \alpha = \alpha' \setminus A \\ \alpha \kappa_A(\alpha' s) & \text{otherwise} \end{cases}$$

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$\kappa_A$  is computable by a finite-state transducer and for  $s \in \mathcal{B}_{A,V}^*$ ,

$$\kappa_A(s) = \begin{cases} d^\square(s) & \text{if } V = \emptyset \\ bc(s) & \text{else if } A = \emptyset \end{cases}$$

# TALK OUTLINE

§1 Transitions from finite automata

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§3 Granularity: sigs & reducts

§4 Deformations: institution as triad (Goguen)

## Finite precision: (Act, Val)-sigs

Given: a function Val from variables  $x$  to sets  $\text{Val}(x)$ , and  
a set Act of acts.

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$(\forall x \in dom(Val)) V(x)$  is a finite partition of Val( $x$ ).

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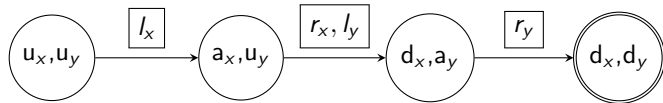
$$(A, V) \preceq (A', V') \iff A \subseteq A' \text{ and } V \leq V'$$

where  $\leq$  allows values (cells) to be refined

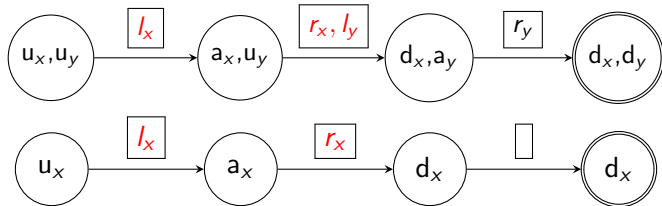
$$V \leq V' \iff (\forall x \in dom(V)) x \in dom(V') \text{ and } V'(x) \text{ refines } V(x)$$

$$\overbrace{(\forall c' \in V'(x))(\exists c \in V(x)) c' \subseteq c}$$

## X meets Y, revisited



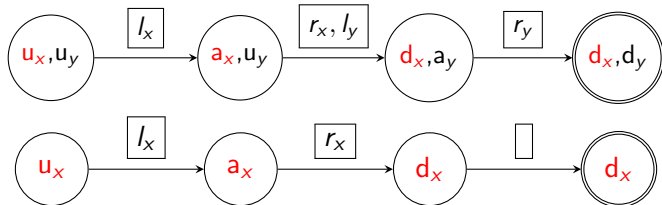
## X meets Y, revisited



$$B\text{-reduct} \quad \rho_B(\alpha_1 \cdots \alpha_n) := (\alpha_1 \cap B) \cdots (\alpha_n \cap B)$$

$$\rho_{\{l_x, r_x\}} \left( \boxed{l_x} \boxed{r_x, l_y} \boxed{r_y} \right) = \boxed{l_x} \boxed{r_x} \boxed{\phantom{r_y}}$$

## X meets Y, revisited



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$$\rho_{\{u_x, a_x, d_x\}} \left( \boxed{u_x, u_y} \boxed{a_x, u_y} \boxed{d_x, a_y} \boxed{d_x, d_y} \right) = \boxed{u_x} \boxed{a_x} \boxed{d_x} \boxed{d_x}$$

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§4 Deformations: institution as triad (Goguen)

# Institution (Goguen & Burstall)

finite automaton	deformation	institution
alphabet $(A, V)$	blur	$\Sigma \in \mathbf{Sig}$

**Joseph Amadee Goguen** (*[/ˈɡɒɡən/](#)* *GOH-gən*; June 28, 1941 – July 3, 2006) was an American [computer scientist](#). He was professor of Computer Science at the [University of California](#) and [University of Oxford](#), and held research positions at [IBM](#) and [SRI International](#).

In the 1960s, along with [Lotfi Zadeh](#), Goguen was one of the earliest researchers in fuzzy logic and made profound contributions to [fuzzy set theory](#).<sup>[1][2]</sup> In the 1970s Goguen's work was one of the earliest approaches to the algebraic characterisation of [abstract data types](#) and he originated and helped develop the [OBJ](#) family of [programming languages](#).<sup>[3][4]</sup> He was author of *A Categorical Manifesto* and founder<sup>[5]</sup> and Editor-in-Chief of the *Journal of Consciousness Studies*. His development of [institution theory](#) impacted the field of [universal logic](#).<sup>[6][7]</sup>

Joseph A. Goguen



Joseph Goguen in 2004



# Institution (Goguen & Burstall)

finite automaton	deformation	institution
alphabet $(A, V)$ string	blur domain warping	$\Sigma \in \mathbf{Sig}$ $\Sigma$ -model

Given  $\Sigma \xrightarrow{\sigma} \Sigma'$ ,  $s' \in \mathbf{Mod}(\Sigma')$ , ,

$\mathbf{Mod}(\sigma) : \mathbf{Mod}(\Sigma') \rightarrow \mathbf{Mod}(\Sigma)$

$\mathbf{Mod}(\sigma)(s') = \kappa_{\sigma}(s')$  reduct ; compression

# Institution (Goguen & Burstall)

finite automaton	deformation	institution
alphabet $(A, V)$	blur	$\Sigma \in \mathbf{Sig}$
string	domain warping	$\Sigma$ -model
regular expression	superposition	$\Sigma$ -sentence

Given  $\Sigma \xrightarrow{\sigma} \Sigma'$ ,  $s' \in \mathbf{Mod}(\Sigma')$ ,  $\varphi \in \mathbf{Sen}(\Sigma)$ ,

**Mod**( $\sigma$ ) : **Mod**( $\Sigma'$ )  $\rightarrow$  **Mod**( $\Sigma$ )

**Mod**( $\sigma$ )( $s'$ ) =  $\kappa_\sigma(s')$     reduct ; compression

contra **Sen**( $\sigma$ ) : **Sen**( $\Sigma$ )  $\rightarrow$  **Sen**( $\Sigma'$ )

**Sen**( $\sigma$ )( $\varphi$ ) =  $\langle \sigma \rangle \varphi$

$s' \models_{\Sigma'} \langle \sigma \rangle \varphi \iff \kappa_\sigma(s') \models_{\Sigma} \varphi$     satisfaction condition

# Institution (Goguen & Burstall)

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**superpose**( $\varphi_1, \varphi_2$ ) as  $\langle \sigma_1 \rangle \varphi_1 \wedge \langle \sigma_2 \rangle \varphi_2$

# Institution (Goguen & Burstall)

finite automaton	deformation	institution
alphabet $(A, V)$	blur	$\Sigma \in \mathbf{Sig}$
string	domain warping	$\Sigma$ -model
regular expression	superposition	$\Sigma$ -sentence
string set	<b>interruption</b>	$\llbracket \varphi \rrbracket_{\Sigma}$

Given  $\Sigma \xrightarrow{\sigma} \Sigma'$ ,  $s' \in \mathbf{Mod}(\Sigma')$ ,  $\varphi \in \mathbf{Sen}(\Sigma)$ ,

**Mod** $(\sigma) : \mathbf{Mod}(\Sigma') \rightarrow \mathbf{Mod}(\Sigma)$

**Mod** $(\sigma)(s') = \kappa_{\sigma}(s')$     reduct ; compression

contra **Sen** $(\sigma) : \mathbf{Sen}(\Sigma) \rightarrow \mathbf{Sen}(\Sigma')$

**Sen** $(\sigma)(\varphi) = \langle \sigma \rangle \varphi$

$s' \models_{\Sigma'} \langle \sigma \rangle \varphi \iff \kappa_{\sigma}(s') \models_{\Sigma} \varphi$     satisfaction condition

superpose $(\varphi_1, \varphi_2)$  as  $\langle \sigma_1 \rangle \varphi_1 \wedge \langle \sigma_2 \rangle \varphi_2$

$\llbracket \varphi \rrbracket_{\Sigma} := \{s \in \mathbf{Mod}(\Sigma) \mid s \models_{\Sigma} \varphi\}$

## Inertia & interruption

For  $a \in \text{Act}$ , let  $\text{af}(a)$  be the set of variables that  $a$  can affect.

An  $(A, V)$ -string  $s$  is  $(A, V, \text{af})$ -inertial if for every  $V$ -pair  $u$ , any  $u$ -change in  $s$  occurs with an act in  $A$  that can affect  $u$

$$\forall i \forall j \quad (iSj \wedge P_u(i) \wedge \neg P_u(j)) \supset \bigvee_{a \in A_u} P_a(i) \quad (\dagger)$$

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Otherwise,  $s$  is  $(A, V, \text{af})$ -*interrupted*.

The  $(A, V)$ -projection of an  $(A', V', \text{af})$ -inertial string can be  $(A, V, \text{af})$ -interrupted because  $(\dagger)$  needs an  $a \in A' \setminus A$ .

Expand  $V$  to  $V'$  for  $(\dagger)$ -converse on *event nuclei* (Moens & Steedman 1988)

<https://web.stanford.edu/~laurik/fsmbook/examples/YaleShooting.html> F & Nairn 2005, IWCS-6 Tilburg 2005

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	{	blur	$\Delta(\text{alphabet})$
<b>string</b>		domain warp	compression
		superposition	language
		interruption	constraints

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*T h a n k      Y o u*

